ESCOLA POLITÉCNICA DA UNIVERSIDADE DE SÃO PAULO

Análise Dinâmica: método de Gibbs-Appell

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Análise Dinâmica: método de Gibbs-Appell

Método de Newton-Euler:

- separação entre os elos da cadeia cinemática
- na formulação consideram-se as forças e momentos reativos
- no. de equações = λ.N

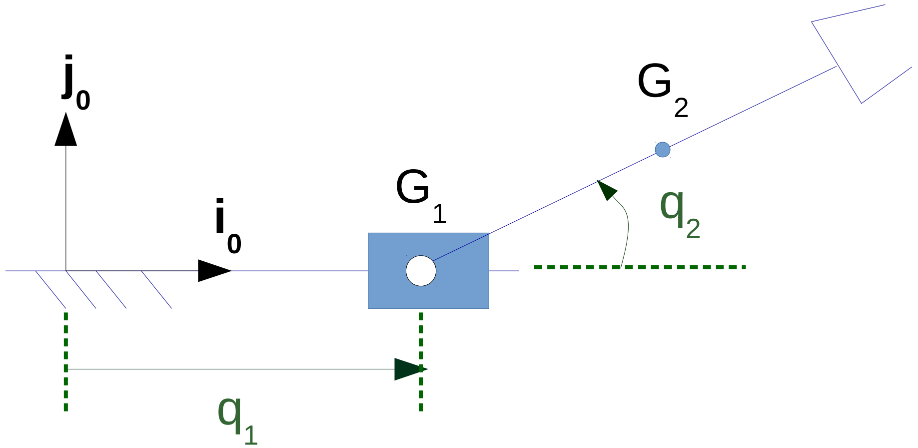
Método de Gibbs-Appel:

- não existe a necessidade da separação dos elos
- forças e momentos reativos não são considerados
- no. de equações = Mobilidade

Análise Dinâmica: método de Gibbs-Appell

$$\boldsymbol{e}_{nx1} = \boldsymbol{0}_{nx1}$$

no. de velocidades generalizadas
ou coordenadas generalizadas



n = 2

N = 2

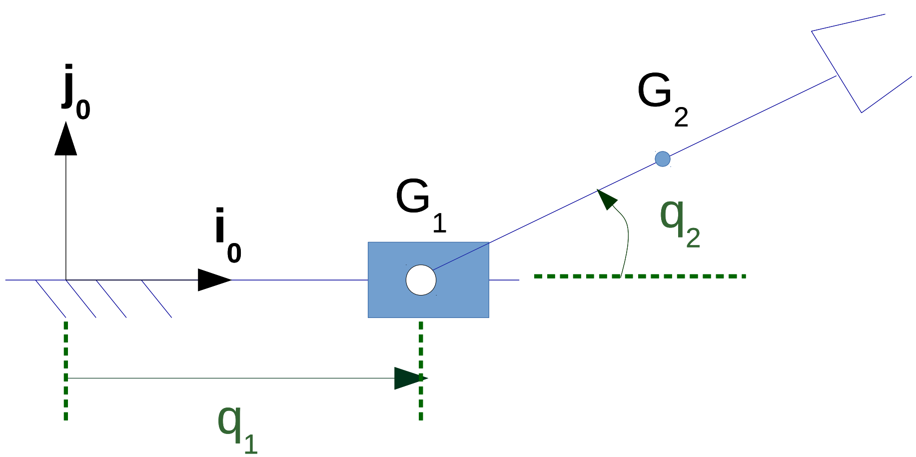
número de elos
móveis

Análise Dinâmica: método de Gibbs-Appell

$$\boldsymbol{e}_{2x1} = \mathbf{0}_{2x1}$$

$$e_i = \sum_{k=1}^{N} \left(\sum_{\square} \vec{F}_{at,k} - m\,\vec{a}_{G_k} \right) . \left(\frac{\partial \vec{V}_{G_K}}{\partial \dot{q}_i} \right) + \left[\sum_{\square} \vec{M}_{at,k} - \left(I_k \dot{\vec{\omega}}_k + \vec{\omega}_k \times \left(I\,\vec{\omega}_k \right) \right) \right] . \left(\frac{\partial \vec{\omega}_K}{\partial \dot{q}_i} \right)$$

$$1 \leqslant i \leqslant 2$$



n = 2

Análise Dinâmica: método de Gibbs-Appell

elo 1

$$e_i = \boxed{\left(\sum \vec{F}_{at,1} - m\vec{a}_{G_1}\right).\left(\frac{\partial \vec{V}_{G_1}}{\partial \dot{q}_i}\right) + \left[\sum \vec{M}_{at,1} - \left(I_1 \dot{\vec{\omega}}_1 + \vec{\omega}_1 \times \left(I\,\vec{\omega}_1\right)\right)\right].\left(\frac{\partial \vec{\omega}_1}{\partial \dot{q}_i}\right)} +$$

$$\left(\sum \vec{F}_{at,2} - m\vec{a}_{G_2}\right).\left(\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_i}\right) + \left[\sum \vec{M}_{at,2} - \left(I_2 \dot{\vec{\omega}}_2 + \vec{\omega}_2 \times \left(I\,\vec{\omega}_2\right)\right)\right].\left(\frac{\partial \vec{\omega}_2}{\partial \dot{q}_i}\right)$$

$$1 \leqslant i \leqslant 2 \quad \Rightarrow \quad \vec{e} = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

Análise Dinâmica: método de Gibbs-Appell

$$e_i = \left(\sum_{\square} \vec{F}_{at,1} - m\vec{a}_{G_1}\right) . \left(\frac{\partial \vec{V}_{G_1}}{\partial \dot{q}_i}\right) + \left[\sum_{\square} \vec{M}_{at,1} - \left(I_1 \dot{\vec{\omega}}_1 + \vec{\omega}_1 \times \left(I \vec{\omega}_1\right)\right)\right] . \left(\frac{\partial \vec{\omega}_1}{\partial \dot{q}_i}\right) \quad +$$

$$\boxed{\left(\sum_{\square} \vec{F}_{at,2} - m\vec{a}_{G_2}\right) . \left(\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_i}\right) + \left[\sum_{\square} \vec{M}_{at,2} - \left(I_2 \dot{\vec{\omega}}_2 + \vec{\omega}_2 \times \left(I \vec{\omega}_2\right)\right)\right] . \left(\frac{\partial \vec{\omega}_2}{\partial \dot{q}_i}\right)}$$

elo 2

$$1 \leqslant i \leqslant 2 \quad \Rightarrow \quad \vec{e} = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

Análise Dinâmica: método de Gibbs-Appell

$$e_1 = \left(\sum_{\square} \vec{F}_{at,1} - m\,\vec{a}_{G_1}\right).\left(\frac{\partial \vec{V}_{G_1}}{\partial \dot{q}_1}\right) + \left[\sum_{\square} \vec{M}_{at,1} - \left(I_1 \dot{\vec{\omega}}_1 + \vec{\omega}_1 \times \left(I\,\vec{\omega}_1\right)\right)\right].\left(\frac{\partial \vec{\omega}_1}{\partial \dot{q}_1}\right) +$$
$$\left(\sum_{\square} \vec{F}_{at,2} - m\,\vec{a}_{G_2}\right).\left(\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_1}\right) + \left[\sum_{\square} \vec{M}_{at,2} - \left(I_2 \dot{\vec{\omega}}_2 + \vec{\omega}_2 \times \left(I\,\vec{\omega}_2\right)\right)\right].\left(\frac{\partial \vec{\omega}_2}{\partial \dot{q}_1}\right)$$

Derivadas parciais em relação a $\dot{q}_1$

$$e_2 = \left(\sum_{\square} \vec{F}_{at,1} - m\,\vec{a}_{G_1}\right).\left(\frac{\partial \vec{V}_{G_1}}{\partial \dot{q}_2}\right) + \left[\sum_{\square} \vec{M}_{at,1} - \left(I_1 \dot{\vec{\omega}}_1 + \vec{\omega}_1 \times \left(I\,\vec{\omega}_1\right)\right)\right].\left(\frac{\partial \vec{\omega}_1}{\partial \dot{q}_2}\right) +$$
$$\left(\sum_{\square} \vec{F}_{at,2} - m\,\vec{a}_{G_2}\right).\left(\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_2}\right) + \left[\sum_{\square} \vec{M}_{at,2} - \left(I_2 \dot{\vec{\omega}}_2 + \vec{\omega}_2 \times \left(I\,\vec{\omega}_2\right)\right)\right].\left(\frac{\partial \vec{\omega}_2}{\partial \dot{q}_2}\right)$$

Análise Dinâmica: método de Gibbs-Appell

$$e_1 = \left(\sum_{\square} \vec{F}_{at,1} - m\vec{a}_{G_1}\right).\left(\frac{\partial \vec{V}_{G_1}}{\partial \dot{q}_1}\right) + \left[\sum_{\square} \vec{M}_{at,1} - \left(I_1 \dot{\vec{\omega}}_1 + \vec{\omega}_1 \times \left(I\vec{\omega}_1\right)\right)\right].\left(\frac{\partial \vec{\omega}_1}{\partial \dot{q}_1}\right) +$$

$$\left(\sum_{\square} \vec{F}_{at,2} - m\vec{a}_{G_2}\right).\left(\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_1}\right) + \left[\sum_{\square} \vec{M}_{at,2} - \left(I_2 \dot{\vec{\omega}}_2 + \vec{\omega}_2 \times \left(I\vec{\omega}_2\right)\right)\right].\left(\frac{\partial \vec{\omega}_2}{\partial \dot{q}_1}\right)$$

Derivadas parciais em relação a $\dot{q}_2$

$$\boxed{\begin{aligned} e_2 = &\left(\sum_{\square} \vec{F}_{at,1} - m\vec{a}_{G_1}\right).\left(\frac{\partial \vec{V}_{G_1}}{\partial \dot{q}_2}\right) + \left[\sum_{\square} \vec{M}_{at,1} - \left(I_1 \dot{\vec{\omega}}_1 + \vec{\omega}_1 \times \left(I\vec{\omega}_1\right)\right)\right].\left(\frac{\partial \vec{\omega}_1}{\partial \dot{q}_2}\right) + \\ &\left(\sum_{\square} \vec{F}_{at,2} - m\vec{a}_{G_2}\right).\left(\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_2}\right) + \left[\sum_{\square} \vec{M}_{at,2} - \left(I_2 \dot{\vec{\omega}}_2 + \vec{\omega}_2 \times \left(I\vec{\omega}_2\right)\right)\right].\left(\frac{\partial \vec{\omega}_2}{\partial \dot{q}_2}\right)\end{aligned}}$$

Análise Dinâmica: método de Gibbs-Appell

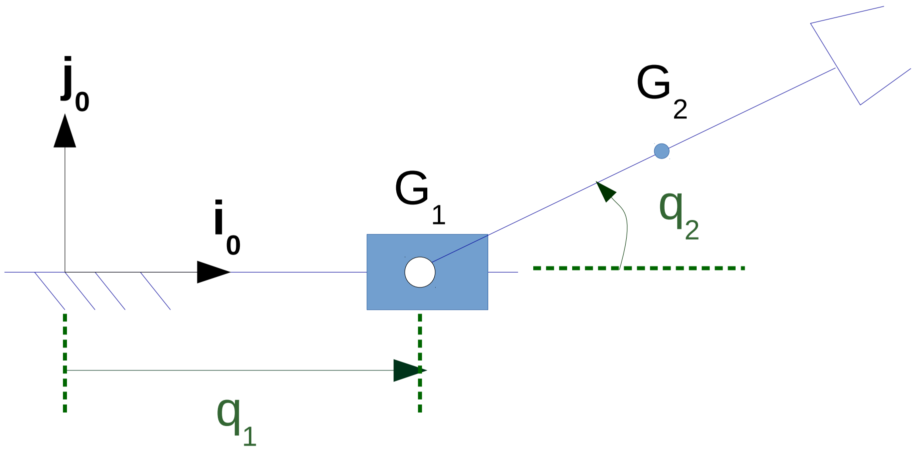
$$e_1 = \left(\sum_{\square} \vec{F}_{at,1} - m\vec{a}_{G_1}\right) . \left(\frac{\partial \vec{V}_{G_1}}{\partial \dot{q}_1}\right) + \left[\sum_{\square} \vec{M}_{at,1} - \left(I_1 \dot{\vec{\omega}}_1 + \vec{\omega}_1 \times (I \vec{\omega}_1)\right)\right] . \left(\frac{\partial \vec{\omega}_1}{\partial \dot{q}_1}\right) +$$

$$\left(\sum_{\square} \vec{F}_{at,2} - m\vec{a}_{G_2}\right) . \left(\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_1}\right) + \left[\sum_{\square} \vec{M}_{at,2} - \left(I_2 \dot{\vec{\omega}}_2 + \vec{\omega}_2 \times (I \vec{\omega}_2)\right)\right] . \left(\frac{\partial \vec{\omega}_2}{\partial \dot{q}_1}\right)$$

Derivadas parciais em relação a $\dot{q}_1$

$$\vec{V}_{G_1} = \dot{q}_1 \boldsymbol{i_0}$$

$$\frac{\partial \vec{V}_{G_1}}{\partial \dot{q}_1} = \boldsymbol{i_0}$$



Análise Dinâmica: método de Gibbs-Appell

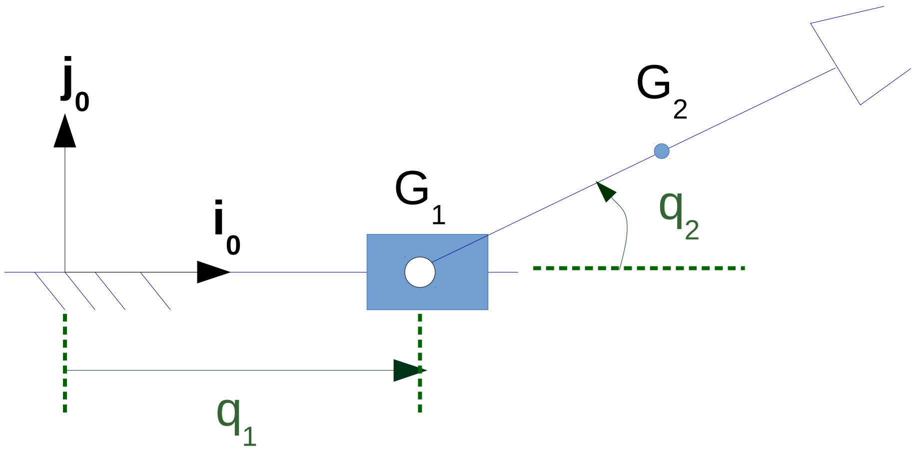
$$e_1 = \left(\sum_{\square} \vec{F}_{at,1} - m\vec{a}_{G_1}\right).\left(\frac{\partial \vec{V}_{G_1}}{\partial \dot{q}_1}\right) + \left[\sum_{\square} \vec{M}_{at,1} - \left(I_1 \dot{\vec{\omega}}_1 + \vec{\omega}_1 \times (I \vec{\omega}_1)\right)\right].\left(\frac{\partial \vec{\omega}_1}{\partial \dot{q}_1}\right) +$$

$$\left(\sum_{\square} \vec{F}_{at,2} - m\vec{a}_{G_2}\right).\left(\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_1}\right) + \left[\sum_{\square} \vec{M}_{at,2} - \left(I_2 \dot{\vec{\omega}}_2 + \vec{\omega}_2 \times (I \vec{\omega}_2)\right)\right].\left(\frac{\partial \vec{\omega}_2}{\partial \dot{q}_1}\right)$$

Derivadas parciais em relação a $\dot{q}_1$

$$\vec{\omega}_1 = \mathbf{0}$$

$$\frac{\partial \vec{\omega}_1}{\partial \dot{q}_1} = \mathbf{0}$$

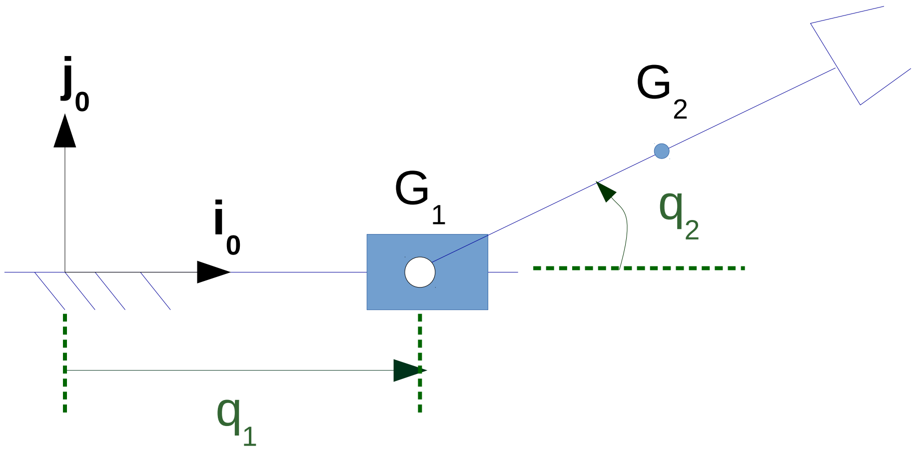


Análise Dinâmica: método de Gibbs-Appell

$$e_1 = \left(\sum_{\square} \vec{F}_{at,1} - m\vec{a}_{G_1}\right).(\mathbf{i_0}) + \left[\sum_{\square} \vec{M}_{at,1} - \left(I_1\dot{\vec{\omega}}_1 + \vec{\omega}_1 \times (I\vec{\omega}_1)\right)\right].(\mathbf{0}) \quad +$$

$$\left(\sum_{\square} \vec{F}_{at,2} - m\vec{a}_{G_2}\right).\left(\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_1}\right) + \left[\sum_{\square} \vec{M}_{at,2} - \left(I_2\dot{\vec{\omega}}_2 + \vec{\omega}_2 \times (I\vec{\omega}_2)\right)\right].\left(\frac{\partial \vec{\omega}_2}{\partial \dot{q}_1}\right)$$

Derivadas parciais em relação a $\dot{q}_1$



Análise Dinâmica: método de Gibbs-Appell

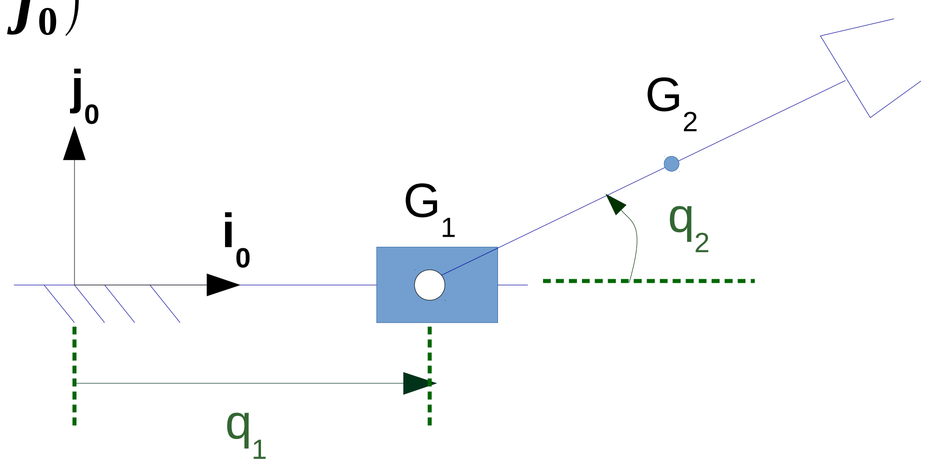
$$e_1 = \left(\sum_{\square} \vec{F}_{at,1} - m\vec{a}_{G_1}\right).(\boldsymbol{i_0}) + \left[\sum_{\square} \vec{M}_{at,1} - \left(I_1\dot{\vec{\omega}}_1 + \vec{\omega}_1 \times (I\vec{\omega}_1)\right)\right].(\boldsymbol{0}) \quad +$$

$$\left(\sum_{\square} \vec{F}_{at,2} - m\vec{a}_{G_2}\right).\left(\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_1}\right) + \left[\sum_{\square} \vec{M}_{at,2} - \left(I_2\dot{\vec{\omega}}_2 + \vec{\omega}_2 \times (I\vec{\omega}_2)\right)\right].\left(\frac{\partial \vec{\omega}_2}{\partial \dot{q}_1}\right)$$

