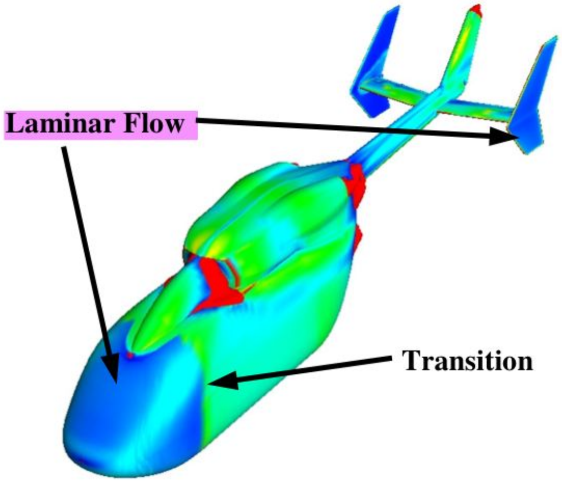
The phenomenon of bypass transition and its modeling

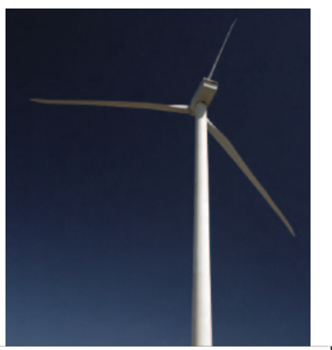
Paul Durbin

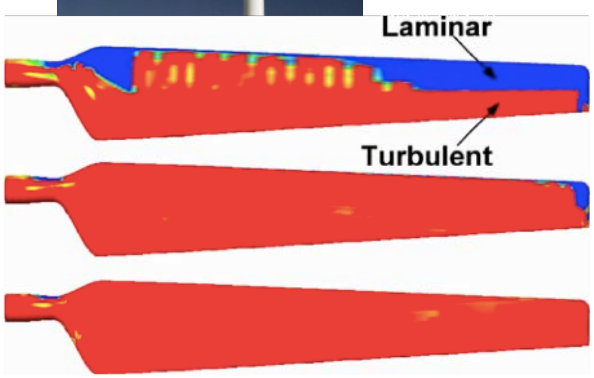
Aerospace Engineering
Iowa State University