Derivadas parciais em relação a $\dot{q}_1$

$$\vec{V}_{G_2} = \dot{q}_1 \boldsymbol{i_0} + \dot{q}_2 L\left(-sen\, q_2 \boldsymbol{i_0} + \cos q_2 \boldsymbol{j_0}\right)$$

$$\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_1} = \boldsymbol{i_0}$$



Análise Dinâmica: método de Gibbs-Appell

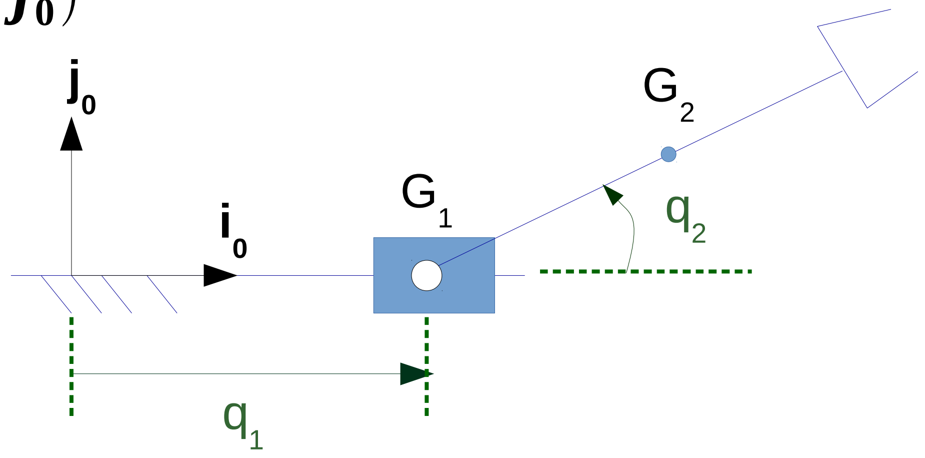
$$e_1 = \left(\sum \vec{F}_{at,1} - m\vec{a}_{G_1}\right).(\boldsymbol{i_0}) + \left[\sum \vec{M}_{at,1} - (I_1\dot{\vec{\omega}}_1 + \vec{\omega}_1 \times (I\vec{\omega}_1))\right].(\boldsymbol{0}) \quad +$$

$$\left(\sum \vec{F}_{at,2} - m\vec{a}_{G_2}\right).(\boldsymbol{i_0}) + \left[\sum \vec{M}_{at,2} - (I_2\dot{\vec{\omega}}_2 + \vec{\omega}_2 \times (I\vec{\omega}_2))\right].\left(\frac{\partial \vec{\omega}_2}{\partial \dot{q}_1}\right)$$

Derivadas parciais em relação a $\dot{q}_1$

$$\vec{V}_{G_2} = \dot{q}_1 \boldsymbol{i_0} + \dot{q}_2 L(-sen\, q_2 \boldsymbol{i_0} + \cos q_2 \boldsymbol{j_0})$$

$$\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_1} = \boldsymbol{i_0}$$



Análise Dinâmica: método de Gibbs-Appell

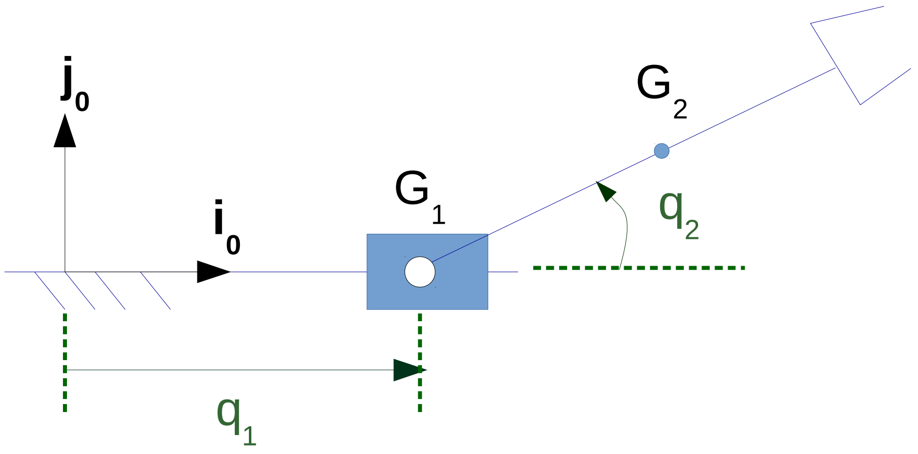
$$e_1 = \left(\sum_{\square} \vec{F}_{at,1} - m\vec{a}_{G_1}\right).(\boldsymbol{i_0}) + \left[\sum_{\square} \vec{M}_{at,1} - \left(I_1\dot{\vec{\omega}}_1 + \vec{\omega}_1 \times (I\vec{\omega}_1)\right)\right].(\mathbf{0}) \quad +$$

$$\left(\sum_{\square} \vec{F}_{at,2} - m\vec{a}_{G_2}\right).(\boldsymbol{i_0}) + \left[\sum_{\square} \vec{M}_{at,2} - \left(I_2\dot{\vec{\omega}}_2 + \vec{\omega}_2 \times (I\vec{\omega}_2)\right)\right].\left(\frac{\partial \vec{\omega}_2}{\partial \dot{q}_1}\right)$$

Derivadas parciais em relação a $\dot{q}_1$

$$\vec{\omega}_2 = \dot{q}_2 \boldsymbol{k_0}$$

$$\frac{\partial \vec{\omega}_2}{\partial \dot{q}_1} = \mathbf{0}$$



Análise Dinâmica: método de Gibbs-Appell

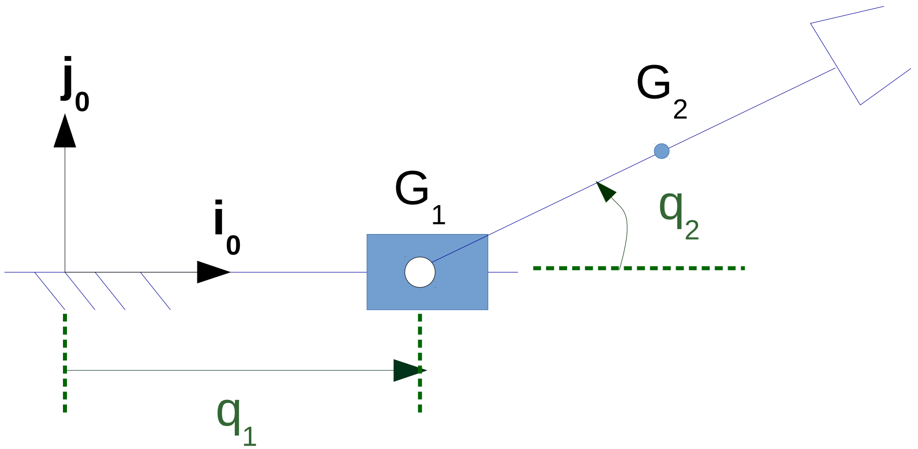
$$e_1 = \left(\sum \vec{F}_{at,1} - m\vec{a}_{G_1}\right).(\boldsymbol{i}_0) + \left[\sum \vec{M}_{at,1} - \left(I_1\dot{\vec{\omega}}_1 + \vec{\omega}_1 \times (I\vec{\omega}_1)\right)\right].(\boldsymbol{0}) \quad +$$

$$\left(\sum \vec{F}_{at,2} - m\vec{a}_{G_2}\right).(\boldsymbol{i}_0) + \left[\sum \vec{M}_{at,2} - \left(I_2\dot{\vec{\omega}}_2 + \vec{\omega}_2 \times (I\vec{\omega}_2)\right)\right].(\boldsymbol{0})$$

Derivadas parciais em relação a $\dot{q}_1$

$$\vec{\omega}_2 = \dot{q}_2 \boldsymbol{k}_0$$

$$\frac{\partial \vec{\omega}_2}{\partial \dot{q}_1} = \boldsymbol{0}$$



Análise Dinâmica: método de Gibbs-Appell

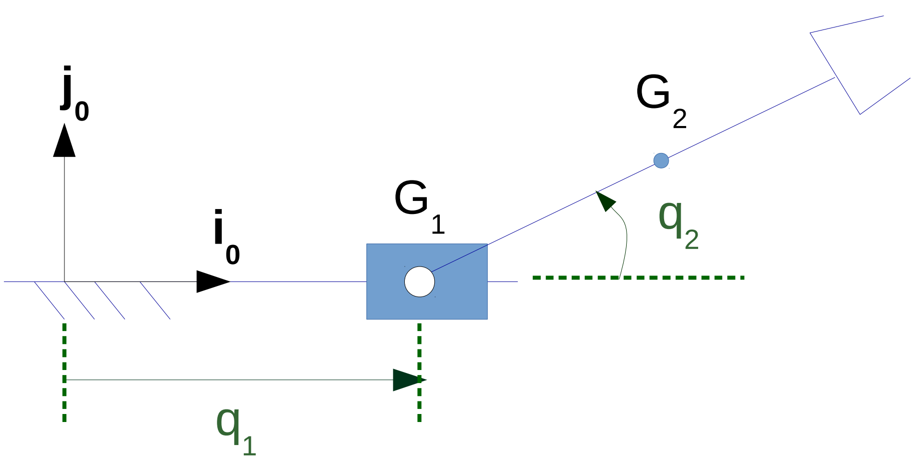
$$e_2 = \left(\sum \vec{F}_{at,1} - m\vec{a}_{G_1}\right) . \left(\frac{\partial \vec{V}_{G_1}}{\partial \dot{q}_2}\right) + \left[\sum \vec{M}_{at,1} - \left(I_1 \dot{\vec{\omega}}_1 + \vec{\omega}_1 \times \left(I \vec{\omega}_1\right)\right)\right] . \left(\frac{\partial \vec{\omega}_1}{\partial \dot{q}_2}\right) +$$

$$\left(\sum \vec{F}_{at,2} - m\vec{a}_{G_2}\right) . \left(\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_2}\right) + \left[\sum \vec{M}_{at,2} - \left(I_2 \dot{\vec{\omega}}_2 + \vec{\omega}_2 \times \left(I \vec{\omega}_2\right)\right)\right] . \left(\frac{\partial \vec{\omega}_2}{\partial \dot{q}_2}\right)$$

Derivadas parciais em relação a $\dot{q}_2$

$$\vec{V}_{G_1} = \dot{q}_1 \boldsymbol{i_0}$$

$$\frac{\partial \vec{V}_{G_1}}{\partial \dot{q}_2} = \mathbf{0}$$



Análise Dinâmica: método de Gibbs-Appell

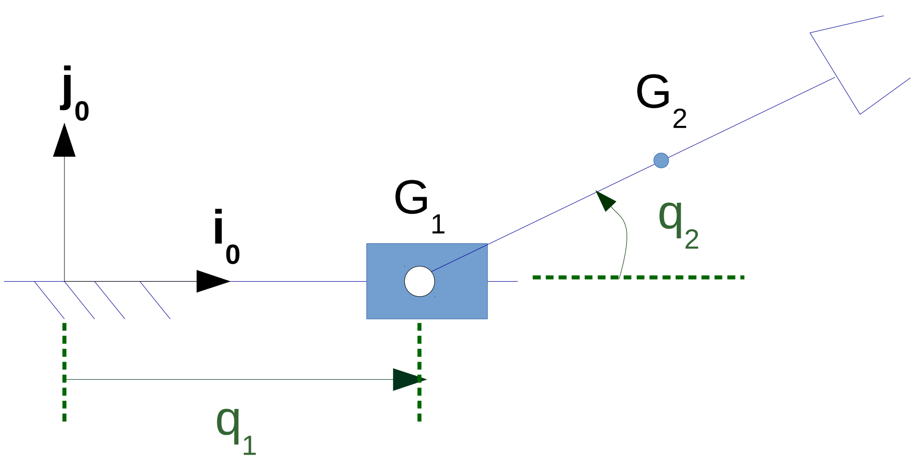
$$e_2=\left(\sum \vec{F}_{at,1}-m\,\vec{a}_{G_1}\right).(\mathbf{0})+\left[\sum \vec{M}_{at,1}-\left(I_1\dot{\vec{\omega}}_1+\vec{\omega}_1\times\left(I\,\vec{\omega}_1\right)\right)\right].\left(\frac{\partial\vec{\omega}_1}{\partial\dot{q}_2}\right)+$$

$$\left(\sum \vec{F}_{at,2}-m\,\vec{a}_{G_2}\right).\left(\frac{\partial\vec{V}_{G_2}}{\partial\dot{q}_2}\right)+\left[\sum \vec{M}_{at,2}-\left(I_2\dot{\vec{\omega}}_2+\vec{\omega}_2\times\left(I\,\vec{\omega}_2\right)\right)\right].\left(\frac{\partial\vec{\omega}_2}{\partial\dot{q}_2}\right)$$

Derivadas parciais em relação a $\dot{q}_2$

$$\vec{V}_{G_1}=\dot{q}_1\,\boldsymbol{i_0}$$

$$\frac{\partial\vec{V}_{G_1}}{\partial\dot{q}_2}=\mathbf{0}$$



Análise Dinâmica: método de Gibbs-Appell

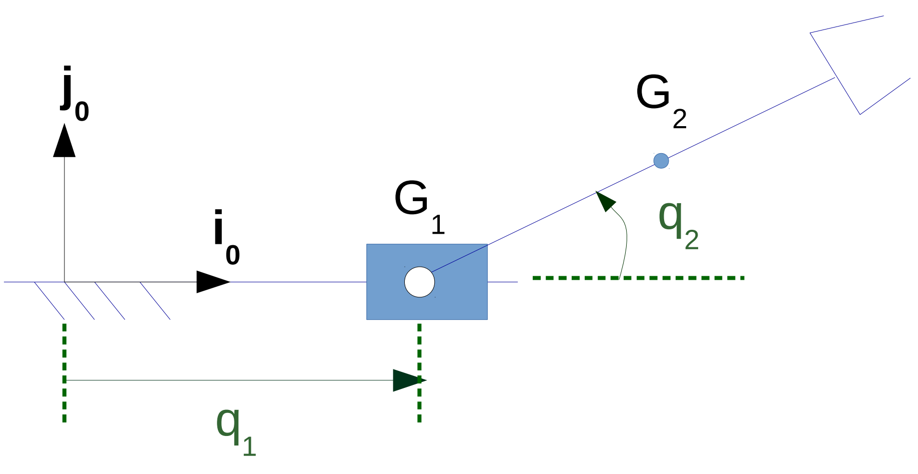
$$e_2=\left(\sum \vec{F}_{at,1}-m\vec{a}_{G_1}\right).(\mathbf{0})+\left[\sum \vec{M}_{at,1}-\left(I_1\dot{\vec{\omega}}_1+\vec{\omega}_1\times(I\,\vec{\omega}_1)\right)\right].\left(\frac{\partial\vec{\omega}_1}{\partial\dot{q}_2}\right)+$$

$$\left(\sum \vec{F}_{at,2}-m\vec{a}_{G_2}\right).\left(\frac{\partial\vec{V}_{G_2}}{\partial\dot{q}_2}\right)+\left[\sum \vec{M}_{at,2}-\left(I_2\dot{\vec{\omega}}_2+\vec{\omega}_2\times(I\,\vec{\omega}_2)\right)\right].\left(\frac{\partial\vec{\omega}_2}{\partial\dot{q}_2}\right)$$

Derivadas parciais em relação a $\dot{q}_2$

$$\vec{\omega}_1=\mathbf{0}$$

$$\frac{\partial\vec{\omega}_1}{\partial\dot{q}_2}=\mathbf{0}$$



Análise Dinâmica: método de Gibbs-Appell

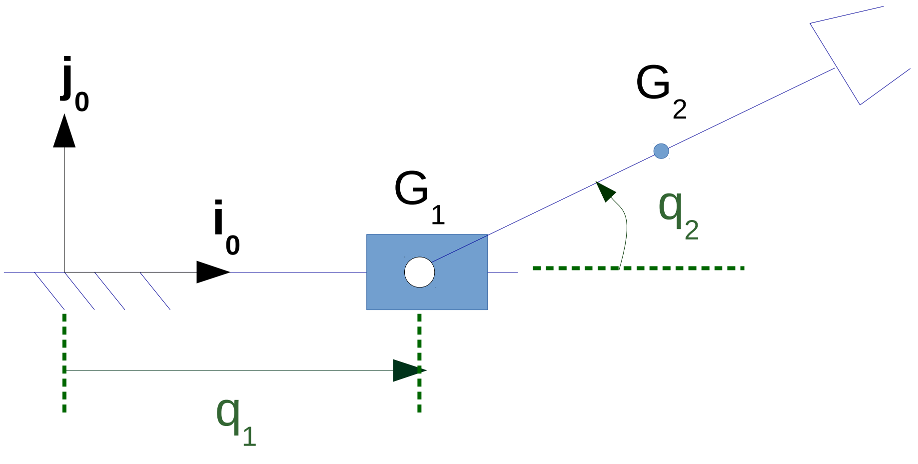
$$e_2 = \left(\sum_{\square} \vec{F}_{at,1} - m\vec{a}_{G_1}\right).(\mathbf{0}) + \left[\sum_{\square} \vec{M}_{at,1} - \left(I_1\dot{\vec{\omega}}_1 + \vec{\omega}_1 \times (I\vec{\omega}_1)\right)\right].(\mathbf{0}) \quad +$$

$$\left(\sum_{\square} \vec{F}_{at,2} - m\vec{a}_{G_2}\right).\left(\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_2}\right) + \left[\sum_{\square} \vec{M}_{at,2} - \left(I_2\dot{\vec{\omega}}_2 + \vec{\omega}_2 \times (I\vec{\omega}_2)\right)\right].\left(\frac{\partial \vec{\omega}_2}{\partial \dot{q}_2}\right)$$

Derivadas parciais em relação a $\dot{q}_2$

$$\vec{\omega}_1 = \mathbf{0}$$

$$\frac{\partial \vec{\omega}_1}{\partial \dot{q}_2} = \mathbf{0}$$



Análise Dinâmica: método de Gibbs-Appell

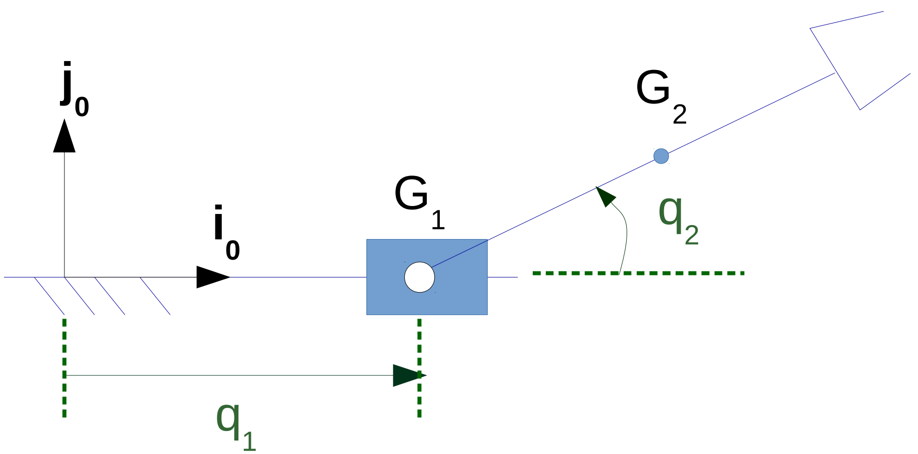
$$e_2 = \left(\sum \vec{F}_{at,1} - m\,\vec{a}_{G_1}\right).(\mathbf{0}) + \left[\sum \vec{M}_{at,1} - \left(I_1\,\dot{\vec{\omega}}_1 + \vec{\omega}_1 \times (I\,\vec{\omega}_1)\right)\right].(\mathbf{0}) +$$

$$\left(\sum \vec{F}_{at,2} - m\,\vec{a}_{G_2}\right).\left(\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_2}\right) + \left[\sum \vec{M}_{at,2} - \left(I_2\,\dot{\vec{\omega}}_2 + \vec{\omega}_2 \times (I\,\vec{\omega}_2)\right)\right].\left(\frac{\partial \vec{\omega}_2}{\partial \dot{q}_2}\right)$$

Derivadas parciais em relação a $\dot{q}_2$

$$\vec{V}_{G_2} = \dot{q}_1\,\boldsymbol{i_0} + \dot{q}_2\,L\left(-sen\,q_2\,\boldsymbol{i_0} + \cos q_2\,\boldsymbol{j_0}\right)$$

$$\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_2} = L\left(-sen\,q_2\,\boldsymbol{i_0} + \cos q_2\,\boldsymbol{j_0}\right)$$



Análise Dinâmica: método de Gibbs-Appell

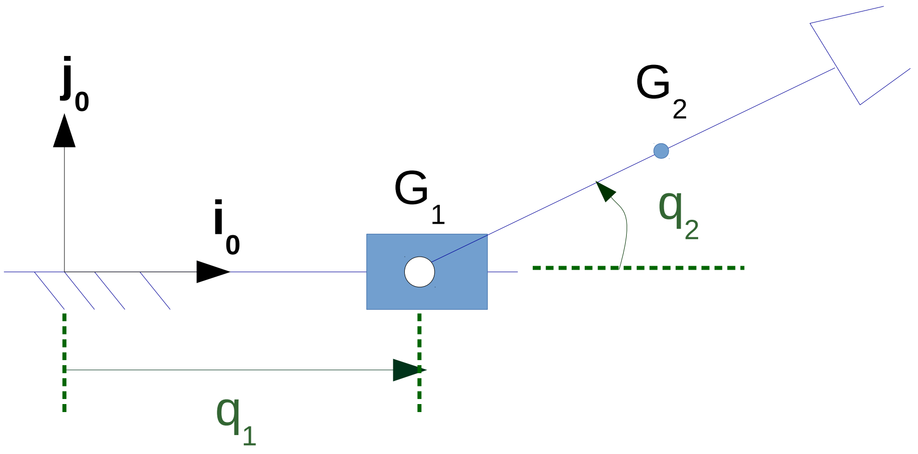
$$e_2 = \left(\sum \vec{F}_{at,1} - m\vec{a}_{G_1}\right).(\mathbf{0}) + \left[\sum \vec{M}_{at,1} - \left(I_1\dot{\vec{\omega}}_1 + \vec{\omega}_1 \times (I_1\vec{\omega}_1)\right)\right].(\mathbf{0}) +$$

$$\left(\sum \vec{F}_{at,2} - m\vec{a}_{G_2}\right).\left(L(-sen\, q_2\, \boldsymbol{i_0} + \cos q_2\, \boldsymbol{j_0})\right) + \left[\sum \vec{M}_{at,2} - \left(I_2\dot{\vec{\omega}}_2 + \vec{\omega}_2 \times (I_2\vec{\omega}_2)\right)\right].\left(\frac{\partial \vec{\omega}_2}{\partial \dot{q}_2}\right)$$

Derivadas parciais em relação a $\dot{q}_2$

$$\vec{V}_{G_2} = \dot{q}_1\boldsymbol{i_0} + \dot{q}_2 L(-sen\, q_2\, \boldsymbol{i_0} + \cos q_2\, \boldsymbol{j_0})$$

$$\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_2} = L(-sen\, q_2\, \boldsymbol{i_0} + \cos q_2\, \boldsymbol{j_0})$$



Análise Dinâmica: método de Gibbs-Appell

$$e_2 = \left(\sum \vec{F}_{at,1} - m\vec{a}_{G_1}\right).(\mathbf{0}) + \left[\sum \vec{M}_{at,1} - \left(I_1\dot{\vec{\omega}}_1 + \vec{\omega}_1 \times (I_1\vec{\omega}_1)\right)\right].(\mathbf{0}) +$$

$$\left(\sum \vec{F}_{at,2} - m\vec{a}_{G_2}\right).\left(L(-sen\, q_2\, \mathbf{i_0} + \cos q_2\, \mathbf{j_0})\right) + \left[\sum \vec{M}_{at,2} - \left(I_2\dot{\vec{\omega}}_2 + \vec{\omega}_2 \times (I_2\vec{\omega}_2)\right)\right].(\mathbf{k_0})$$

Derivadas parciais em relação a $\dot{q}_2$

$$\vec{e} = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -L\,senq_2 & L\,cosq_2 & 1 \end{array}\right] \begin{bmatrix} \left(\sum \vec{F}_{at,1} - m\vec{a}_{G_1}\right) \\ \left[\sum \vec{M}_{at,1} - \left(I_1\dot{\vec{\omega}}_1 + \vec{\omega}_1 \times (I_1\vec{\omega}_1)\right)\right] \\ \left(\sum \vec{F}_{at,2} - m\vec{a}_{G_2}\right) \\ \left[\sum \vec{M}_{at,2} - \left(I_2\dot{\vec{\omega}}_1 + \vec{\omega}_2 \times (I_2\vec{\omega}_2)\right)\right] \end{bmatrix}$$

Análise Dinâmica: método de Gibbs-Appell

$$e_2=\left(\sum \vec{F}_{at,1}-m\vec{a}_{G_1}\right).(\mathbf{0})+\left[\sum \vec{M}_{at,1}-\left(I_1\dot{\vec{\omega}}_1+\vec{\omega}_1\times(I_1\vec{\omega}_1)\right)\right].(\mathbf{0})\;+$$

$$\left(\sum \vec{F}_{at,2}-m\vec{a}_{G_2}\right).\left(L(-sen\,q_2\,\boldsymbol{i_0}+\cos q_2\,\boldsymbol{j_0})\right)+\left[\sum \vec{M}_{at,2}-\left(I_2\dot{\vec{\omega}}_2+\vec{\omega}_2\times(I_2\vec{\omega}_2)\right)\right].(\boldsymbol{k_0})$$

Derivadas parciais em relação a $\dot{q}_2$

$$\vec{e}=\begin{bmatrix}e_1\\e_2\end{bmatrix}=\left[\begin{array}{ccc|ccc}1 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & -L\,senq_2 & L\,cosq_2 & 1\end{array}\right]\begin{bmatrix}\left(\sum \vec{F}_{at,1}-m\vec{a}_{G_1}\right)\\ \left[\sum \vec{M}_{at,1}-\left(I_1\dot{\vec{\omega}}_1+\vec{\omega}_1\times(I_1\vec{\omega}_1)\right)\right]\\ \left(\sum \vec{F}_{at,2}-m\vec{a}_{G_2}\right)\\ \left[\sum \vec{M}_{at,2}-\left(I_2\dot{\vec{\omega}}_1+\vec{\omega}_2\times(I_2\vec{\omega}_2)\right)\right]\end{bmatrix}$$

Análise Dinâmica: método de Gibbs-Appell

$$e_2 = \Big(\sum \vec{F}_{at,1} - m\vec{a}_{G_1}\Big).(\mathbf{0}) + \Big[\sum \vec{M}_{at,1} - \big(I_1\dot{\vec{\omega}}_1 + \vec{\omega}_1 \times (I_1\vec{\omega}_1)\big)\Big].(\mathbf{0}) +$$

$$\Big(\sum \vec{F}_{at,2} - m\vec{a}_{G_2}\Big).\big(L(-\operatorname{sen} q_2\,\boldsymbol{i_0} + \cos q_2\,\boldsymbol{j_0})\big) + \Big[\sum \vec{M}_{at,2} - \big(I_2\dot{\vec{\omega}}_2 + \vec{\omega}_2 \times (I_2\vec{\omega}_2)\big)\Big].(\boldsymbol{k_0})$$

Derivadas parciais em relação a $\dot{q}_2$

$$\vec{e} = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -L\,senq_2 & L\,cosq_2 & 1 \end{array}\right] \begin{bmatrix} \big(\sum \vec{F}_{at,1} - m\vec{a}_{G_1}\big) \\ \big[\sum \vec{M}_{at,1} - \big(I_1\dot{\vec{\omega}}_1 + \vec{\omega}_1 \times (I_1\vec{\omega}_1)\big)\big] \\ \big(\sum \vec{F}_{at,2} - m\vec{a}_{G_2}\big) \\ \big[\sum \vec{M}_{at,2} - \big(I_2\dot{\vec{\omega}}_1 + \vec{\omega}_2 \times (I_2\vec{\omega}_2)\big)\big] \end{bmatrix}$$

Análise Dinâmica: método de Gibbs-Appell

$$e_2=\left(\sum \vec{F}_{at,1}-m\vec{a}_{G_1}\right).(\mathbf{0})+\left[\sum \vec{M}_{at,1}-\left(I_1\dot{\vec{\omega}}_1+\vec{\omega}_1\times\left(I_1\vec{\omega}_1\right)\right)\right].(\mathbf{0})\quad +$$

$$\left(\sum \vec{F}_{at,2}-m\vec{a}_{G_2}\right).\left(L\left(-sen\, q_2\,\boldsymbol{i}_0+\cos q_2\,\boldsymbol{j}_0\right)\right)+\left[\sum \vec{M}_{at,2}-\left(I_2\dot{\vec{\omega}}_2+\vec{\omega}_2\times\left(I_2\vec{\omega}_2\right)\right)\right].(\boldsymbol{k}_0)$$

Derivadas parciais em relação a $\dot{q}_2$

$$\vec{e}=\begin{bmatrix}e_1\\ e_2\end{bmatrix}=\begin{bmatrix}1 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & -L\,sen q_2 & L\,cos q_2 & 1\end{bmatrix}\begin{bmatrix}\left(\sum \vec{F}_{at,1}-m\vec{a}_{G_1}\right)\\ \left[\sum \vec{M}_{at,1}-\left(I_1\dot{\vec{\omega}}_1+\vec{\omega}_1\times\left(I_1\vec{\omega}_1\right)\right)\right]\\ \left(\sum \vec{F}_{at,2}-m\vec{a}_{G_2}\right)\\ \left[\sum \vec{M}_{at,2}-\left(I_2\dot{\vec{\omega}}_1+\vec{\omega}_2\times\left(I_2\vec{\omega}_2\right)\right)\right]\end{bmatrix}$$

Análise Dinâmica: método de Gibbs-Appell

$$e_1 = \left(\sum \vec{F}_{at,1} - m\vec{a}_{G_1}\right).(\boldsymbol{i_0}) + \left[\sum \vec{M}_{at,1} - \left(I_1\dot{\vec{\omega}}_1 + \vec{\omega}_1 \times (I\,\vec{\omega}_1)\right)\right].(\boldsymbol{0}) \; +$$

$$\left(\sum \vec{F}_{at,2} - m\vec{a}_{G_2}\right).(\boldsymbol{i_0}) + \left[\sum \vec{M}_{at,2} - \left(I_2\dot{\vec{\omega}}_2 + \vec{\omega}_2 \times (I\,\vec{\omega}_2)\right)\right].(\boldsymbol{0})$$

Derivadas parciais em relação a $\dot{q}_2$

$$\vec{e} = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -L\,sen q_2 & L\,cos q_2 & 1 \end{array}\right] \begin{bmatrix} \left(\sum \vec{F}_{at,1} - m\vec{a}_{G_1}\right) \\ \left[\sum \vec{M}_{at,1} - \left(I_1\dot{\vec{\omega}}_1 + \vec{\omega}_1 \times (I_1\vec{\omega}_1)\right)\right] \\ \left(\sum \vec{F}_{at,2} - m\vec{a}_{G_2}\right) \\ \left[\sum \vec{M}_{at,2} - \left(I_2\dot{\vec{\omega}}_1 + \vec{\omega}_2 \times (I_2\vec{\omega}_2)\right)\right] \end{bmatrix}$$

Análise Dinâmica: método de Gibbs-Appell

$$e_1 = \left(\sum \vec{F}_{at,1} - m\vec{a}_{G_1}\right).(\boldsymbol{i_0}) + \left[\sum \vec{M}_{at,1} - \left(I_1\dot{\vec{\omega}}_1 + \vec{\omega}_1 \times (I\,\vec{\omega}_1)\right)\right].(\mathbf{0}) +$$

$$\left(\sum \vec{F}_{at,2} - m\vec{a}_{G_2}\right).(\boldsymbol{i_0}) + \left[\sum \vec{M}_{at,2} - \left(I_2\dot{\vec{\omega}}_2 + \vec{\omega}_2 \times (I\,\vec{\omega}_2)\right)\right].(\mathbf{0})$$

Derivadas parciais em relação a $\dot{q}_2$

$$\vec{e} = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -L\,sen q_2 & L\,cos q_2 & 1 \end{array}\right] \begin{bmatrix} \left(\sum \vec{F}_{at,1} - m\vec{a}_{G_1}\right) \\ \left[\sum \vec{M}_{at,1} - \left(I_1\dot{\vec{\omega}}_1 + \vec{\omega}_1 \times (I_1\vec{\omega}_1)\right)\right] \\ \left(\sum \vec{F}_{at,2} - m\vec{a}_{G_2}\right) \\ \left[\sum \vec{M}_{at,2} - \left(I_2\dot{\vec{\omega}}_1 + \vec{\omega}_2 \times (I_2\vec{\omega}_2)\right)\right] \end{bmatrix}$$

Análise Dinâmica: método de Gibbs-Appell

$$e_1 = \Big(\sum_{\square} \vec{F}_{at,1} - m\,\vec{a}_{G_1}\Big).(\boldsymbol{i_0}) + \Big[\sum_{\square} \vec{M}_{at,1} - \big(I_1\dot{\vec{\omega}}_1 + \vec{\omega}_1 \times (I\,\vec{\omega}_1)\big)\Big].(\mathbf{0}) \; +$$

$$\Big(\sum_{\square} \vec{F}_{at,2} - m\,\vec{a}_{G_2}\Big).(\boldsymbol{i_0}) + \Big[\sum_{\square} \vec{M}_{at,2} - \big(I_2\dot{\vec{\omega}}_2 + \vec{\omega}_2 \times (I\,\vec{\omega}_2)\big)\Big].(\mathbf{0})$$

Derivadas parciais em relação a $\dot{q}_2$

$$\vec{e} = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \left[\begin{array}{ccc|ccc} 1 & 0 & \boxed{0} & 1 & 0 & 0 \\ 0 & 0 & 0 & -L\,sen q_2 & L\,cos q_2 & 1 \end{array}\right] \begin{bmatrix} \Big(\sum_{\square} \vec{F}_{at,1} - m\,\vec{a}_{G_1}\Big) \\ \Big[\sum_{\square} \vec{M}_{at,1} - \big(I_1\dot{\vec{\omega}}_1 + \vec{\omega}_1 \times (I_1\vec{\omega}_1)\big)\Big] \\ \Big(\sum_{\square} \vec{F}_{at,2} - m\,\vec{a}_{G_2}\Big) \\ \Big[\sum_{\square} \vec{M}_{at,2} - \big(I_2\dot{\vec{\omega}}_1 + \vec{\omega}_2 \times (I_2\vec{\omega}_2)\big)\Big] \end{bmatrix}$$

Análise Dinâmica: método de Gibbs-Appell

$$e_1=(\sum_{\square}\vec{F}_{at,1}-m\vec{a}_{G_1}).(\boldsymbol{i_0})+[\sum_{\square}\vec{M}_{at,1}-(I_1\dot{\vec{\omega}}_1+\vec{\omega}_1\times(I\,\vec{\omega}_1))].(\boldsymbol{0})\ +$$

$$(\sum_{\square}\vec{F}_{at,2}-m\vec{a}_{G_2}).(\boldsymbol{i_0})+[\sum_{\square}\vec{M}_{at,2}-(I_2\dot{\vec{\omega}}_2+\vec{\omega}_2\times(I\,\vec{\omega}_2))].(\boldsymbol{0})$$

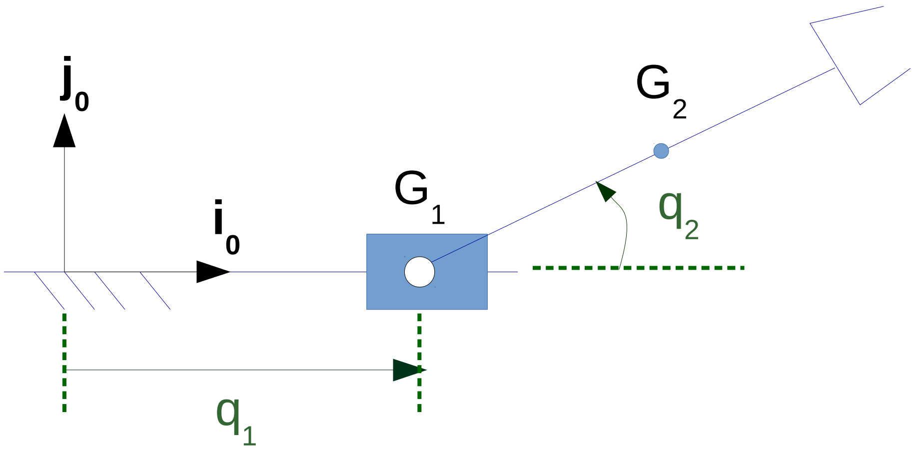
Derivadas parciais em relação a $\dot{q}_2$

$$\vec{e}=\begin{bmatrix}e_1\\e_2\end{bmatrix}=\left[\begin{array}{ccc|ccc}1&0&0&1&0&0\\0&0&0&-L\,senq_2&L\,cosq_2&1\end{array}\right]\begin{bmatrix}(\sum_{\square}\vec{F}_{at,1}-m_1\vec{a}_{G_1})\\ [\sum_{\square}\vec{M}_{at,1}-(I_1\dot{\vec{\omega}}_1+\vec{\omega}_1\times(I_1\vec{\omega}_1))]\\ (\sum_{\square}\vec{F}_{at,2}-m_2\vec{a}_{G_2})\\ [\sum_{\square}\vec{M}_{at,2}-(I_2\dot{\vec{\omega}}_1+\vec{\omega}_2\times(I_2\vec{\omega}_2))]\end{bmatrix}$$