Transition affects performance, aerodynamics, mixing, heat transfer

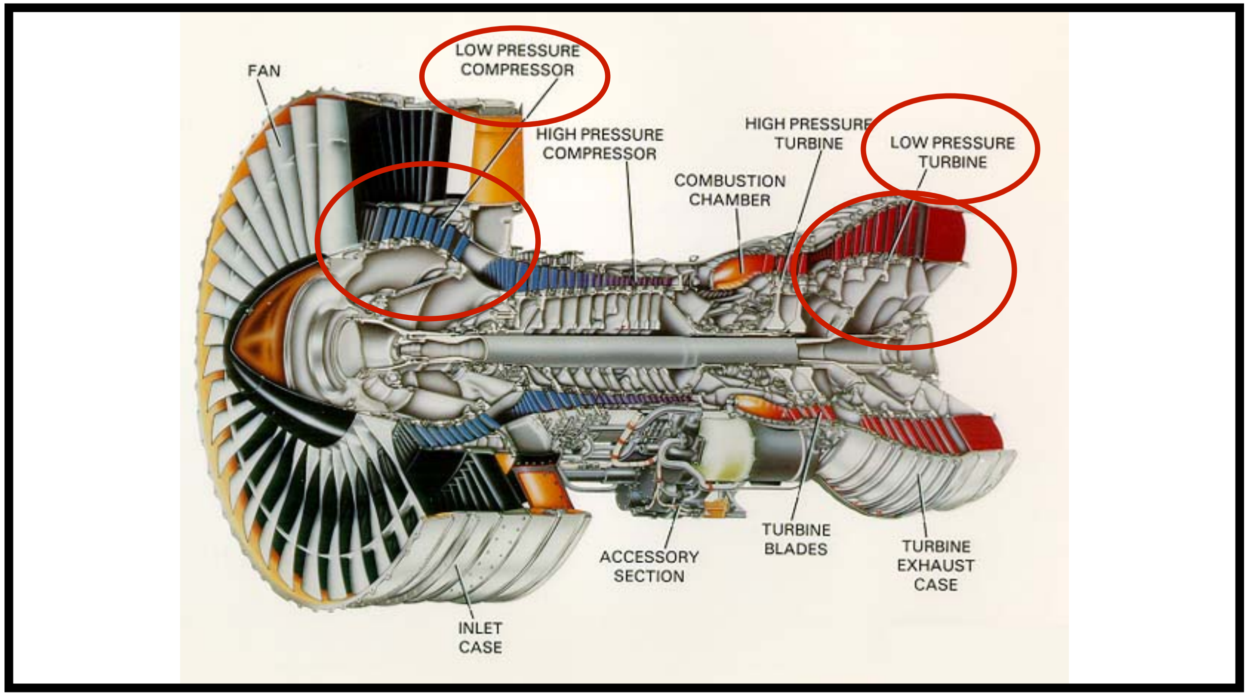


From Lantry-Menter

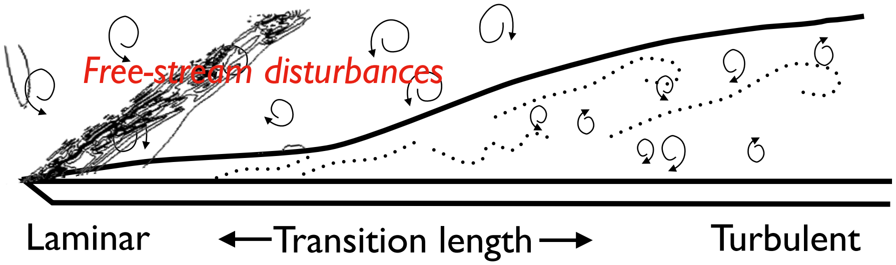




Engine Blades

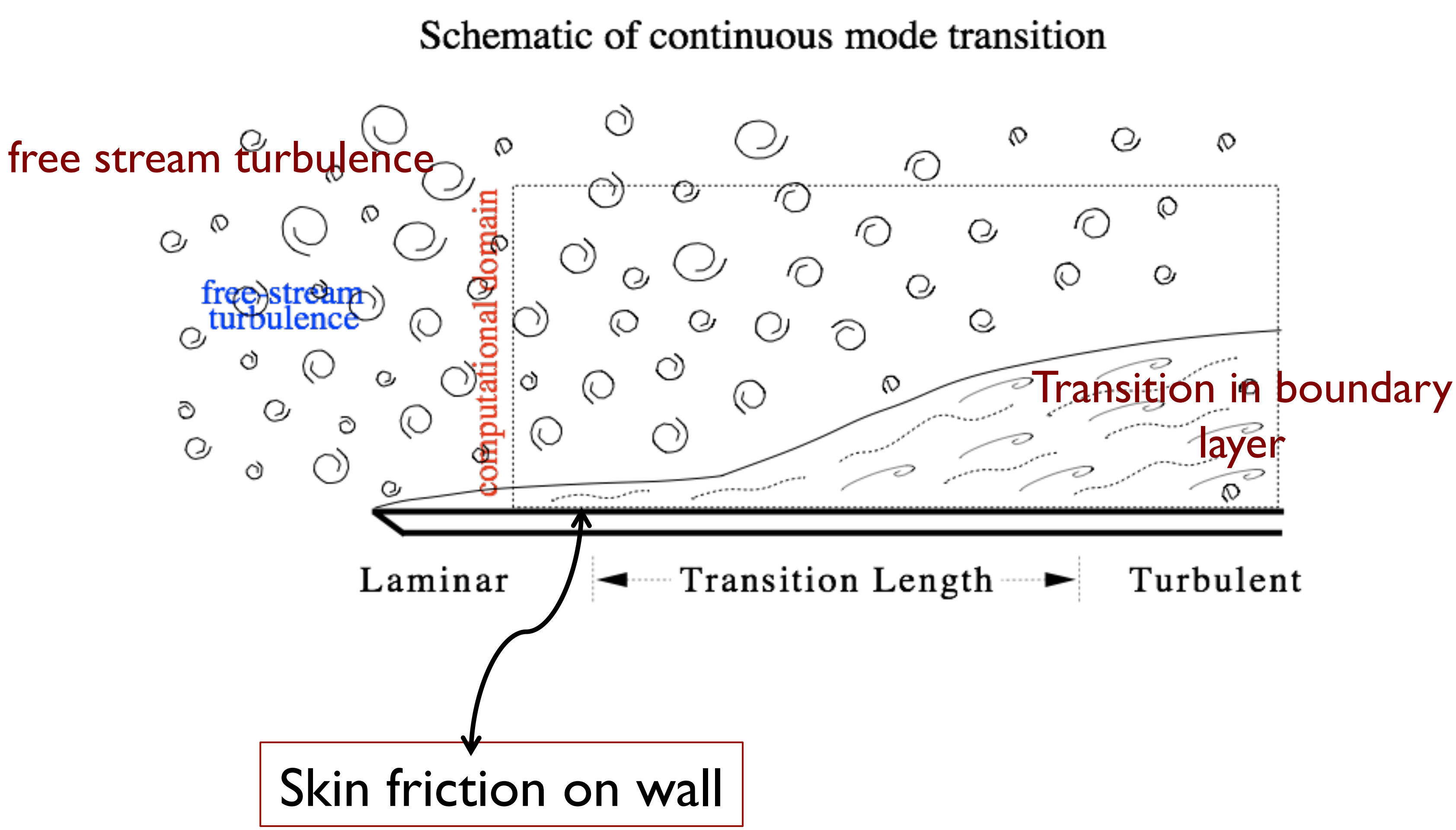


Considerations

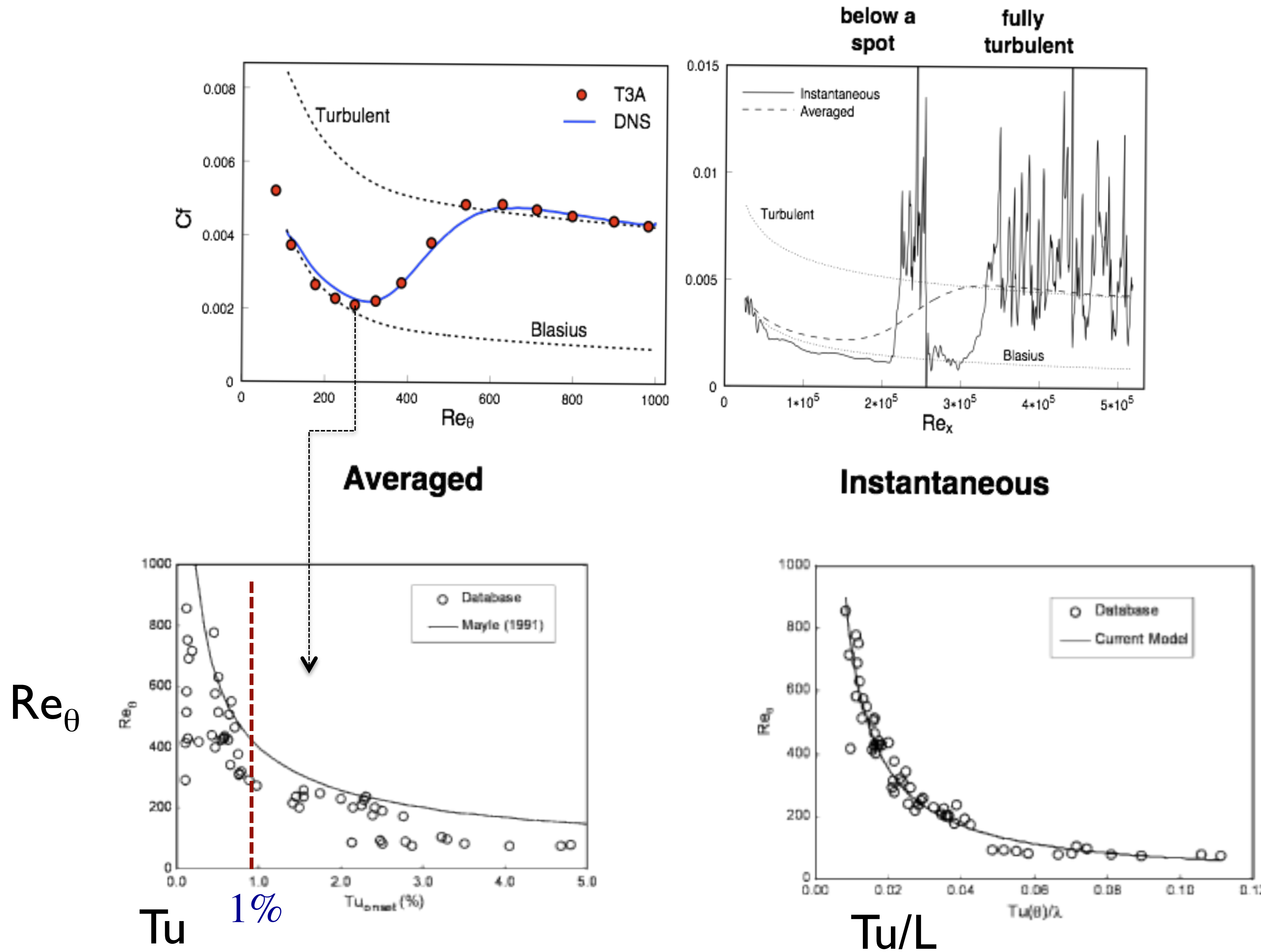


- Transition refers to development of *small-scale*, *self-sustained* turbulence within the boundary layer
- *Free-stream* disturbances (turbulence; wakes) are large scale and have little direct effect on skin friction, heat transfer
- External disturbances diffuse into the boundary layer, create low frequency perturbations that break down to turbulence
- Transition occurs over an extended length
- Practical modeling represents averaged properties