Análise Dinâmica: método de Gibbs-Appell

$$\vec{e}=\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}=\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -L\,sen q_2 & L\,cos q_2 & 1 \end{array}\right]\begin{bmatrix} \left(\sum \vec{F}_{at,1}-m\vec{a}_{G_1}\right) \\ \left[\sum \vec{M}_{at,1}-\left(I_1\dot{\vec{\omega}}_1+\vec{\omega}_1\times\left(I_1\vec{\omega}_1\right)\right)\right] \\ \left(\sum \vec{F}_{at,2}-m\vec{a}_{G_2}\right) \\ \left[\sum \vec{M}_{at,2}-\left(I_2\dot{\vec{\omega}}_1+\vec{\omega}_2\times\left(I_2\vec{\omega}_2\right)\right)\right] \end{bmatrix}$$

$$\vec{F}_{at,1}-m\vec{a}_{G_1}$$

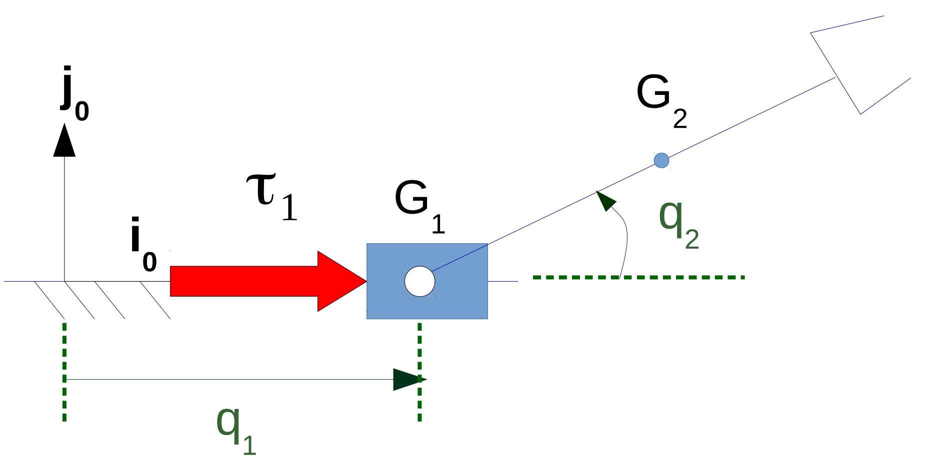


Análise Dinâmica: método de Gibbs-Appell

$$\vec{e}=\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}=\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -L\,sen q_2 & L\,cos q_2 & 1 \end{array}\right]\left[\begin{array}{c} \boxed{\left(\sum \vec{F}_{at,1}-m_1\vec{a}_{G_1}\right)} \\ \left[\sum \vec{M}_{at,1}-\left(I_1\dot{\vec{\omega}}_1+\vec{\omega}_1\times\left(I_1\vec{\omega}_1\right)\right)\right] \\ \hline \left(\sum \vec{F}_{at,2}-m_2\vec{a}_{G_2}\right) \\ \left[\sum \vec{M}_{at,2}-\left(I_2\dot{\vec{\omega}}_1+\vec{\omega}_2\times\left(I_2\vec{\omega}_2\right)\right)\right] \end{array}\right]$$

$$\vec{F}_{at,1}-m_1\vec{a}_{G_1}$$

$$\boxed{\tau_1\boldsymbol{i_0}}$$

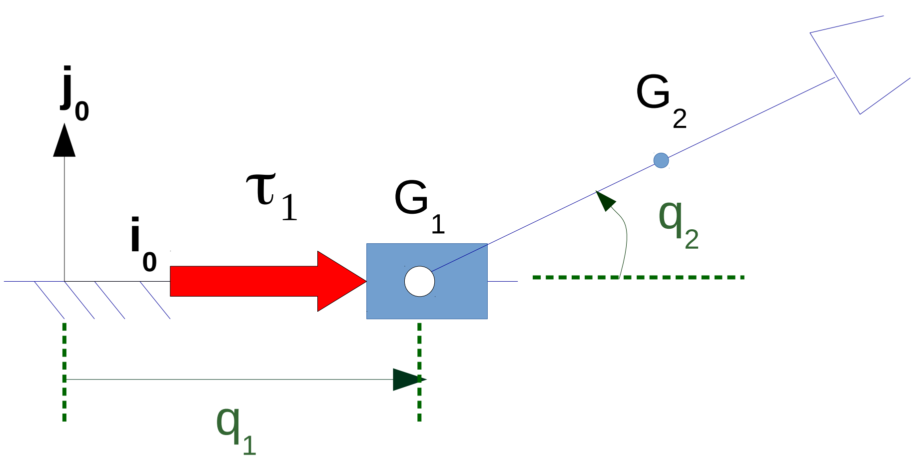


Análise Dinâmica: método de Gibbs-Appell

$$\vec{e}=\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}=\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -L\,sen q_2 & L\,cos q_2 & 1 \end{array}\right]\left[\begin{array}{c} \boxed{\left(\sum \vec{F}_{at,1}-m_1\,\vec{a}_{G_1}\right)} \\ \left[\sum \vec{M}_{at,1}-\left(I_1\dot{\vec{\omega}}_1+\vec{\omega}_1\times\left(I_1\vec{\omega}_1\right)\right)\right] \\ \hline \left(\sum \vec{F}_{at,2}-m_2\,\vec{a}_{G_2}\right) \\ \left[\sum \vec{M}_{at,2}-\left(I_2\dot{\vec{\omega}}_2+\vec{\omega}_2\times\left(I_2\vec{\omega}_2\right)\right)\right] \end{array}\right]$$

$$\vec{F}_{at,1}-m_1\,\vec{a}_{G_1}$$

$$\boxed{\vec{a}_{G_1}=\ddot{q}_1\,\boldsymbol{i_0}}$$

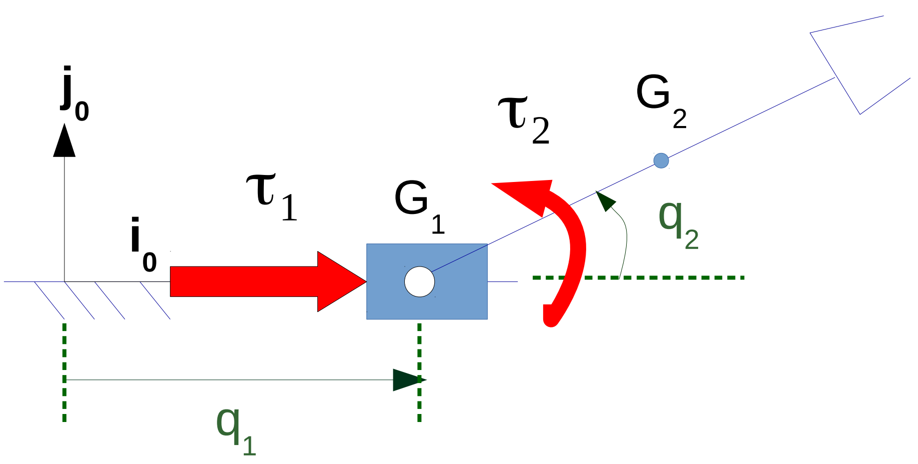


Análise Dinâmica: método de Gibbs-Appell

$$\vec{e}=\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}=\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -L\,sen q_2 & L\,cos q_2 & 1 \end{array}\right]\begin{bmatrix} (\tau_1 - m_1 \ddot{q}_1) \\ 0 \\ \left[\sum \vec{M}_{at,1} - (I_1 \dot{\vec{\omega}}_1 + \vec{\omega}_1 \times (I_1 \vec{\omega}_1))\right] \\ \left(\sum \vec{F}_{at,2} - m_2 \vec{a}_{G_2}\right) \\ \left[\sum \vec{M}_{at,2} - (I_2 \dot{\vec{\omega}}_2 + \vec{\omega}_2 \times (I_2 \vec{\omega}_2))\right] \end{bmatrix}$$

$$\left[\sum \vec{M}_{at,1} - (I_1 \dot{\vec{\omega}}_1 + \vec{\omega}_1 \times (I_1 \vec{\omega}_1))\right]$$

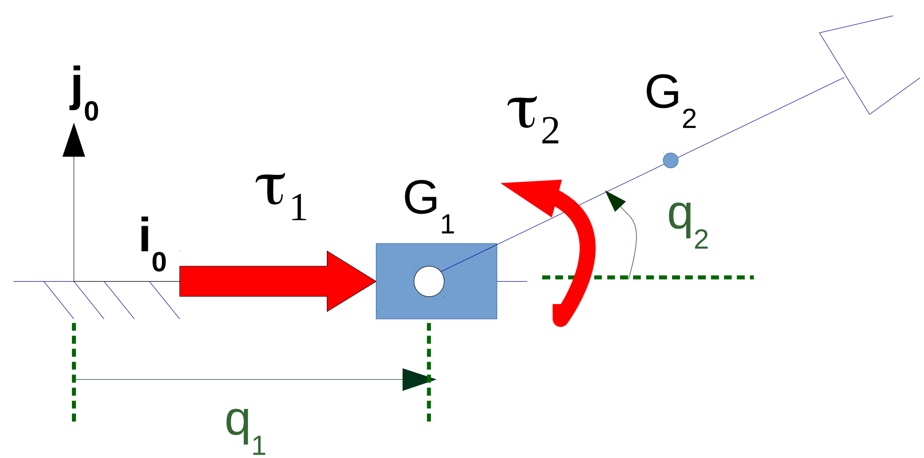
$$-\tau_2 \boldsymbol{k_0} \qquad \mathbf{0}$$



Análise Dinâmica: método de Gibbs-Appell

$$\vec{e}=\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}=\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -L\,senq_2 & L\,cosq_2 & 1 \end{array}\right]\left[\begin{array}{c} (\tau_1 - m_1\ddot{q}_1) \\ 0 \\ -\tau_2 \\ \hline (\sum \vec{F}_{at,2} - m_2\vec{a}_{G_2}) \\ [\sum \vec{M}_{at,2} - (I_2\dot{\vec{\omega}}_2 + \vec{\omega}_2 \times (I_2\vec{\omega}_2))] \end{array}\right]$$

$$[\underbrace{\sum \vec{M}_{at,1}}_{-\tau_2 \boldsymbol{k_0}} - (\underbrace{I_1\dot{\vec{\omega}}_1 + \vec{\omega}_1 \times (I_1\vec{\omega}_1)}_{\mathbf{0}})]$$



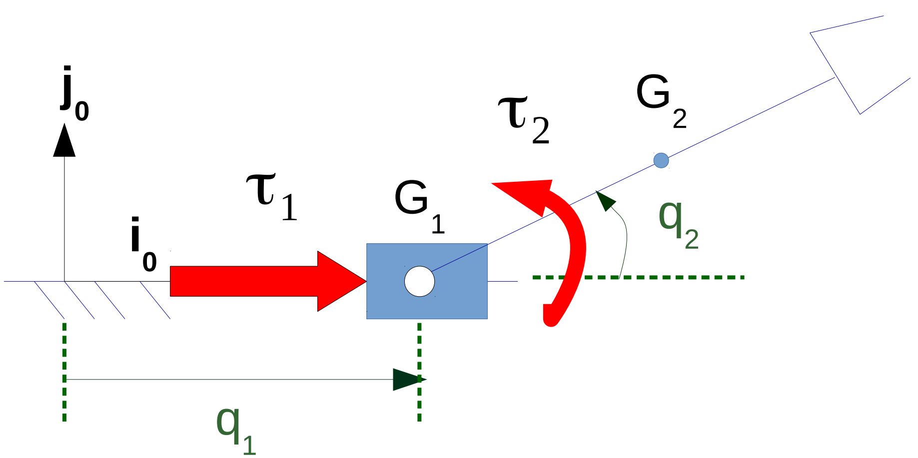
Análise Dinâmica: método de Gibbs-Appell

$$\vec{e}=\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}=\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -L\,sen q_2 & L\,cos q_2 & 1 \end{array}\right]\left[\begin{array}{c} (\tau_1 - m_1 \ddot{q}_1) \\ 0 \\ -\tau_2 \\ \hline \left(\sum \vec{F}_{at,2} - m_2 \vec{a}_{G_2}\right) \\ \left[\sum \vec{M}_{at,2} - \left(I_2 \dot{\vec{\omega}}_2 + \vec{\omega}_2 \times (I_2 \vec{\omega}_2)\right)\right] \end{array}\right]$$

$$\left[\sum \vec{M}_{at,2} - \left(I_2 \dot{\vec{\omega}}_2 + \vec{\omega}_2 \times (I_2 \vec{\omega}_2)\right)\right]$$

$$\tau_2 \boldsymbol{k_0}$$

$$-I_2 \ddot{q}_2 \boldsymbol{k_0}$$



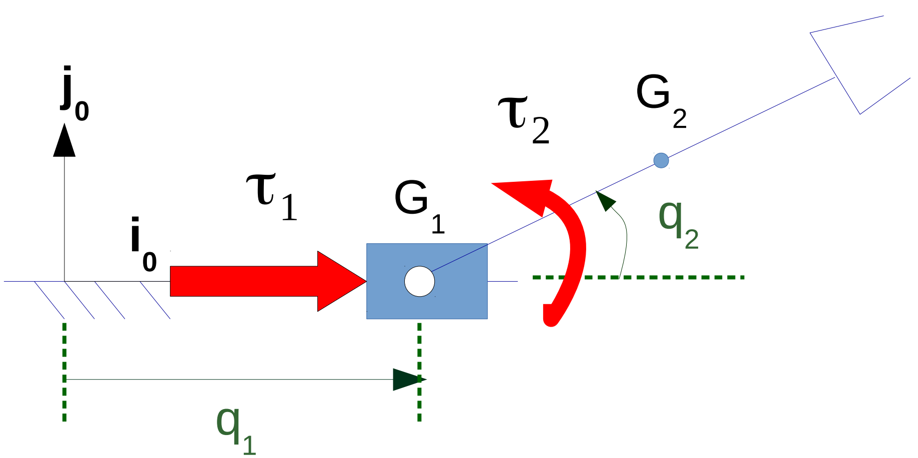
Análise Dinâmica: método de Gibbs-Appell

$$\vec{e}=\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}=\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -L\,senq_2 & L\,cosq_2 & 1 \end{array}\right]\begin{bmatrix} (\tau_1-m_1\ddot{q}_1) \\ 0 \\ -\tau_2 \\ \hline \left(\sum \vec{F}_{at,2}-m_2\vec{a}_{G_2}\right) \\ \tau_2-I_2\ddot{q}_2 \end{bmatrix}$$

$$\left[\sum \vec{M}_{at,2}-\left(I_2\dot{\vec{\omega}}_2+\vec{\omega}_2\times\left(I_2\vec{\omega}_2\right)\right)\right]$$

$$\tau_2\boldsymbol{k_0}$$

$$-I_2\ddot{q}_2\boldsymbol{k_0}$$

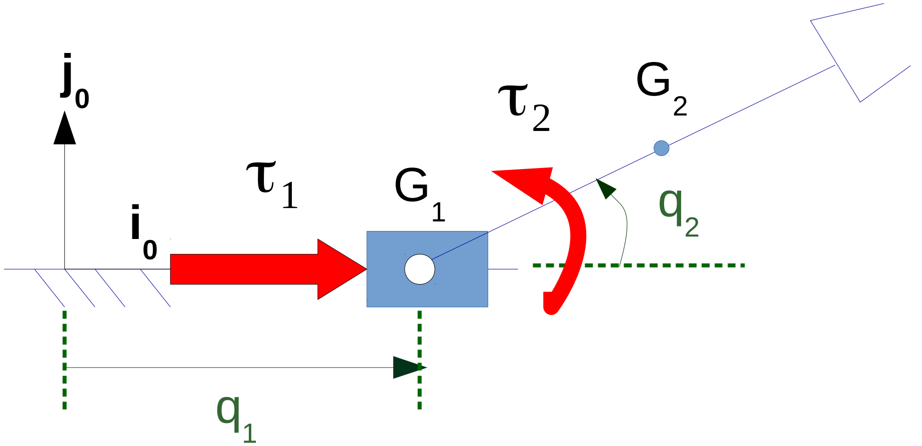


Análise Dinâmica: método de Gibbs-Appell

$$\vec{e}=\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}=\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -L\,senq_2 & L\,cosq_2 & 1 \end{array}\right]\begin{bmatrix} (\tau_1-m_1\ddot{q}_1) \\ 0 \\ -\tau_2 \\ \hline \left(\sum \vec{F}_{at,2}-m_2\vec{a}_{G_2}\right) \\ \tau_2-I_2\ddot{q}_2 \end{bmatrix}$$

$$\vec{F}_{at,2}-m_2\vec{a}_{G_2}$$

$$\mathbf{0}$$



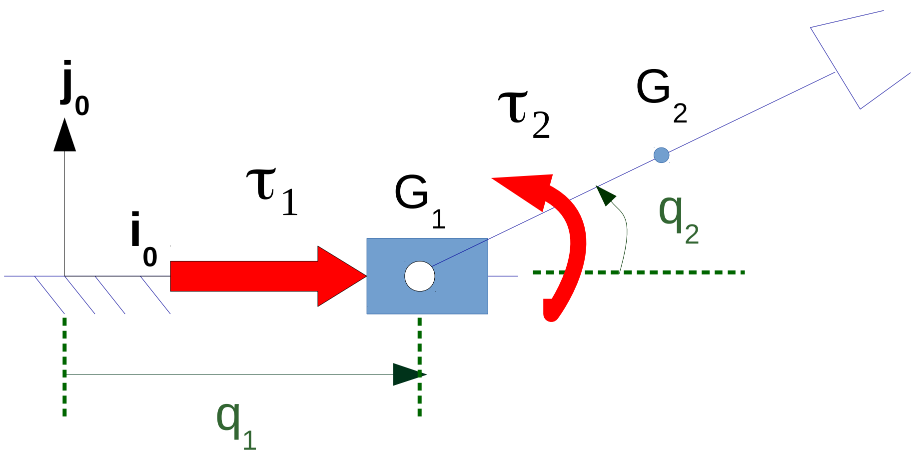
$$\vec{a}_{G_2}=\ddot{q}_1\,\boldsymbol{i_0}+\ddot{q}_2\,L\left(-sen\,q_2\,\boldsymbol{i_0}+\cos q_2\,\boldsymbol{j_0}\right)-\dot{q}_2^{\,2}\,L\left(\cos q_2\,\boldsymbol{i_0}+sen\,q_2\,\boldsymbol{j_0}\right)$$

Análise Dinâmica: método de Gibbs-Appell

$$\vec{e}=\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}=\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -L\,senq_2 & L\,cosq_2 & 1 \end{array}\right]\begin{bmatrix} (\tau_1-m_1\ddot{q}_1) \\ 0 \\ -\tau_2 \\ \hline -m_2[\ddot{q}_1-L(\ddot{q}_2 senq_2+\dot{q}_2^{\,2} cosq_2)] \\ -m_2[L(\ddot{q}_2 cosq_2-\dot{q}_2^{\,2} senq_2)] \\ \tau_2-I_2\ddot{q}_2 \end{bmatrix}$$

$$\vec{F}_{at,2}-m_2\vec{a}_{G_2}$$

$$\mathbf{0}$$



$$\vec{a}_{G_2}=\ddot{q}_1\,\boldsymbol{i_0}+\ddot{q}_2\,L(-sen\,q_2\,\boldsymbol{i_0}+\cos q_2\,\boldsymbol{j_0})-\dot{q}_2^{\,2}\,L(\cos q_2\,\boldsymbol{i_0}+sen\,q_2\,\boldsymbol{j_0})$$

Análise Dinâmica: método de Gibbs-Appell

$$\vec{e}=\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}=\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -L\,senq_2 & L\,cosq_2 & 1 \end{array}\right]\left[\begin{array}{c} (\tau_1-m_1\ddot{q}_1) \\ 0 \\ -\tau_2 \\ \hline -m_2[\ddot{q}_1-L(\ddot{q}_2 senq_2+\dot{q}_2{}^2 cosq_2)] \\ -m_2[L(\ddot{q}_2 cosq_2-\dot{q}_2{}^2 senq_2)] \\ \tau_2-I_2\dot{\omega}_2 \end{array}\right]$$

$$\begin{bmatrix} \tau_1-m_1\ddot{q}_1-m_2[\ddot{q}_1-L(\ddot{q}_2 senq_2+\dot{q}_2{}^2 cosq_2)] \\ m_2[\ddot{q}_1-L(\ddot{q}_2 senq_2+\dot{q}_2{}^2 cosq_2)]Lsenq_2-m_2[L(\ddot{q}_2 cosq_2-\dot{q}_2{}^2 senq_2)]Lcosq_2+\tau_2-I_2\ddot{q}_2 \end{bmatrix}$$

Análise Dinâmica: método de Gibbs-Appell

$$\vec{e}=\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}=\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -L\,senq_2 & L\,cosq_2 & 1 \end{array}\right]\left[\begin{array}{c} (\tau_1-m_1\ddot{q}_1) \\ 0 \\ -\tau_2 \\ \hline -m_2[\ddot{q}_1-L(\ddot{q}_2\,senq_2+\dot{q}_2^{\,2}cosq_2)] \\ -m_2[L(\ddot{q}_2\,cosq_2-\dot{q}_2^{\,2}senq_2)] \\ \tau_2-I_2\dot{\omega}_2 \end{array}\right]$$

$$\begin{bmatrix} \tau_1-m_1\ddot{q}_1-m_2[\ddot{q}_1-L(\ddot{q}_2\,senq_2+\dot{q}_2^{\,2}cosq_2)] \\ m_2[\ddot{q}_1-L(\ddot{q}_2\,senq_2+\dot{q}_2^{\,2}cosq_2)]\,Lsenq_2-m_2[L(\ddot{q}_2\,cosq_2-\dot{q}_2^{\,2}senq_2)]\,Lcosq_2+\tau_2-I_2\ddot{q}_2 \end{bmatrix}$$

$$\begin{bmatrix} \tau_1-(m_1+m_2)\ddot{q}_1+(m_2L\,senq_2)\ddot{q}_2+(m_2L\,cosq_2)\dot{q}_2^{\,2} \\ m_2Lsenq_2\ddot{q}_1-(m_2L^2+I_2)\ddot{q}_2+\tau_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Análise Dinâmica: método de Gibbs-Appell

$$\vec{e}=\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}=\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -L\,senq_2 & L\,cosq_2 & 1 \end{array}\right]\left[\begin{array}{c} (\tau_1 - m_1\ddot{q}_1) \\ 0 \\ -\tau_2 \\ \hline -m_2[\ddot{q}_1 - L(\ddot{q}_2\,senq_2 + \dot{q}_2^{\,2}\,cosq_2)] \\ -m_2[L(\ddot{q}_2\,cosq_2 - \dot{q}_2^{\,2}\,senq_2)] \\ \tau_2 - I_2\dot{\omega}_2 \end{array}\right]$$

$$\begin{bmatrix} \tau_1 - (m_1+m_2)\ddot{q}_1 + (m_2L\,senq_2)\ddot{q}_2 + (m_2L\,cosq_2)\dot{q}_2^{\,2} \\ m_2Lsenq_2\ddot{q}_1 - (m_2L^2+I_2)\ddot{q}_2 + \tau_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

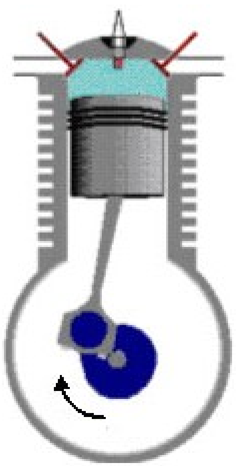
$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}=\begin{bmatrix} (m_1+m_2)\ddot{q}_1 - (m_2L\,senq_2)\ddot{q}_2 - (m_2L\,cosq_2)\dot{q}_2^{\,2} \\ -m_2Lsenq_2\ddot{q}_1 + (m_2L^2+I_2)\ddot{q}_2 \end{bmatrix}$$

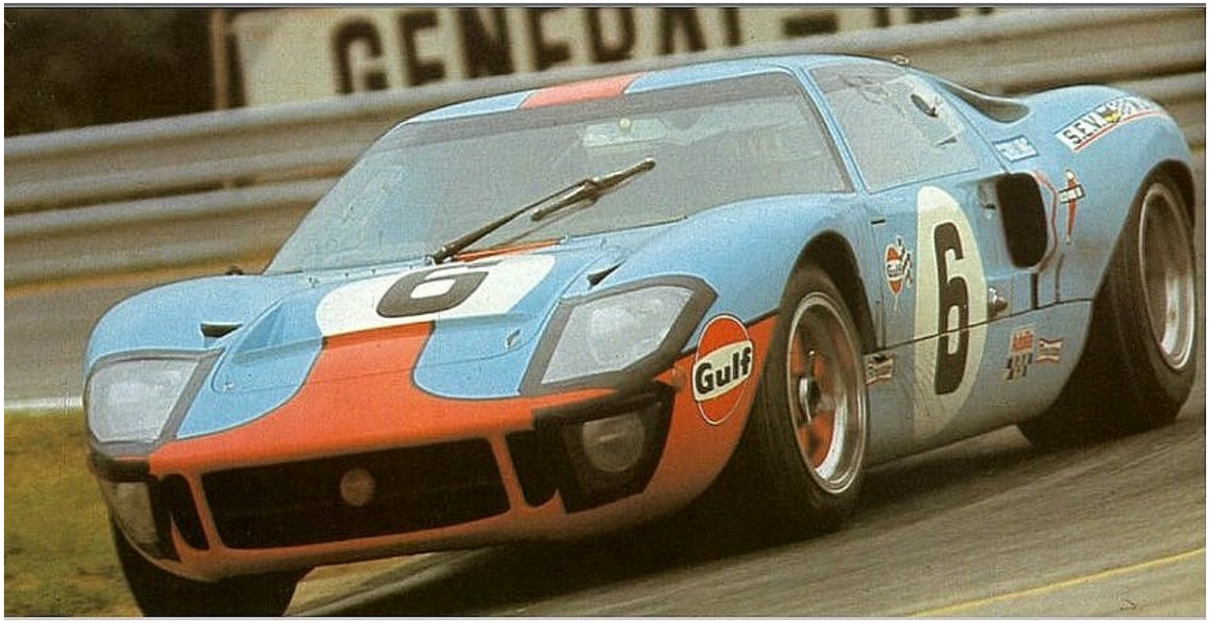
Tipos de carregamentos sobre o mecanismo

Motor de combustão interna Ford GT 40 Mk I

Rotação 5000 rpm = 523,6 rad/s (torque máximo 447 Nm)
Curso 73mm, potência 385 HP

Aceleração pistão $A\,\omega^2 = 10.000\ m/s^2 = 1000\ G$ → Predomínio das forças de inércia





Tipos de carregamentos sobre o mecanismo

Veloc. Máx. = 10 m/s

Acel. Máx. = 150 m/s^2 = 15 G

Predomínio das forças de inércia

Tipos de carregamentos sobre o mecanismo

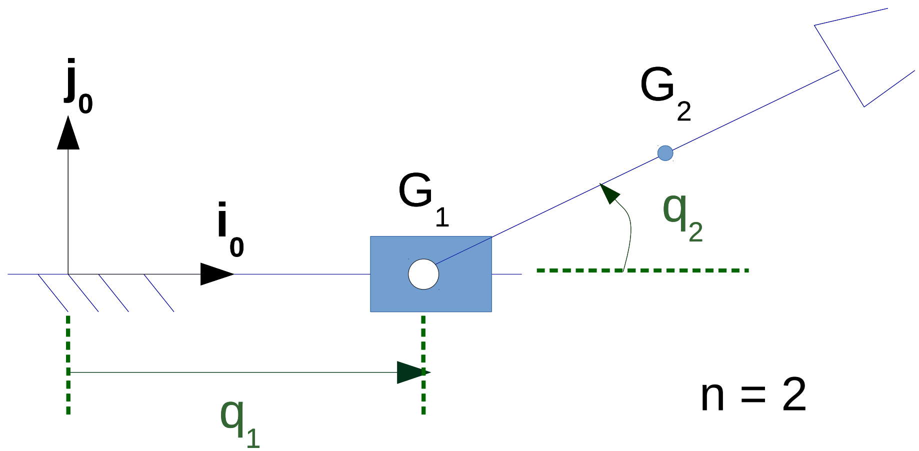
Predomínio da força de campo gravitacional

Análise Dinâmica: método de Gibbs-Appell

$$\vec{e}_{nx1} = D_{nx\lambda N} \boldsymbol{f}_{\boldsymbol{\lambda N x 1}} = \mathbf{0}_{nx1}$$

$$D_{nx\lambda N} = \left[\begin{array}{cc|cc|c|cc} \frac{\partial \mathbf{v}_{G_1}^T}{\partial u_1} & \frac{\partial \omega_1^T}{\partial u_1} & \frac{\partial \mathbf{v}_{G_2}^T}{\partial u_1} & \frac{\partial \omega_2^T}{\partial u_1} & \cdots & \frac{\partial \mathbf{v}_{G_N}^T}{\partial u_1} & \frac{\partial \omega_N^T}{\partial u_1} \\ \frac{\partial \mathbf{v}_{G_1}^T}{\partial u_2} & \frac{\partial \omega_1^T}{\partial u_2} & \frac{\partial \mathbf{v}_{G_2}^T}{\partial u_2} & \frac{\partial \omega_2^T}{\partial u_2} & \cdots & \frac{\partial \mathbf{v}_{G_N}^T}{\partial u_2} & \frac{\partial \omega_N^T}{\partial u_2} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{\partial \mathbf{v}_{G_1}^T}{\partial u_n} & \frac{\partial \omega_1^T}{\partial u_n} & \frac{\partial \mathbf{v}_{G_2}^T}{\partial u_n} & \frac{\partial \omega_2^T}{\partial u_n} & \cdots & \frac{\partial \mathbf{v}_{G_N}^T}{\partial u_n} & \frac{\partial \omega_N^T}{\partial u_n} \end{array} \right]$$

$$u_i = \dot{q}_i$$

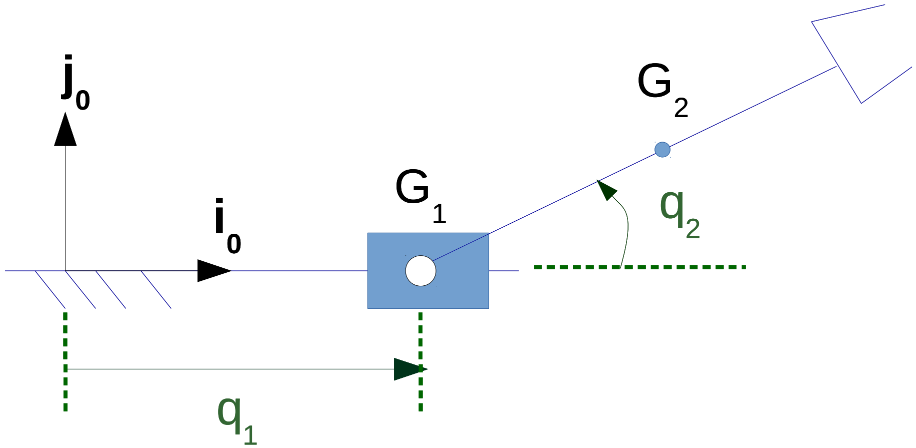


Análise Dinâmica: método de Gibbs-Appell

$$\vec{e}_{nx1} = D_{nx\lambda N} \boldsymbol{f}_{\boldsymbol{\lambda N x 1}} = \mathbf{0}_{nx1}$$

$$\mathbf{f}_{\lambda N x1} = \begin{bmatrix} \sum \mathbf{F}_{at,1} - m_1 \mathbf{a}_{G_1} \\ \sum \mathbf{M}_{at,1} - (I_k \dot{\omega}_1 + \omega_1 \times (I_1 \omega_1)) \\ \sum \mathbf{F}_{at,2} - m_2 \mathbf{a}_{G_2} \\ \sum \mathbf{M}_{at,2} - (I_2 \dot{\omega}_2 + \omega_2 \times (I_2 \omega_2)) \\ \vdots \\ \sum \mathbf{F}_{at,N} - m_N \mathbf{a}_{G_N} \\ \sum \mathbf{M}_{at,N} - (I_N \dot{\omega}_N + \omega_N \times (I_N \omega_N)) \end{bmatrix}$$

Válida para mecanismos de CADEIA ABERTA !!



Análise Dinâmica: método de Gibbs-Appell

$$C_{mxn}{}^{T}\vec{e}_{nx1}=C_{mxn}{}^{T}D_{nx\lambda N}\boldsymbol{f}_{\boldsymbol{\lambda N x 1}}=\boldsymbol{0}_{mx1}$$

m = Mobilidade

Válida para mecanismos
de CADEIA FECHADA !!

Análise Dinâmica: método de Gibbs-Appell

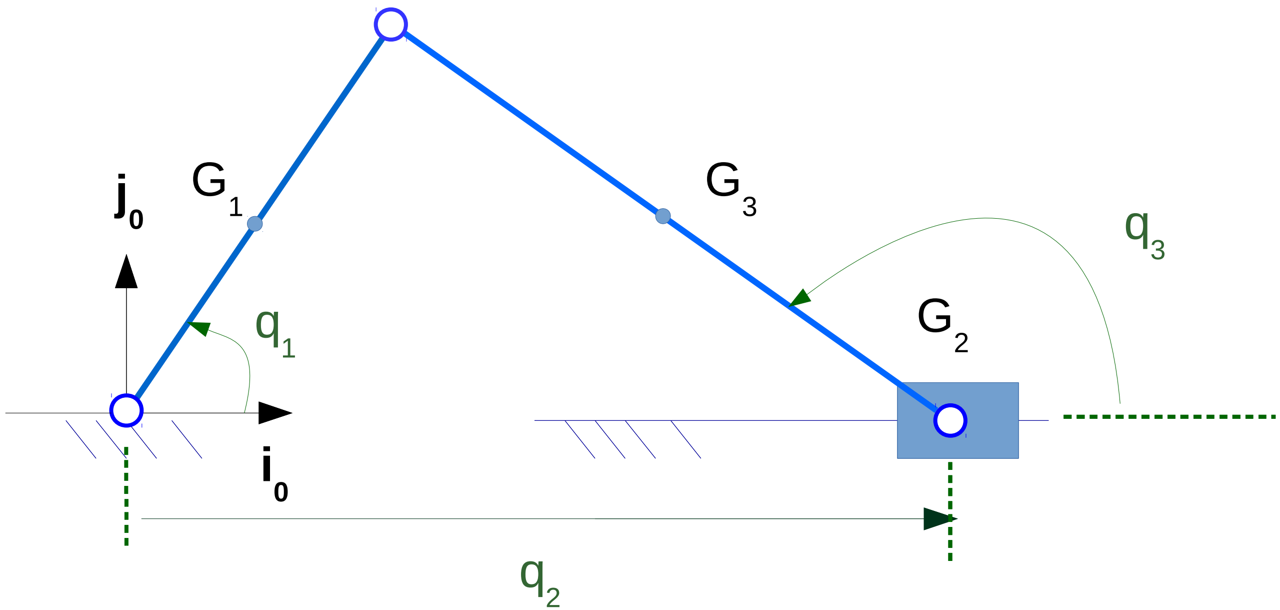
$$C_{mxn}^{T}\vec{e}_{nx1}=C_{mxn}^{T}D_{nx\lambda N}\boldsymbol{f}_{\boldsymbol{\lambda N x 1}}=\boldsymbol{0}_{mx1}$$

$$\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \\ u_{m+1} \\ \vdots \\ u_n \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{1}_{mxm} \\ A_{n-mxm} \end{bmatrix}}_{C_{nxm}} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix}$$

$$u_i=\dot{q}_i$$

Análise Dinâmica: método de Gibbs-Appell

$$C_{mxn}{}^{T}\vec{e}_{nx\,1}=C_{mxn}{}^{T}D_{nx\,\lambda\,N}\boldsymbol{f}_{\boldsymbol{\lambda}\,\boldsymbol{N}\,\boldsymbol{x}\,\boldsymbol{1}}=\boldsymbol{0}_{mx\,1}$$



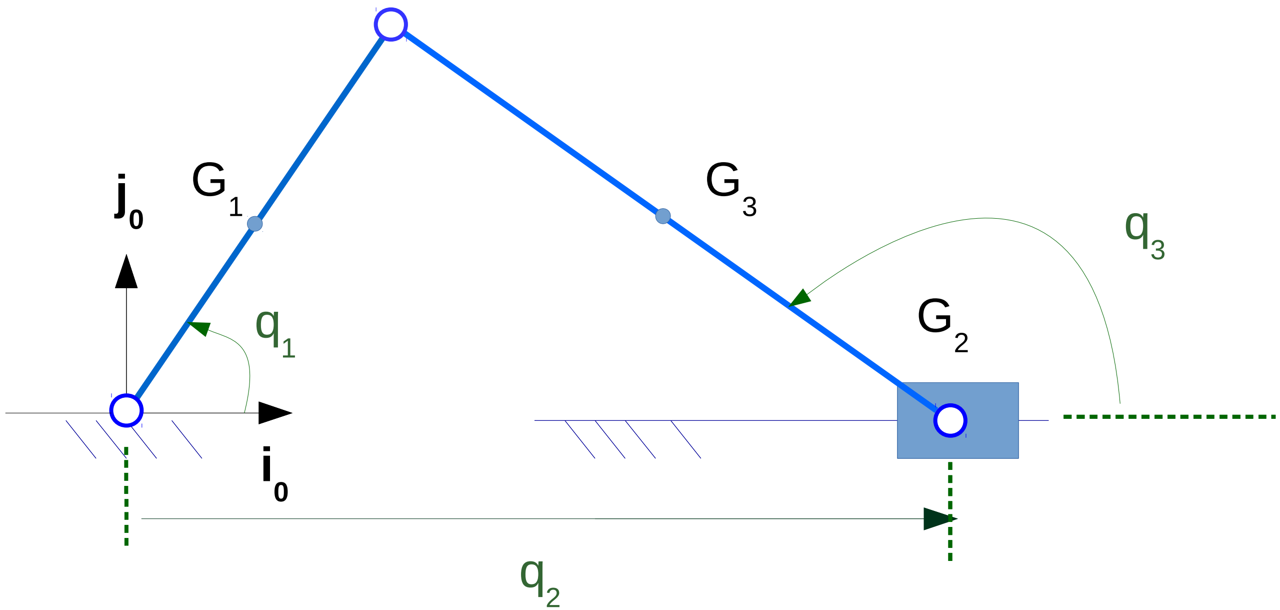
n = 3

N = 3

m = 1

Análise Dinâmica: método de Gibbs-Appell

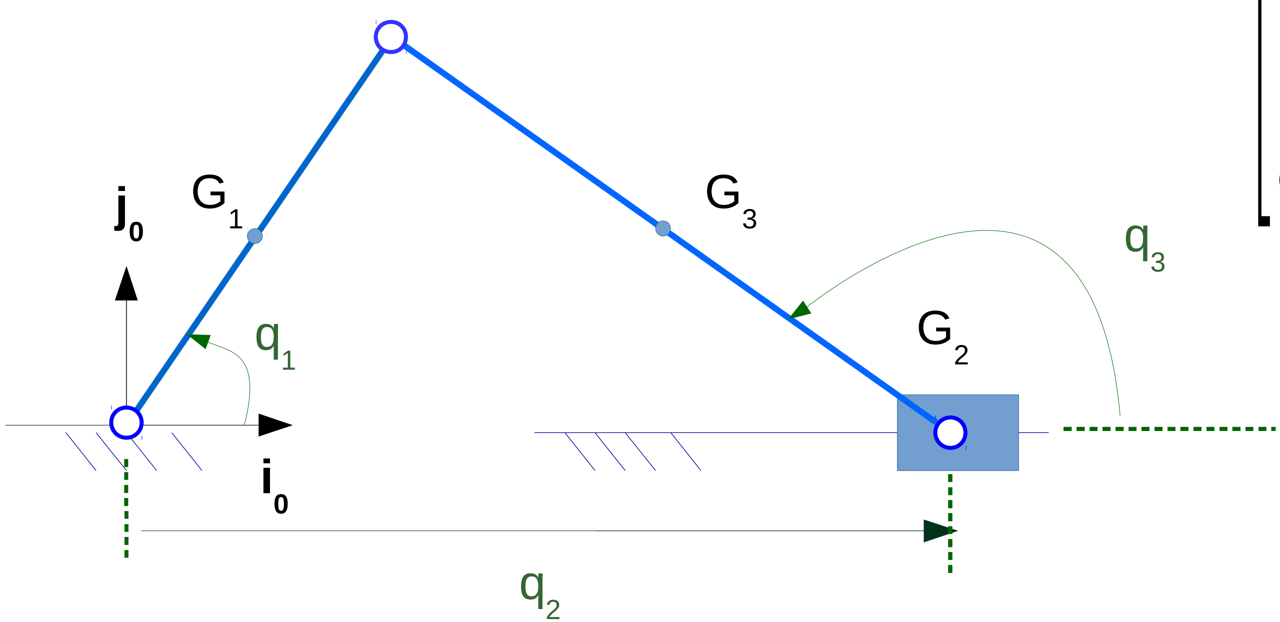
$$C_{mxn}{}^{T}\vec{e}_{nx\,1}=C_{mxn}{}^{T}D_{nx\,\lambda\,N}\boldsymbol{f}_{\boldsymbol{\lambda}\,\boldsymbol{N}\,\boldsymbol{x}\,\boldsymbol{1}}=\boldsymbol{0}_{mx\,1}$$