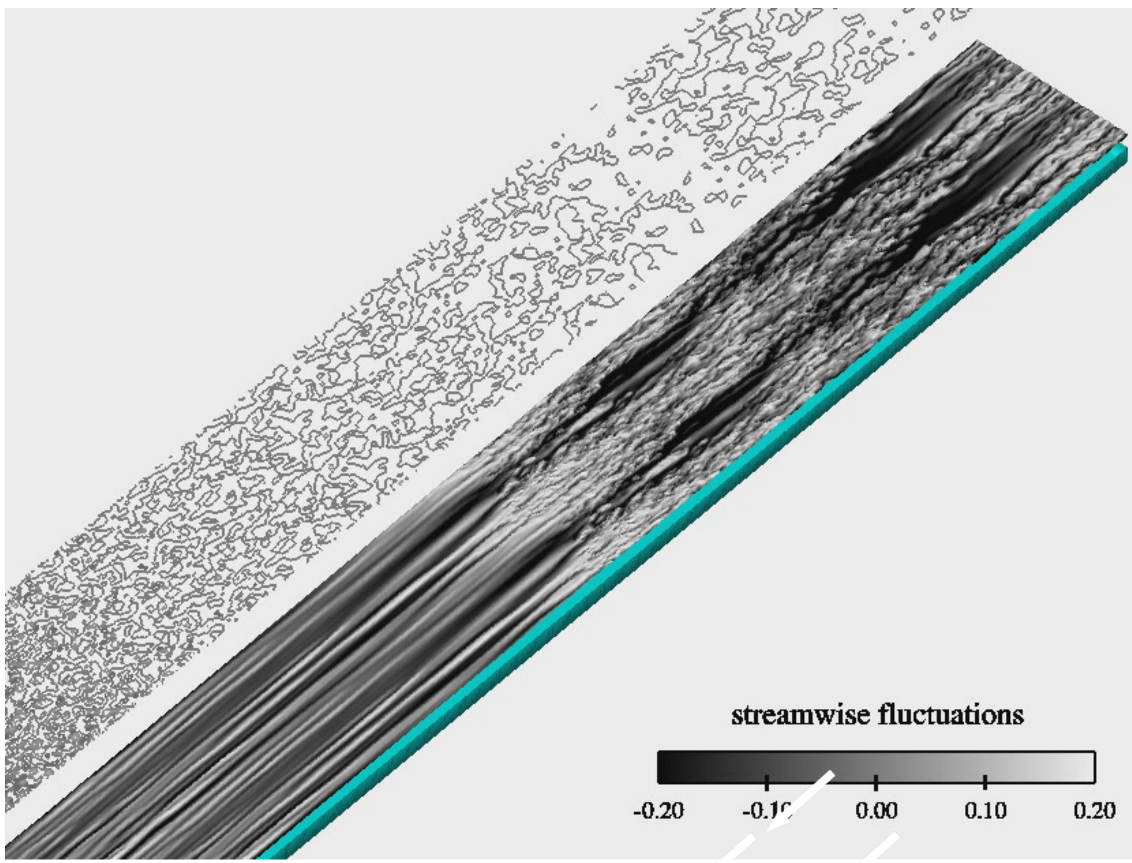
Bypass transition



Averaged view



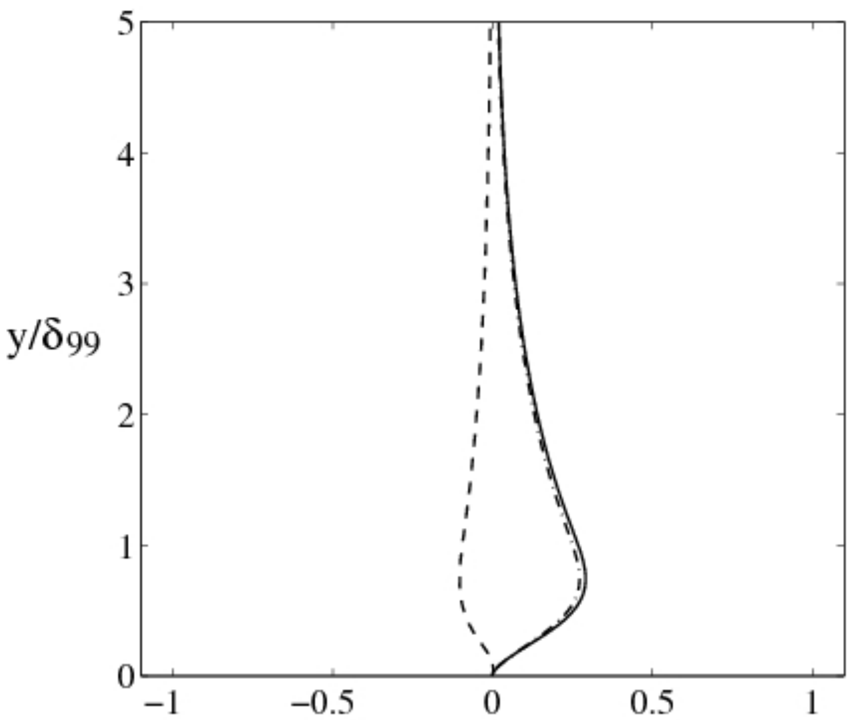
Boundary layer response to Free-stream turbulence