$C_{nxm} = C_{3x1}$

Análise Dinâmica: método de Gibbs-Appell

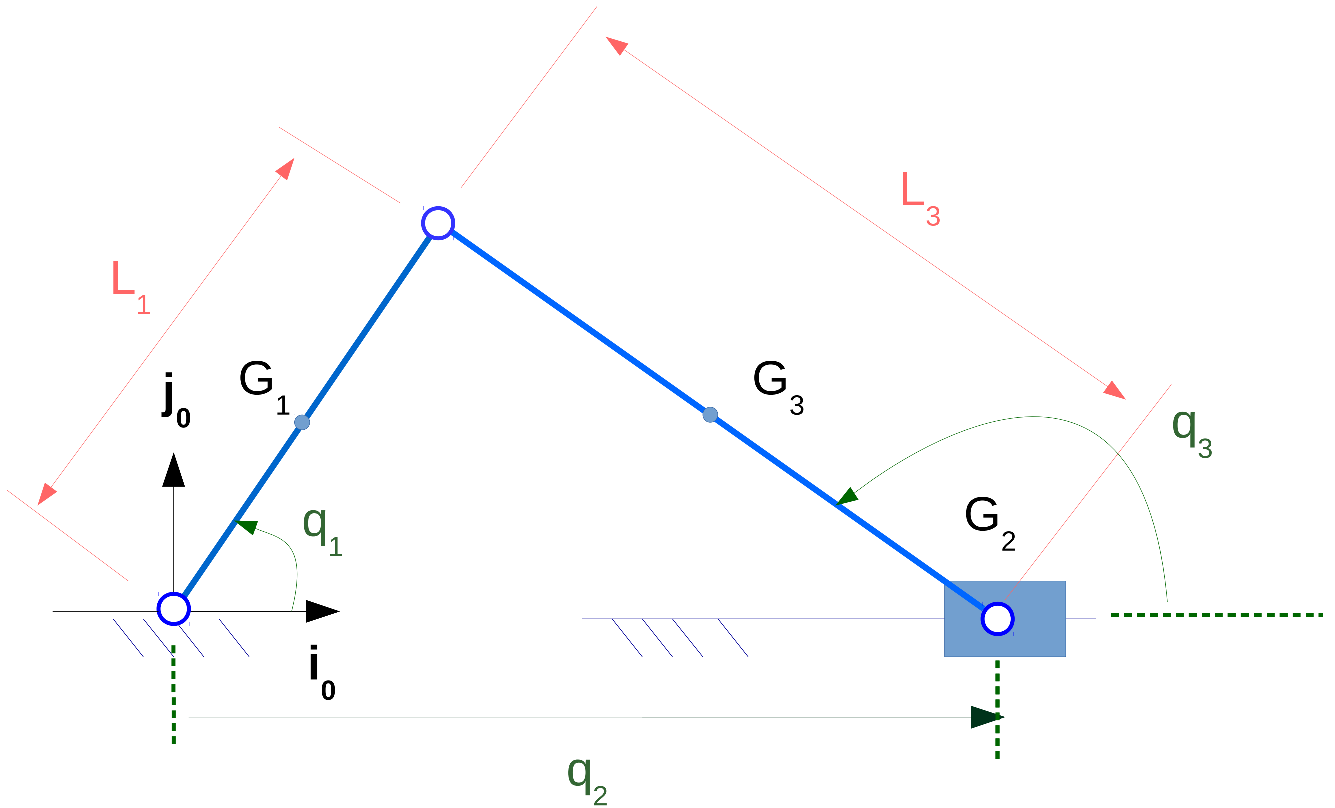
$$C_{mxn}^{\ T}\vec{e}_{nx1} = C_{mxn}^{\ T} D_{nx\lambda N}\boldsymbol{f}_{\boldsymbol{\lambda N x1}} = \boldsymbol{0}_{mx1}$$



$$\underbrace{\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\\ \dot{q}_3\end{bmatrix}=\begin{bmatrix}1\\ A_{2x1}\end{bmatrix}}_{\boldsymbol{C_{3x1}}}[\dot{q}_1]$$

Análise Dinâmica: método de Gibbs-Appell

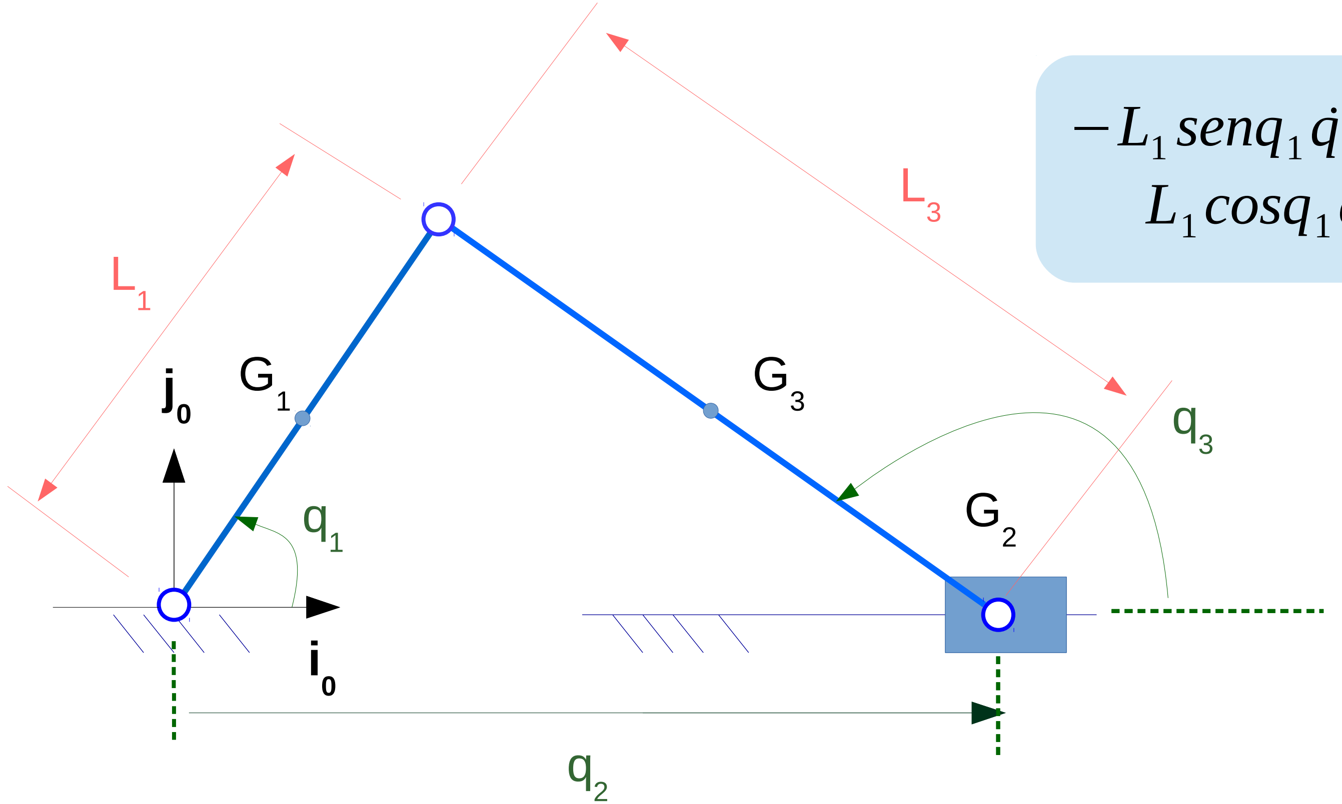
$$L_1 cosq_1 = q_2 + L_3 cosq_3$$
$$L_1 senq_1 = L_3 senq_3$$



Análise Dinâmica: método de Gibbs-Appell

$$L_1 cosq_1 = q_2 + L_3 cosq_3$$
$$L_1 senq_1 = L_3 senq_3$$

$$-L_1 senq_1 \dot{q}_1 = \dot{q}_2 - L_3 senq_3 \dot{q}_3$$
$$L_1 cosq_1 \dot{q}_1 = L_3 cosq_3 \dot{q}_3$$



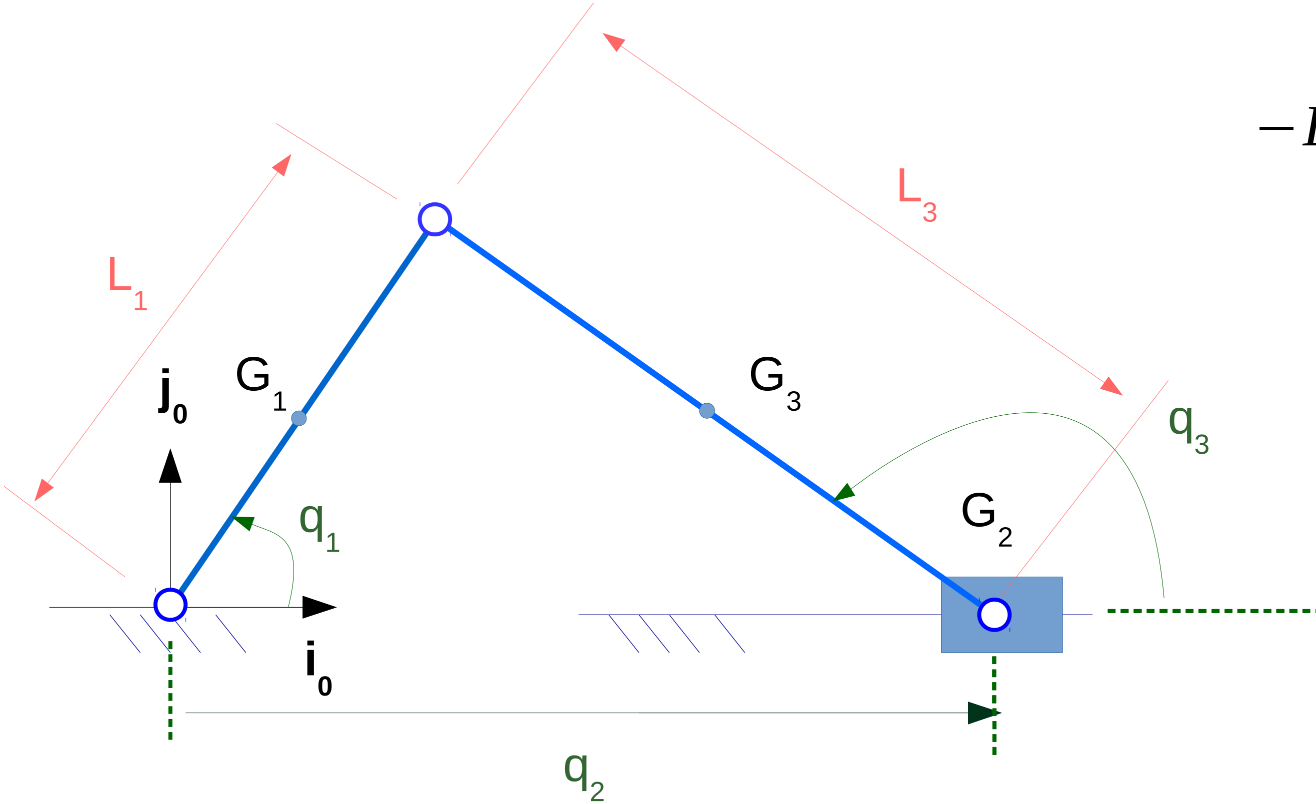
Análise Dinâmica: método de Gibbs-Appell

$$L_1 cosq_1 = q_2 + L_3 cosq_3$$
$$L_1 senq_1 = L_3 senq_3$$

$$\Downarrow$$

$$-L_1 senq_1 \dot{q}_1 = \dot{q}_2 - L_3 senq_3 \dot{q}_3$$
$$L_1 cosq_1 \dot{q}_1 = L_3 cosq_3 \dot{q}_3$$

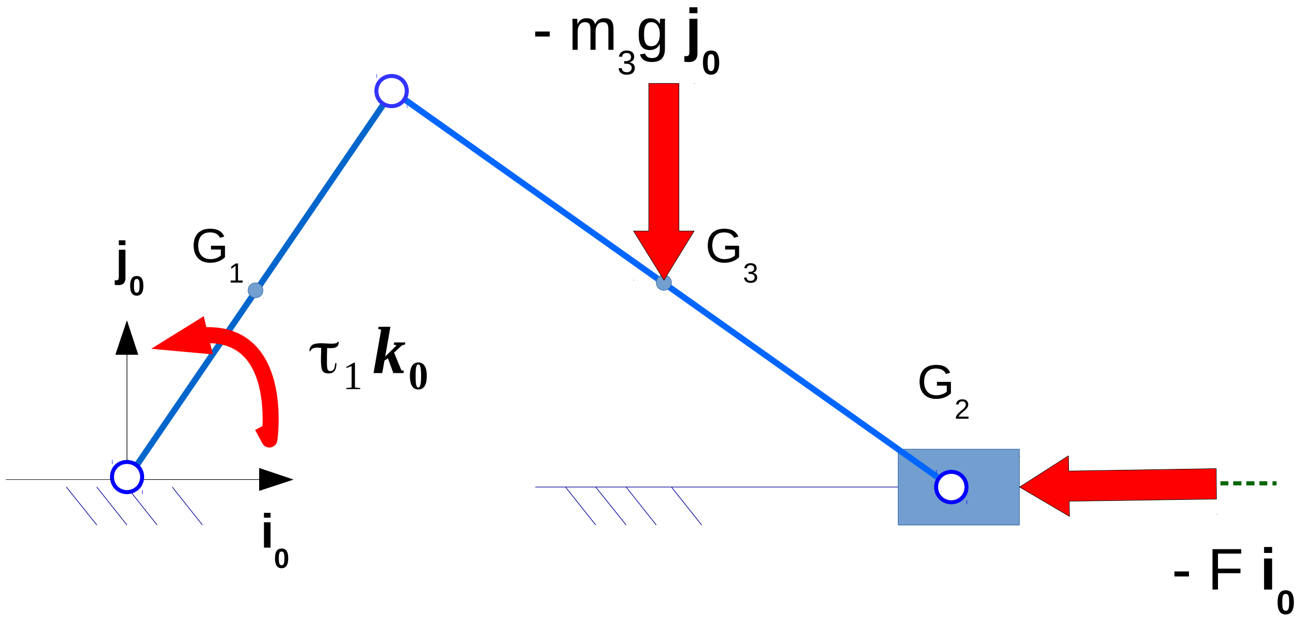
$$\Downarrow$$



$$\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\\ \dot{q}_3\end{bmatrix} = \begin{bmatrix} 1 \\ L_1 \dfrac{sen(q_3 - q_1)}{cosq_3} \\ \left(\dfrac{L_1}{L_3}\right)\left(\dfrac{cosq_1}{cosq_3}\right)\end{bmatrix} [\dot{q}_1]$$

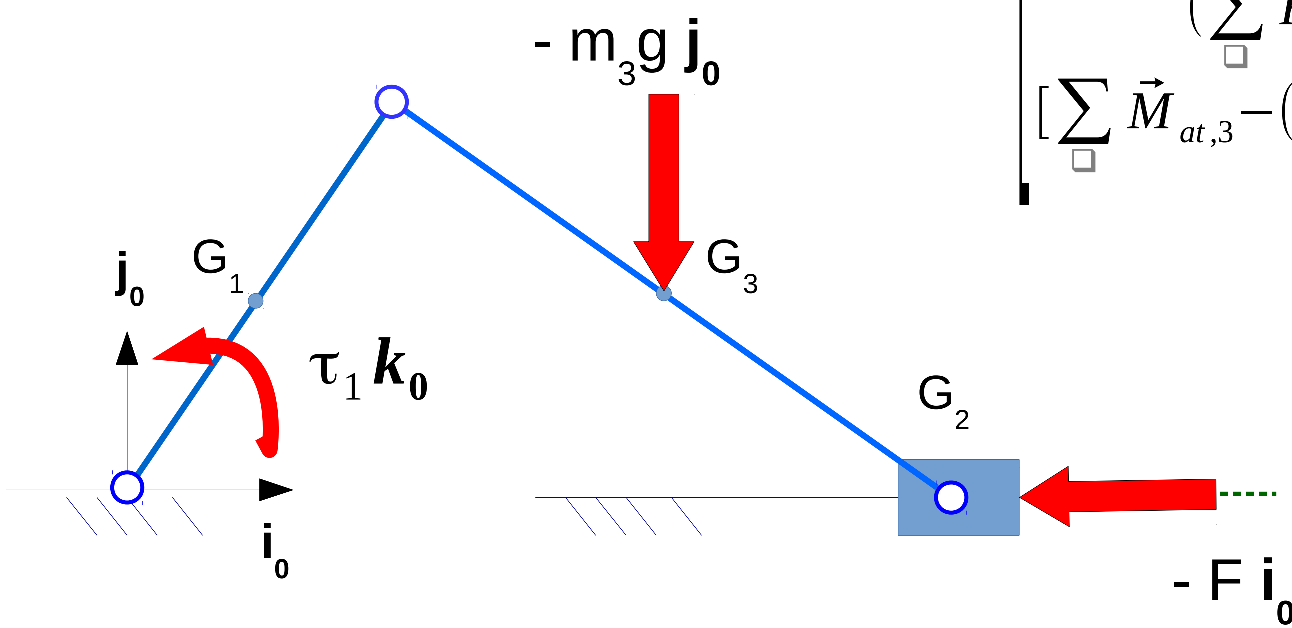
Análise Dinâmica: método de Gibbs-Appell

$$C_{1x3}{}^{T}\vec{e}_{3x1}=C_{1x3}{}^{T}D_{3x9}\boldsymbol{f}_{\mathbf{9x1}}=0_{1x1}$$



Análise Dinâmica: método de Gibbs-Appell

$$f_{9x1}=\begin{bmatrix} \left(\sum \vec{F}_{at,1}-m_1\vec{a}_{G_1}\right) \\ \left[\sum \vec{M}_{at,1}-\left(I_1\dot{\vec{\omega}}_1+\vec{\omega}_1\times\left(I_1\vec{\omega}_1\right)\right)\right] \\ \hline \left(\sum \vec{F}_{at,2}-m_2\vec{a}_{G_2}\right) \\ \left[\sum \vec{M}_{at,2}-\left(I_2\dot{\vec{\omega}}_2+\vec{\omega}_2\times\left(I_2\vec{\omega}_2\right)\right)\right] \\ \hline \left(\sum \vec{F}_{at,3}-m_3\vec{a}_{G_3}\right) \\ \left[\sum \vec{M}_{at,3}-\left(I_3\dot{\vec{\omega}}_3+\vec{\omega}_2\times\left(I_3\vec{\omega}_3\right)\right)\right] \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ \tau_1 \\ \hline -F \\ 0 \\ 0 \\ \hline 0 \\ -m_3 g \\ 0 \end{bmatrix}$$