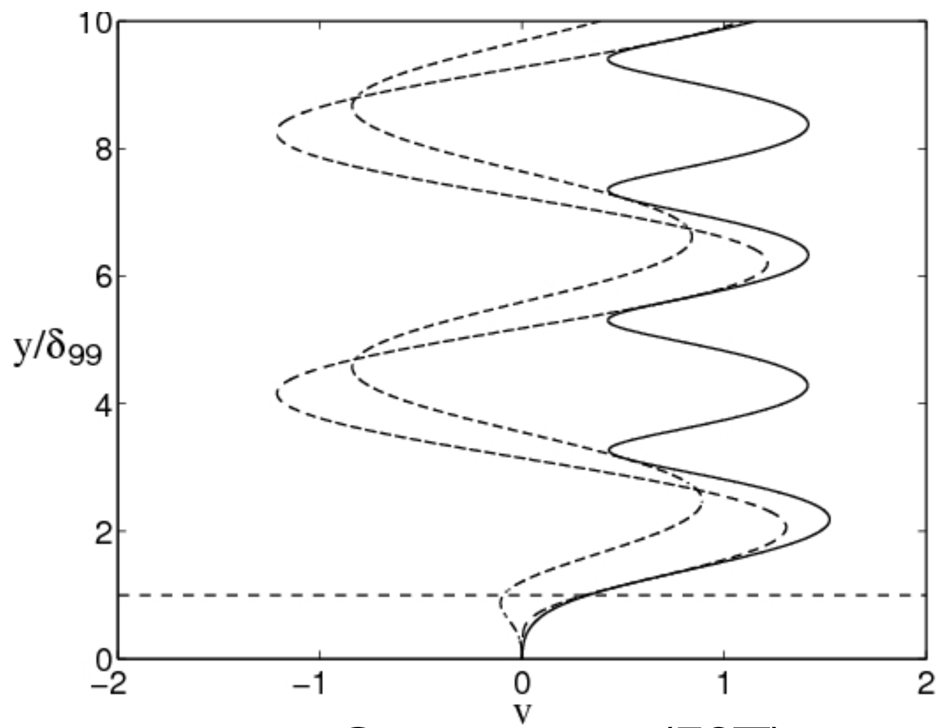
A bypass transition primer

- Continuous modes; discrete modes
- Klebanoff modes; ‘streaks’ or ‘jets’
- Turbulent spots
- Intermittency
- Free-stream turbulence intensity (*Tu*) u'/U (in %)
- Shear sheltering – or filtering
- Bypass and natural transition

Discrete and Continuous mode shapes



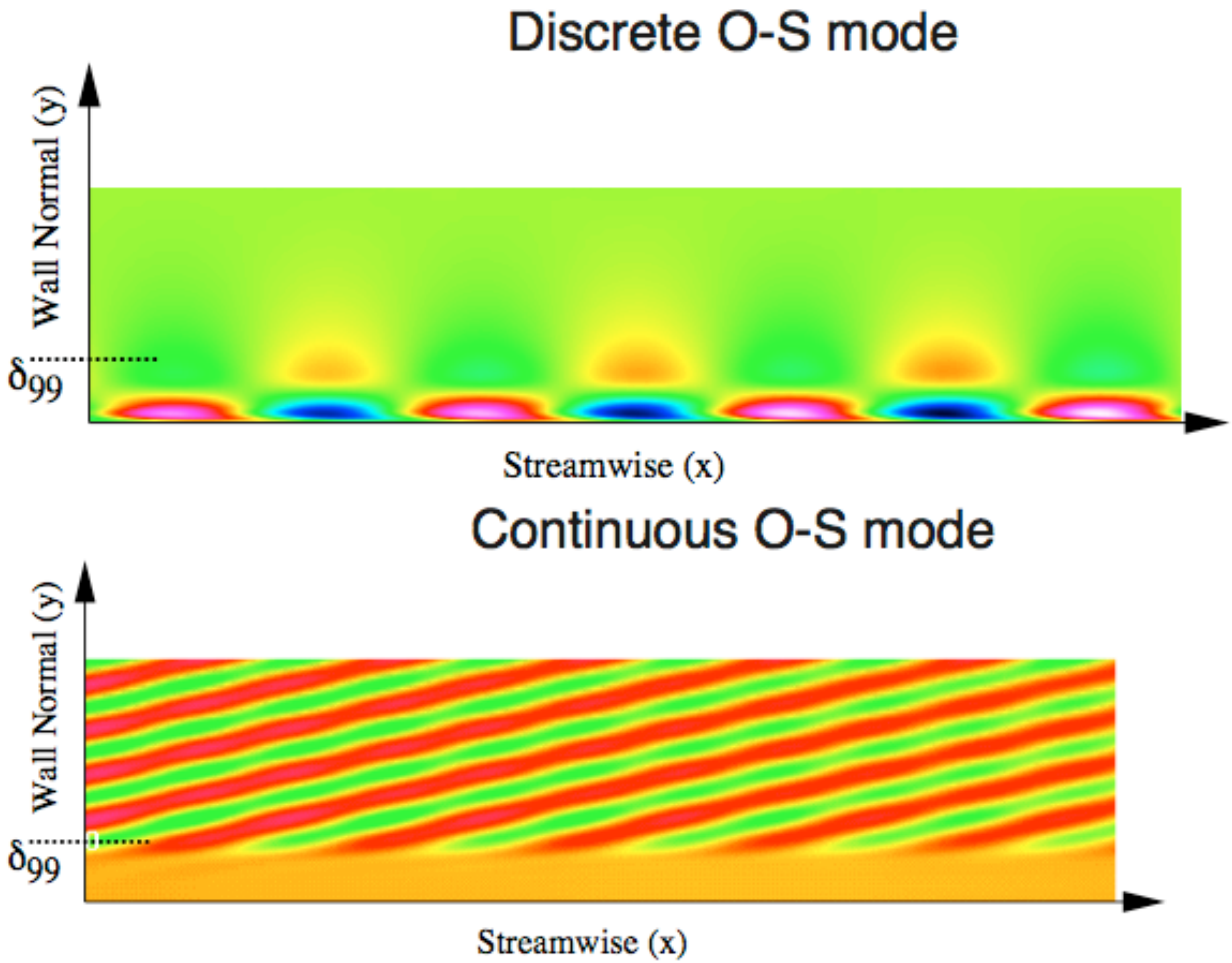
Discrete (T-S)
`bound'