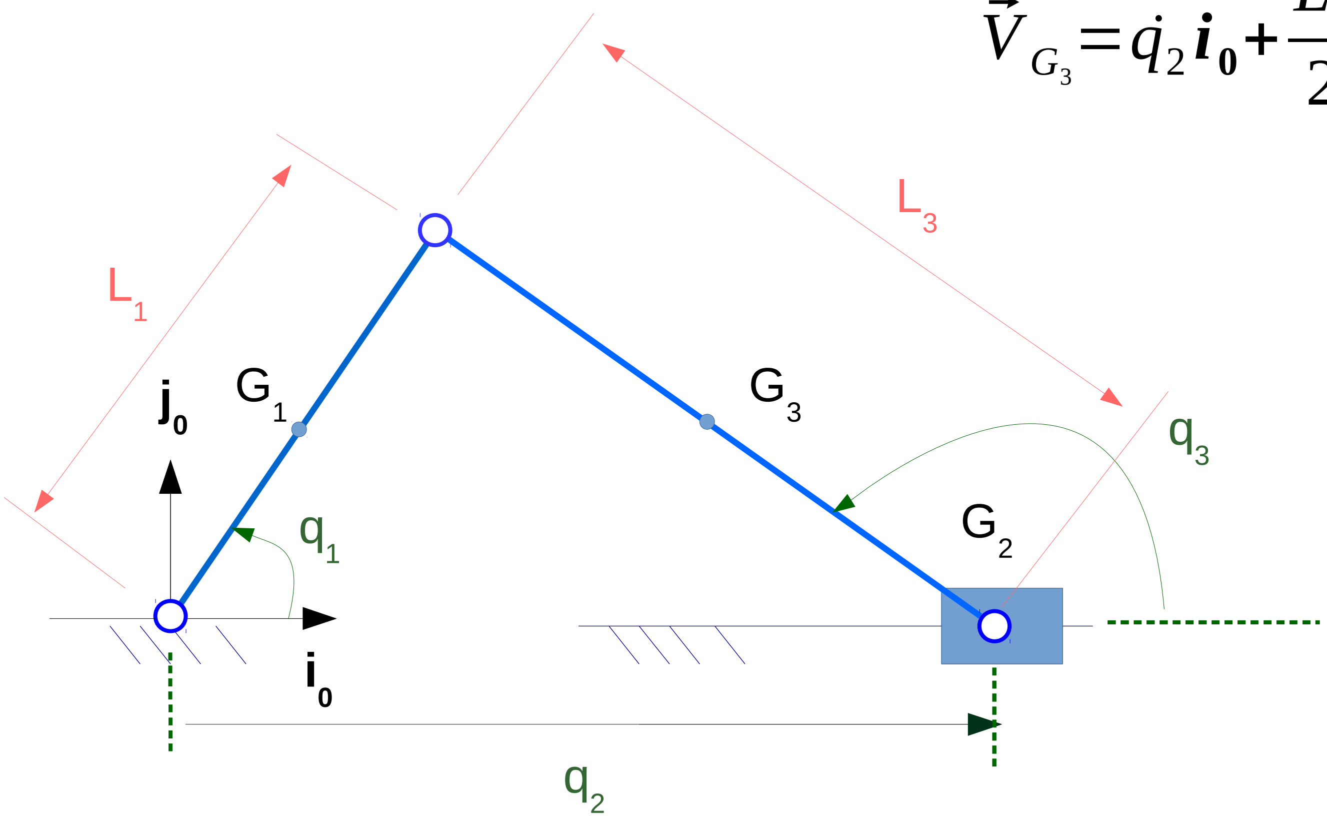


Análise Dinâmica: método de Gibbs-Appell

$$\vec{V}_{G_1}=\frac{L_1}{2}\dot{q}_1\left(-senq_1\,\boldsymbol{i_0}+cosq_1\,\boldsymbol{j_0}\right)$$

$$\vec{V}_{G_2}=\dot{q}_2\,\boldsymbol{i_0}$$

$$\vec{V}_{G_3}=\dot{q}_2\,\boldsymbol{i_0}+\frac{L_3}{2}\dot{q}_3\left(-senq_3\,\boldsymbol{i_0}+cosq_3\,\boldsymbol{j_0}\right)$$



$$\vec{\omega}_1=\dot{q}_1\,\boldsymbol{k_0}$$

$$\vec{\omega}_2=\boldsymbol{0}$$

$$\vec{\omega}_3=\dot{q}_3\,\boldsymbol{k_0}$$

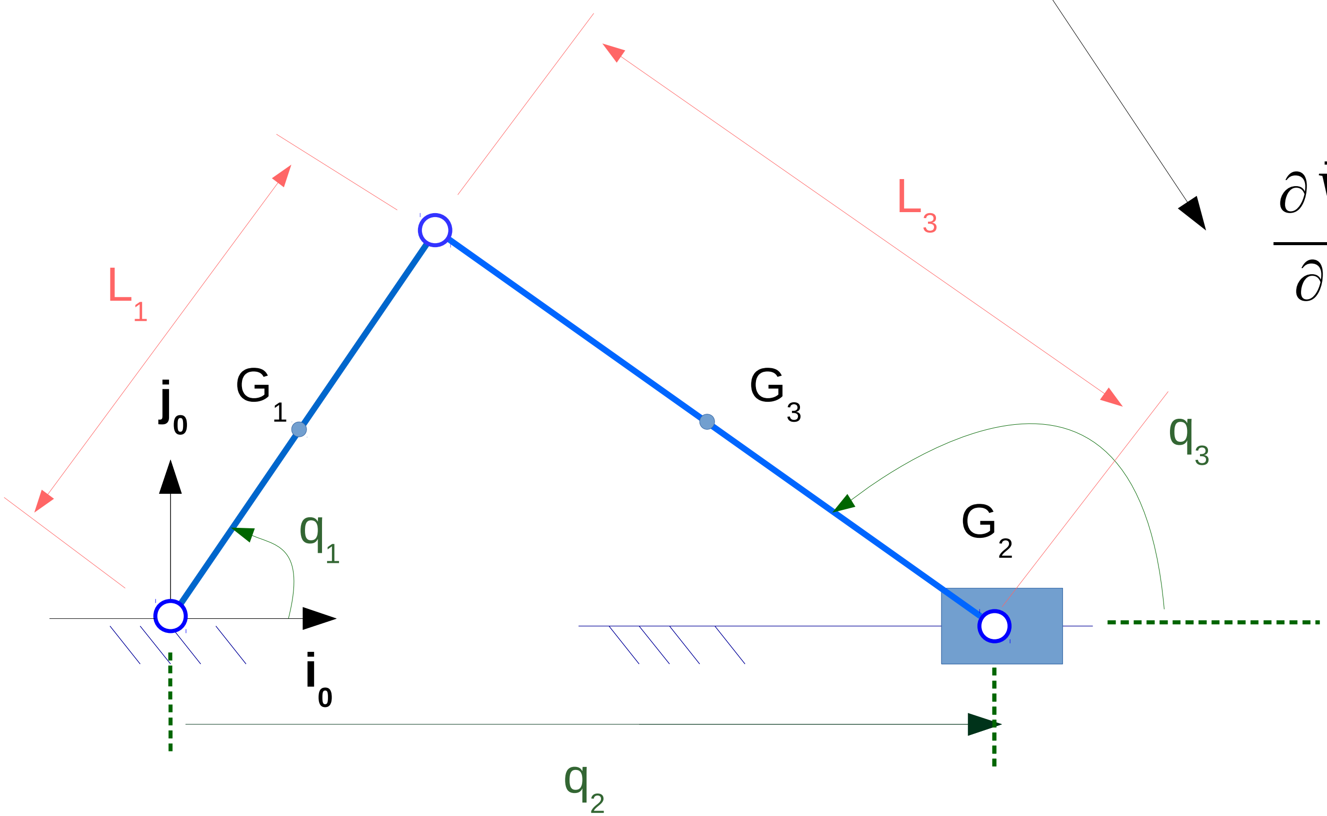
Análise Dinâmica: método de Gibbs-Appell

$$\vec{V}_{G_1} = \frac{L_1}{2}\dot{q}_1(-senq_1\boldsymbol{i_0} + cosq_1\boldsymbol{j_0})$$

$$\frac{\partial \vec{V}_{G_1}}{\partial \dot{q}_1} = \frac{L_1}{2}(-senq_1\boldsymbol{i_0} + cosq_1\boldsymbol{j_0})$$

$$\frac{\partial \vec{V}_{G_1}}{\partial \dot{q}_2} = \boldsymbol{0}$$

$$\frac{\partial \vec{V}_{G_1}}{\partial \dot{q}_3} = \boldsymbol{0}$$



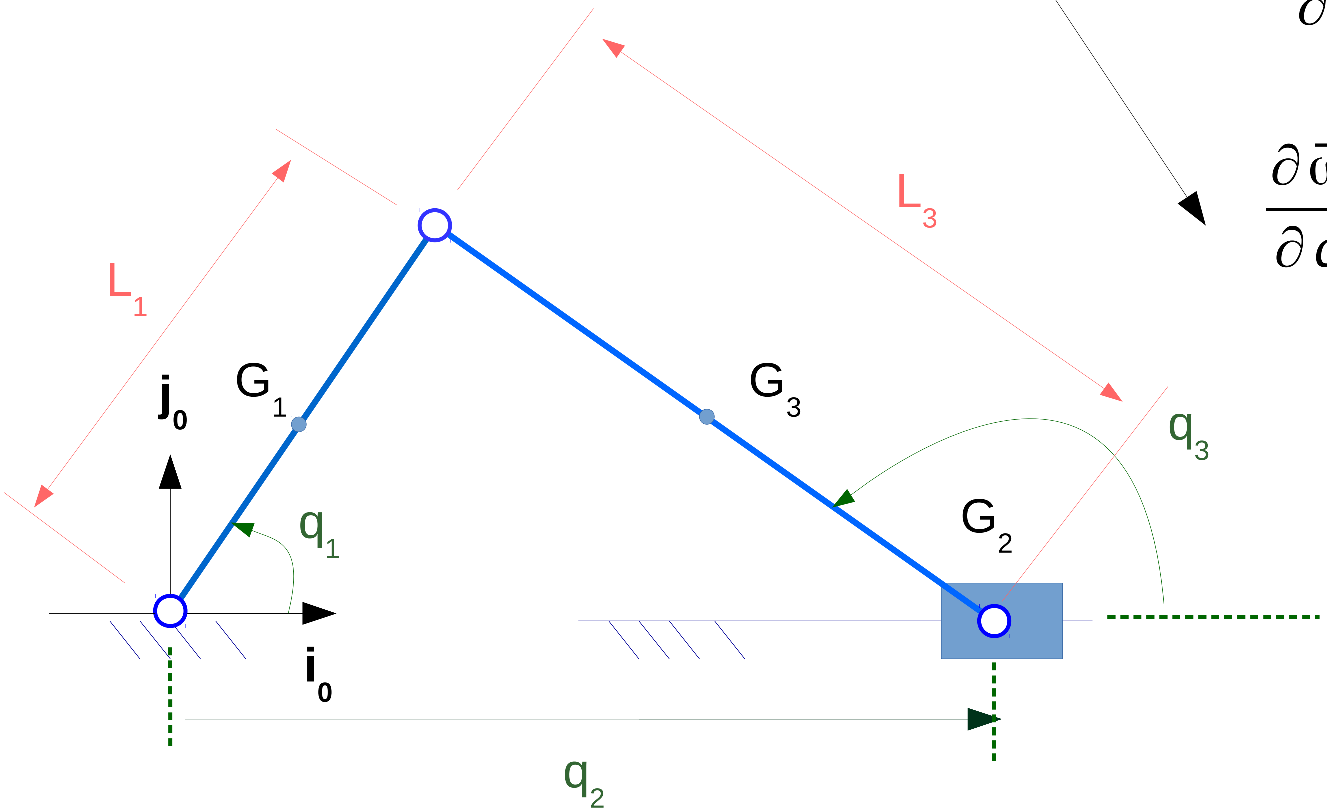
Análise Dinâmica: método de Gibbs-Appell

$$\vec{\omega}_1 = \dot{q}_1 \boldsymbol{k_0}$$

$$\frac{\partial \vec{\omega}_1}{\partial \dot{q}_1} = \boldsymbol{k_0}$$

$$\frac{\partial \vec{\omega}_1}{\partial \dot{q}_2} = \mathbf{0}$$

$$\frac{\partial \vec{\omega}_1}{\partial \dot{q}_3} = \mathbf{0}$$



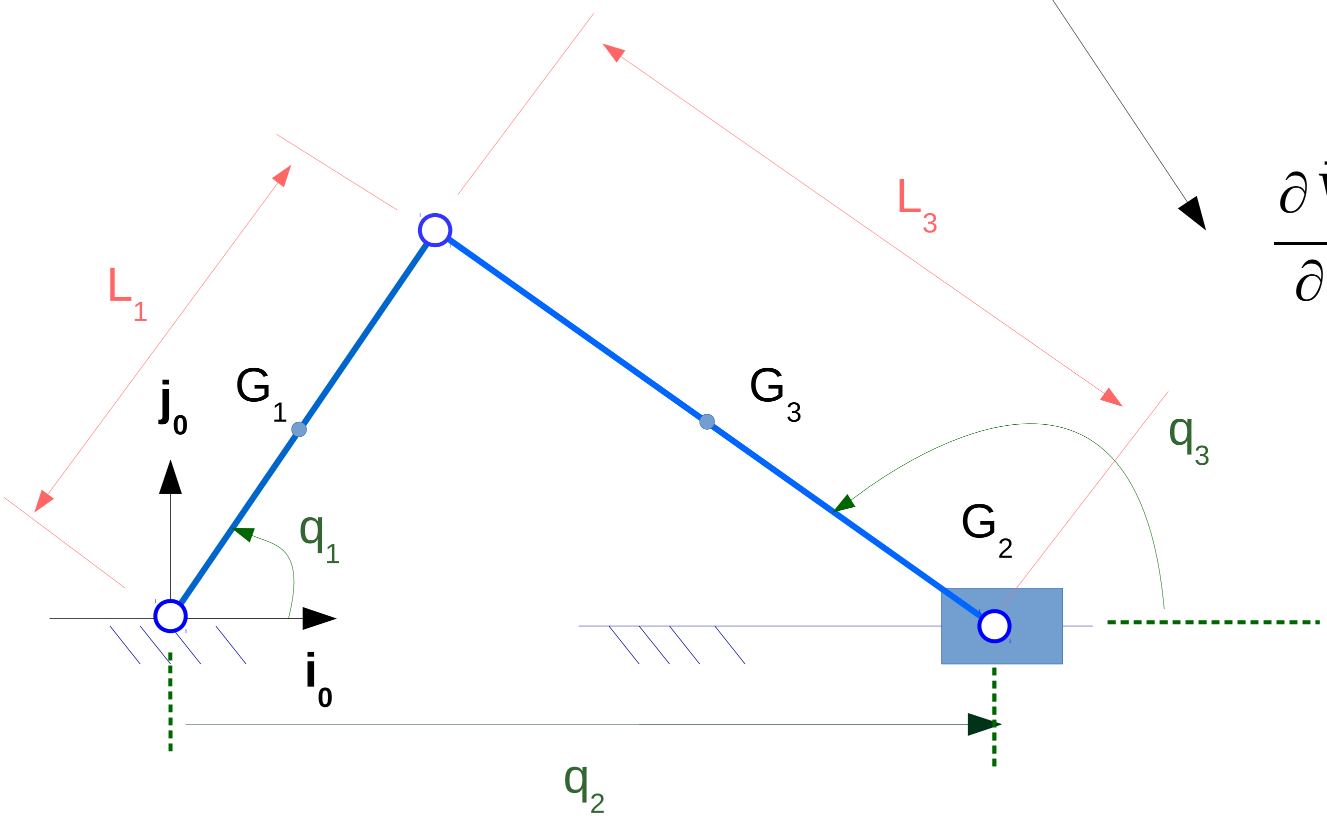
Análise Dinâmica: método de Gibbs-Appell

$$\vec{V}_{G_2} = \dot{q}_2 \boldsymbol{i_0}$$

$$\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_1} = \boldsymbol{0}$$

$$\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_2} = \boldsymbol{i_0}$$

$$\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_3} = \boldsymbol{0}$$



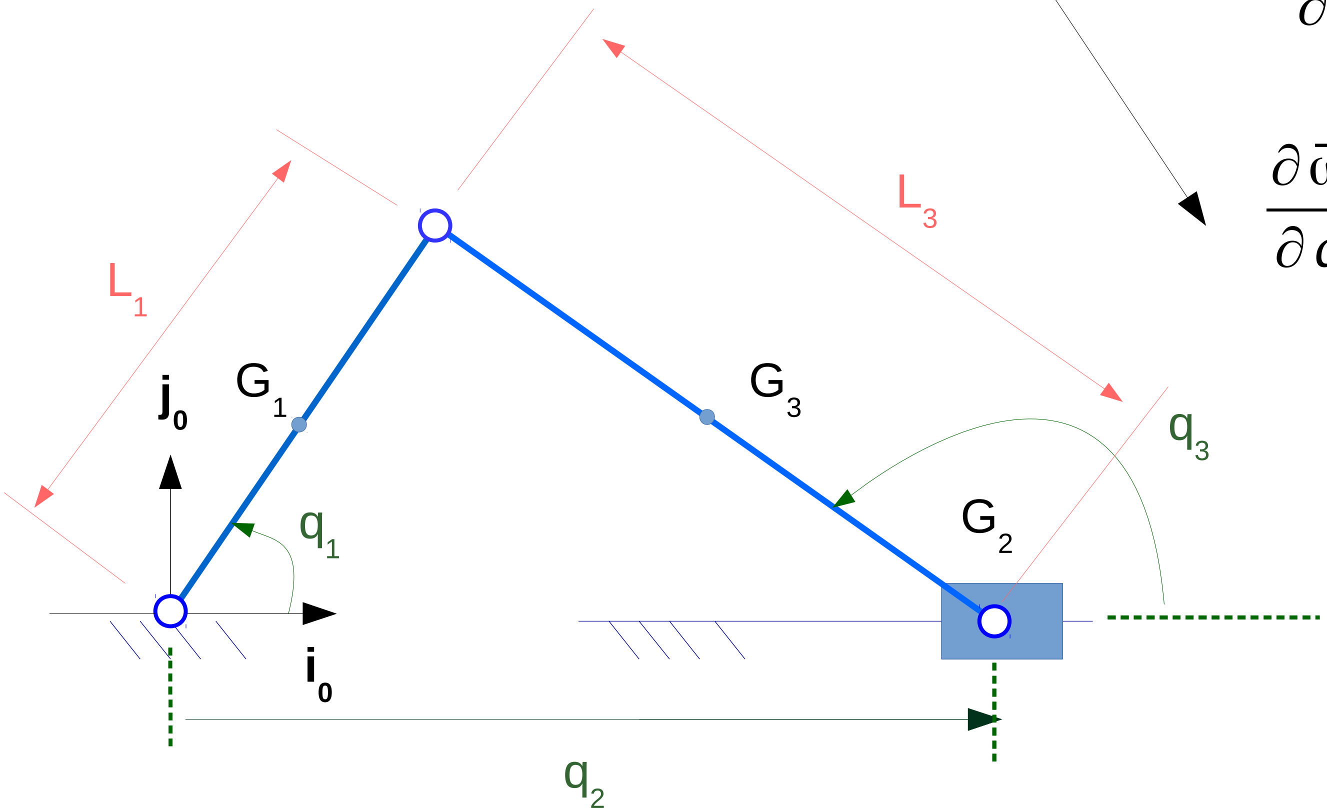
Análise Dinâmica: método de Gibbs-Appell

$$\vec{\omega}_2 = \mathbf{0}$$

$$\frac{\partial \vec{\omega}_2}{\partial \dot{q}_1} = \mathbf{0}$$

$$\frac{\partial \vec{\omega}_2}{\partial \dot{q}_2} = \mathbf{0}$$

$$\frac{\partial \vec{\omega}_2}{\partial \dot{q}_3} = \mathbf{0}$$



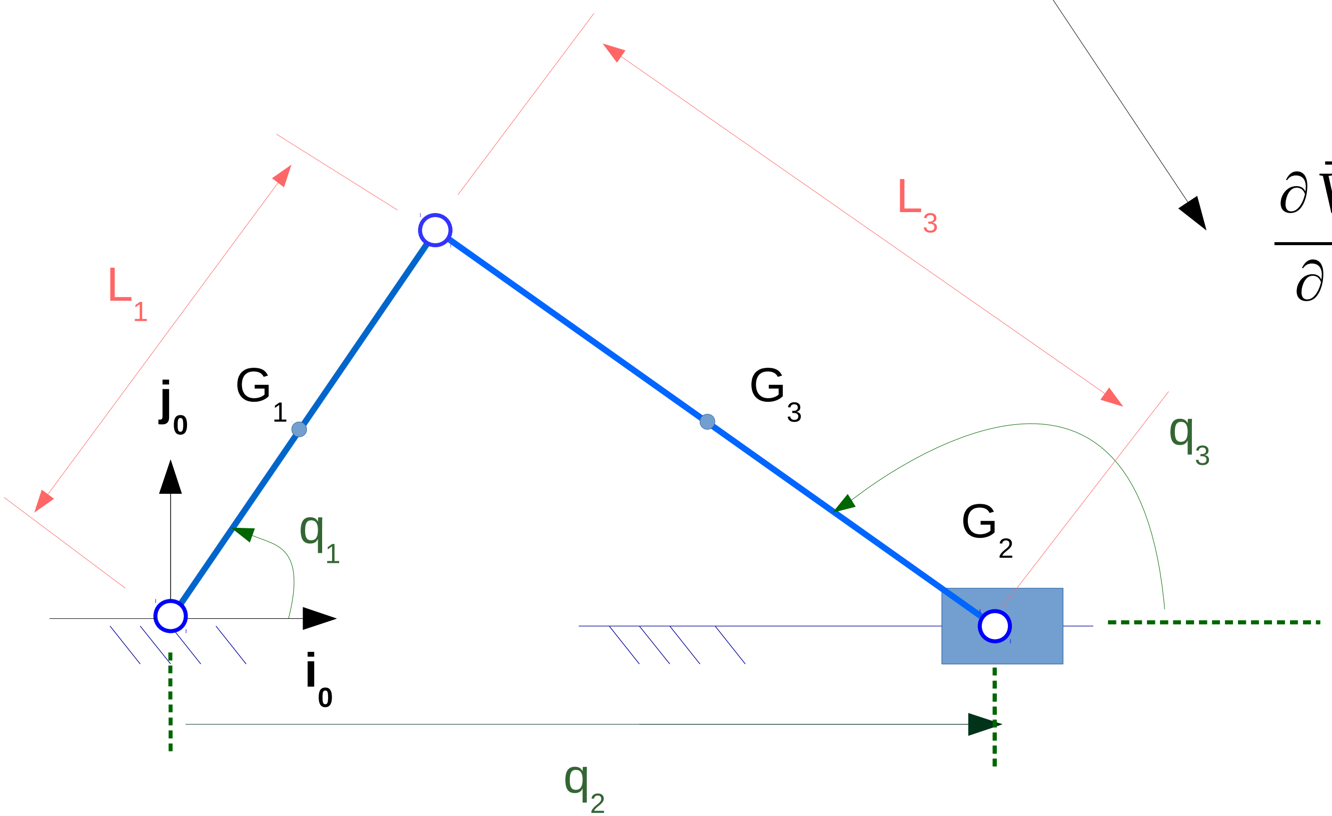
Análise Dinâmica: método de Gibbs-Appell

$$\vec{V}_{G_3} = \dot{q}_2 \boldsymbol{i_0} + \frac{L_3}{2} \dot{q}_3 \left(-senq_3 \boldsymbol{i_0} + cosq_3 \boldsymbol{j_0}\right)$$

$$\frac{\partial \vec{V}_{G_3}}{\partial \dot{q}_1} = \mathbf{0}$$

$$\frac{\partial \vec{V}_{G_3}}{\partial \dot{q}_2} = \boldsymbol{i_0}$$

$$\frac{\partial \vec{V}_{G_3}}{\partial \dot{q}_3} = \frac{L_3}{2} \left(-senq_3 \boldsymbol{i_0} + cosq_3 \boldsymbol{j_0}\right)$$



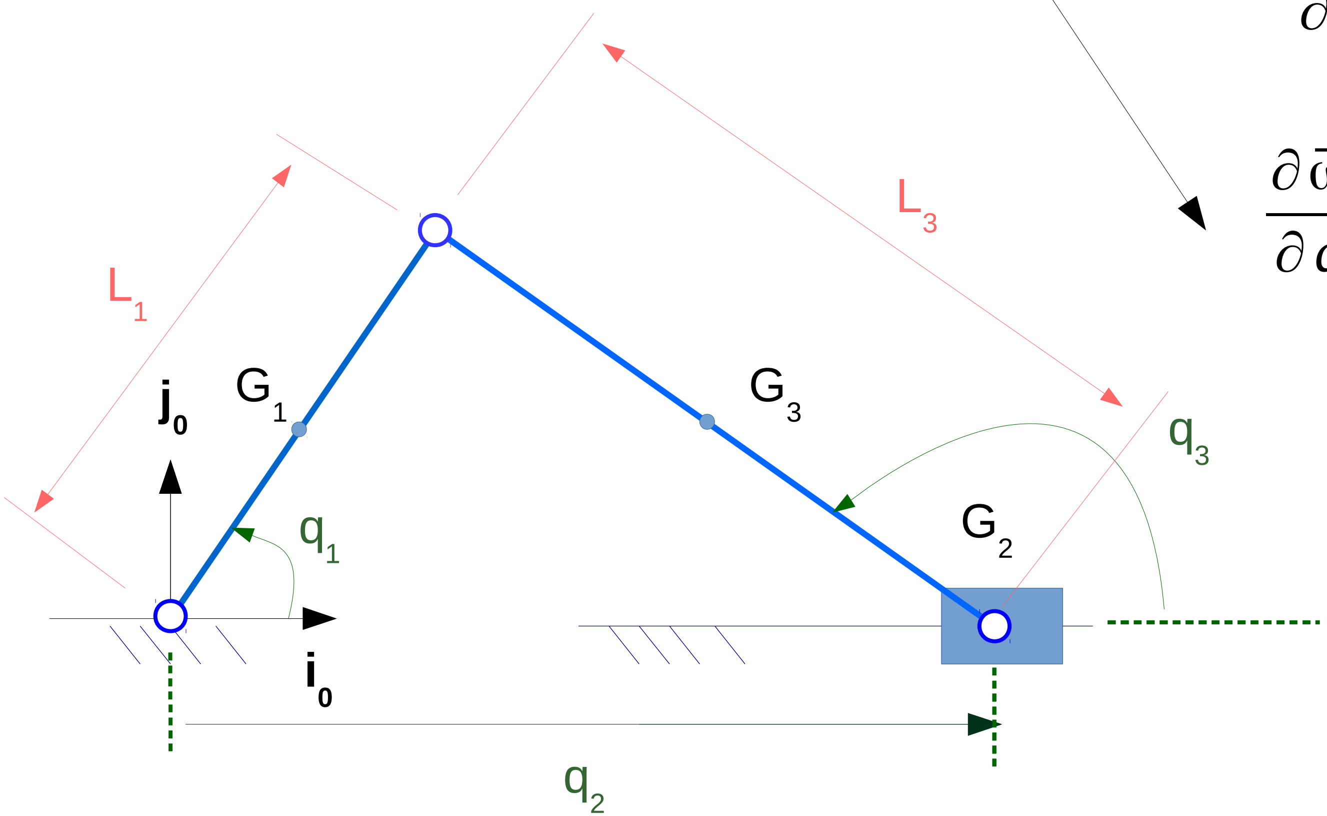
Análise Dinâmica: método de Gibbs-Appell

$$\vec{\omega}_3 = \dot{q}_3 \boldsymbol{k}_{\mathbf{0}}$$

$$\frac{\partial \vec{\omega}_3}{\partial \dot{q}_1} = \mathbf{0}$$

$$\frac{\partial \vec{\omega}_3}{\partial \dot{q}_2} = \mathbf{0}$$

$$\frac{\partial \vec{\omega}_3}{\partial \dot{q}_3} = \boldsymbol{k}_{\mathbf{0}}$$



Análise Dinâmica: método de Gibbs-Appell

$$D_{3x9}=\left[\begin{array}{ccc|ccc|ccc} -\frac{L_1}{2}senq_1 & \frac{L_1}{2}cosq_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{L_3}{2}senq_3 & \frac{L_3}{2}cosq_3 & 1 \end{array}\right]$$

Análise Dinâmica: método de Gibbs-Appell

$$\begin{bmatrix} -\frac{L_1}{2} senq_1 & \frac{L_1}{2} cosq_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{L_3}{2} senq_3 & \frac{L_3}{2} cosq_3 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \tau_1 \\ -F \\ 0 \\ 0 \\ 0 \\ -m_3 g \\ 0 \end{bmatrix}$$

$$D\boldsymbol{f} = \begin{bmatrix} \tau_1 \\ -F \\ -\left(m_3 \frac{L_3}{2} g\right) cosq_3 \end{bmatrix}$$

Análise Dinâmica: método de Gibbs-Appell

$$C^T D\boldsymbol{f} = \begin{bmatrix} 1 & L_1 \dfrac{sen(q_3 - q_1)}{cosq_3} & (\dfrac{L_1}{L_3})(\dfrac{cosq_1}{cosq_3}) \end{bmatrix} \begin{bmatrix} \tau_1 \\ -F \\ -(m_3 \dfrac{L_3}{2} g) cosq_3 \end{bmatrix} = 0$$

$$C^T D\boldsymbol{f} = \tau_1 - F L_1 \frac{sen(q_3 - q_1)}{cosq_3} - (m_3 \frac{L_3}{2} g) cosq_3 (\frac{L_1}{L_3})(\frac{cosq_1}{cosq_3}) = 0$$

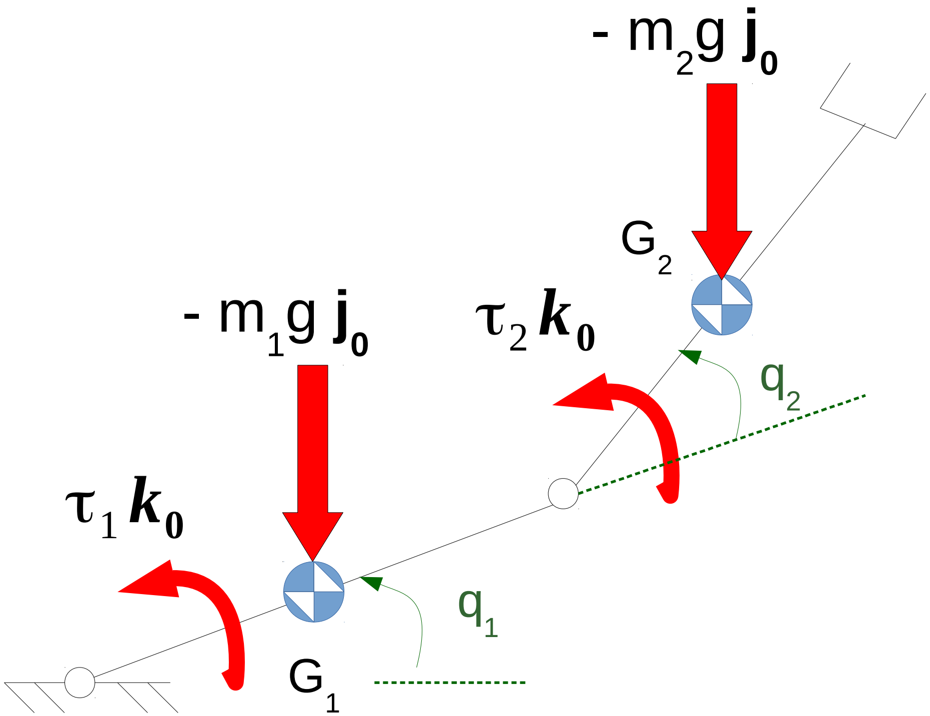
Análise Dinâmica: método de Gibbs-Appell

$$C^T D \boldsymbol{f} = \tau_1 - F L_1 \frac{sen(q_3 - q_1)}{cos q_3} - (m_3 \frac{L_3}{2} g) cos q_3 (\frac{L_1}{L_3})(\frac{cos q_1}{cos q_3}) = 0$$

$$C^T D \boldsymbol{f} = \tau_1 - F L_1 \frac{sen(q_3 - q_1)}{cos q_3} - (m_3 \frac{L_1}{2} g) cos q_1 = 0$$

$$\boxed{\tau_1 = F L_1 \frac{sen(q_3 - q_1)}{cos q_3} + (m_3 \frac{L_1}{2} g) cos q_1}$$

Análise Dinâmica: método de Gibbs-Appell



$\lambda = 3$

$N = 2, \;\; M = 2$

$n = 6 \;\; (\omega_1, \;\; \omega_2, V_{G1x}, V_{G1y}, V_{G2x}, V_{G2y})$

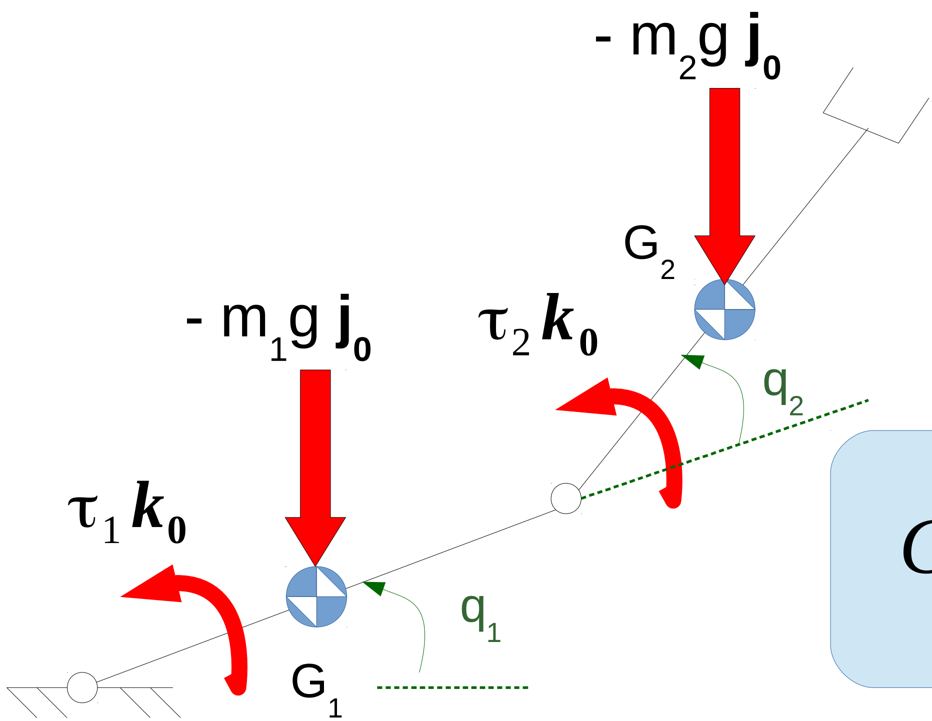
Redundância

Análise Dinâmica: método de Gibbs-Appell

$\lambda = 3$

$N = 2, \quad M = m = 2$

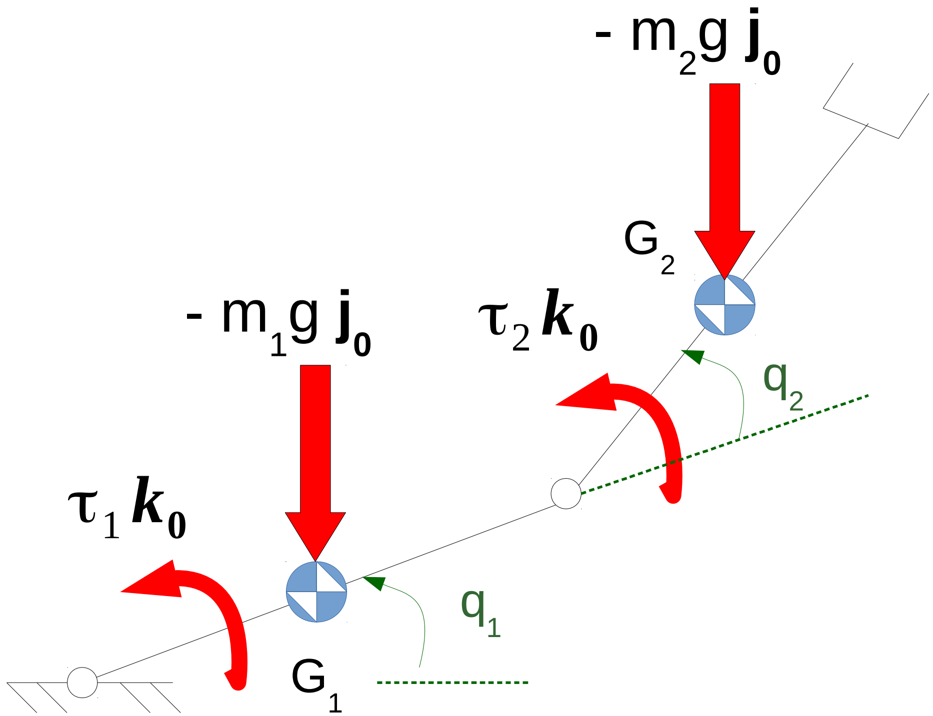
$n = 6 \; (\omega_1, \; \omega_2, V_{G1x}, V_{G1y}, V_{G2x}, V_{G2y})$



$$C_{2x6}^{\;T} \vec{e}_{6x1} = C_{2x6}^{\;T} D_{6x6} \boldsymbol{f}_{\mathbf{6x1}} = 0_{2x1}$$

Análise Dinâmica: método de Gibbs-Appell

$$C_{2x6}^{T}\vec{e}_{6x1}=C_{2x6}^{T}D_{6x6}\boldsymbol{f}_{\boldsymbol{6x1}}=0_{2x1}$$



$$\begin{bmatrix}\omega_1\\ \omega_2\\ V_{G1x}\\ V_{G1y}\\ V_{G2x}\\ V_{G2y}\end{bmatrix}=\overbrace{\begin{bmatrix}1 & 0\\ 0 & 1\\ -L_1 s_1 & 0\\ -L_1 c_1 & 0\\ -L_1 s_1 & -L_2 s_{12}\\ L_1 c_1 & L_2 c_{12}\end{bmatrix}}^{C_{6x2}}\begin{bmatrix}\omega_1\\ \omega_2\end{bmatrix}$$

$$\begin{bmatrix}\omega_1\\ \omega_2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix}$$

Análise Dinâmica: método de Gibbs-Appell

$$C_{2x6}{}^{T}\vec{e}_{6x1}=C_{2x6}{}^{T}D_{6x6}\boldsymbol{f}_{\boldsymbol{6x1}}=0_{2x1}$$

$$\boldsymbol{f}_{\boldsymbol{6x1}}=\begin{bmatrix} -m_1\dot{V}_{G1x} \\ -m_1\dot{V}_{G1y}-m_1 g \\ \tau_1-\tau_2-I_1\dot{\omega}_1 \\ -m_2\dot{V}_{G2x} \\ -m_2\dot{V}_{G2y}-m_2 g \\ \tau_2-I_2\dot{\omega}_2 \end{bmatrix}$$

Análise Dinâmica: método de Gibbs-Appell

$$\begin{array}{cc}
 & \begin{matrix} \dfrac{\partial \vec{V}_{G_1}}{\partial u_i} & \dfrac{\partial \vec{\omega}_1}{\partial u_i} & \dfrac{\partial \vec{V}_{G_2}}{\partial u_i} & \dfrac{\partial \vec{\omega}_2}{\partial u_i} \end{matrix} \\
D_{6x6} = & \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{matrix} \omega_1 \\ \omega_2 \\ V_{G1x} \\ V_{G1y} \\ V_{G2x} \\ V_{G2y} \end{matrix}
\end{array}$$

Elo 1   Elo 2

u_i

Análise Dinâmica: método de Gibbs-Appell

$$C_{2x6}^{T}\vec{e}_{6x1}=C_{2x6}^{T}D_{6x6}\boldsymbol{f}_{\boldsymbol{6x1}}=0_{2x1}$$

$$\begin{bmatrix}\tau_1-\tau_2-I_1\dot{\omega}_1+L_1 s_1 m_1\dot{V}_{G1x}-L_1 c_1\left(m_1\dot{V}_{G1y}+m_1 g\right)+L_1 s_1 m_2\dot{V}_{G2x}-L_1 c_1\left(m_2\dot{V}_{G2y}+m_2 g\right)\\ \tau_2-I_2\dot{\omega}_2+L_2 s_{12} m_2\dot{V}_{G2x}-L_2 c_{12}\left(m_2\dot{V}_{G2y}+m_2 g\right)\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$