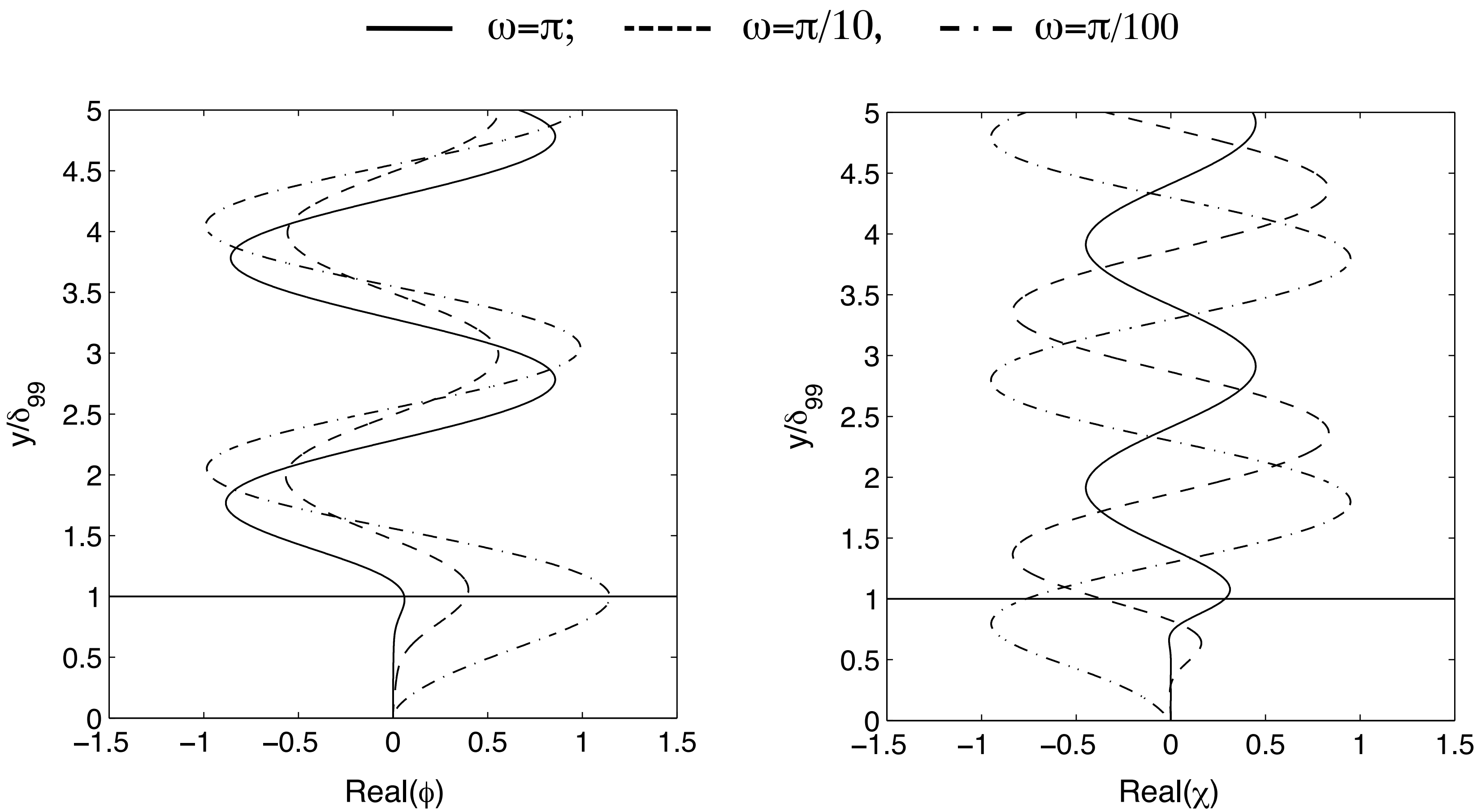
Continuous (FST)
`free'

➔ Natural transition

➔ Bypass transition


Discrete O-S mode
Wall Normal (y)
δ99
Streamwise (x)
Continuous O-S mode
Wall Normal (y)
δ99
Streamwise (x)

Continuous modes: Shear filtering and penetration depth



Theory

O-S + forced Squire equations

O-S=horizontal, Squire=vertical vorticity

$$(\mathcal{L} - \partial_t)v = 0$$

$$(\mathcal{S} - \partial_t)\eta = \mathcal{C}v(\tilde{k_y}, \tilde{\omega})$$

$$\mathcal{L} = \Delta^{-1}\left\{-ik_xU\Delta + ik_xU'' + \left[\Delta^2/R\right]\right\},$$
$$\mathcal{S} = \left[-ik_xU + (\Delta/R)\right], \qquad \mathcal{C} = ik_zU'$$

Discrete modes (TS waves), $v \to 0,\ y \to \infty$

Continuous modes, v bounded, $y \to \infty$

Dispersion relation: temporal (non-dim on δ; $U_\infty = 1$)

$$\omega = \underbrace{k_x}_{c=1} - \underbrace{\frac{i}{R}\left(k_x^2 + k_z^2 + k_y^2\right)}_{decay}$$

O-S and Squire

$\exists$ exact resonance between continuous O-S and Squire modes

Basic idea of algebraic growth: for $k_x \to 0$ (or $\lambda_x \to \infty$)

$$\nabla^2 p = \partial_y U \partial_x v \sim 0$$

Then

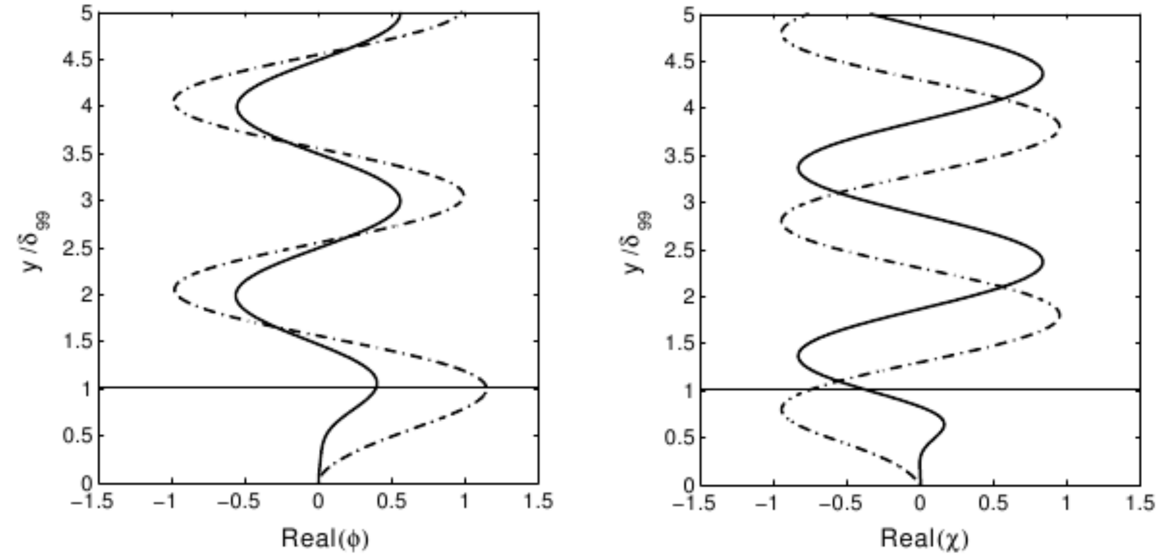
$$d_t v = 0 \to v = v(0)$$

$$d_t u = -v d_y U \to u \sim -v(0) d_y U \, t$$

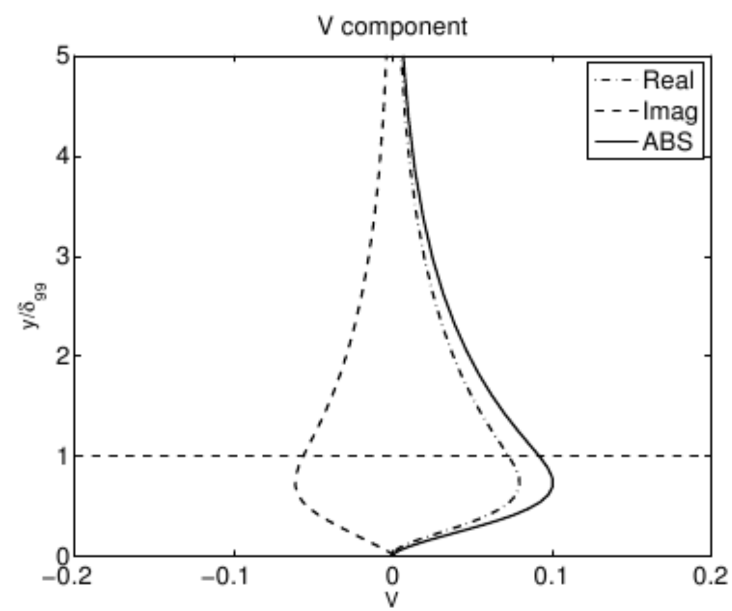
(RDT; c.f. Prandtl)

i.e., streamwise elongated disturbances (jets) grow within the boundary layer

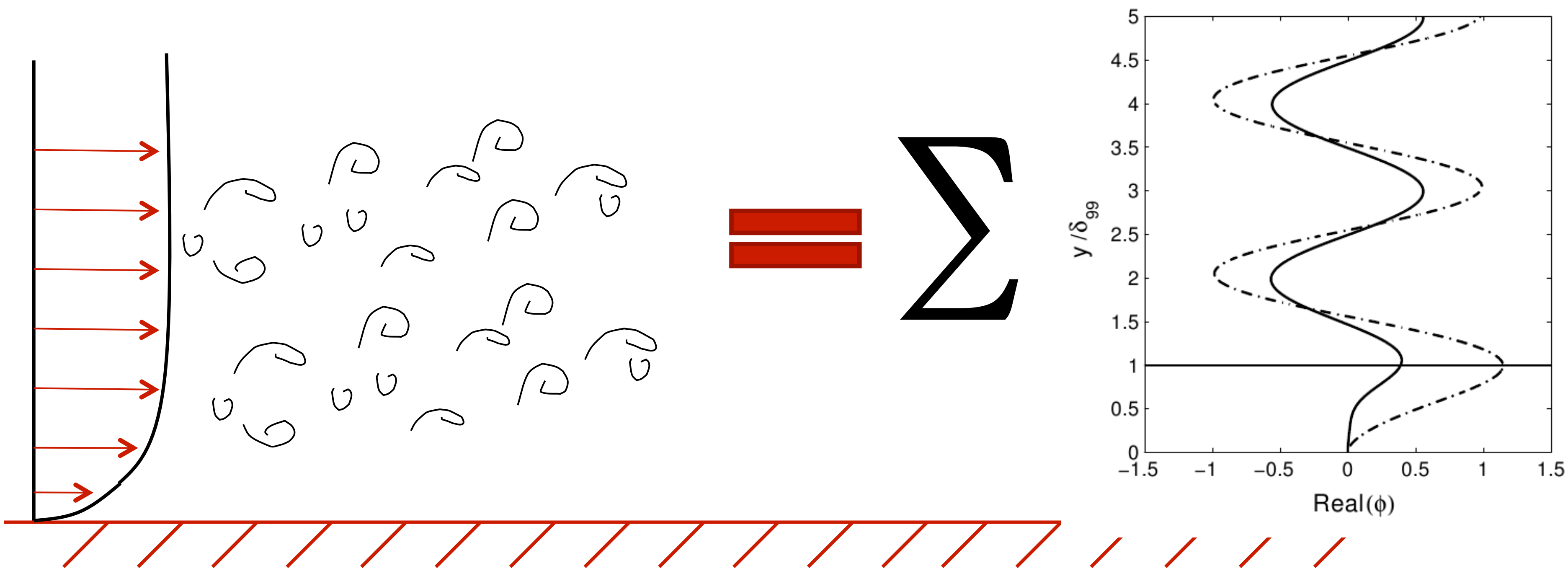
Typical continuous modes



Typical discrete mode

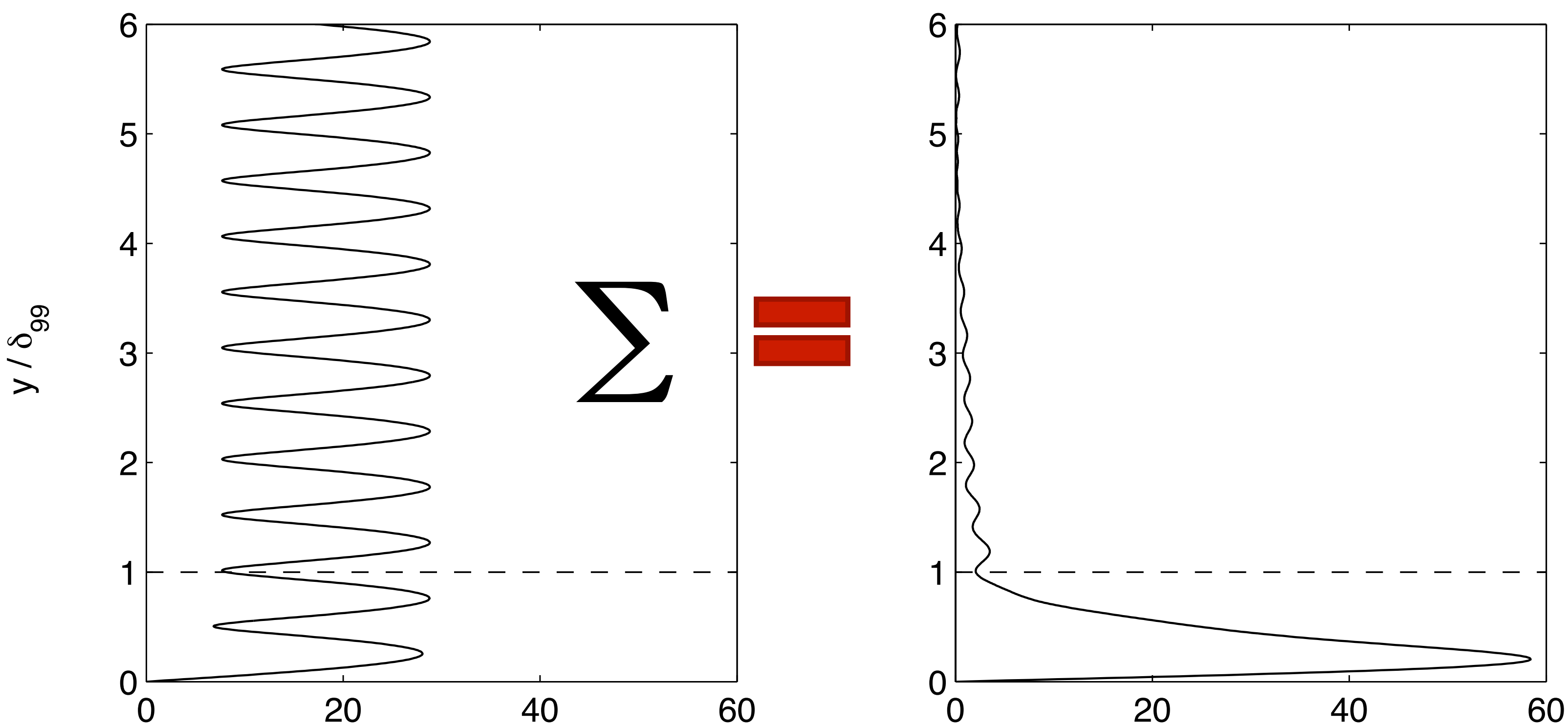


Continuous mode transition



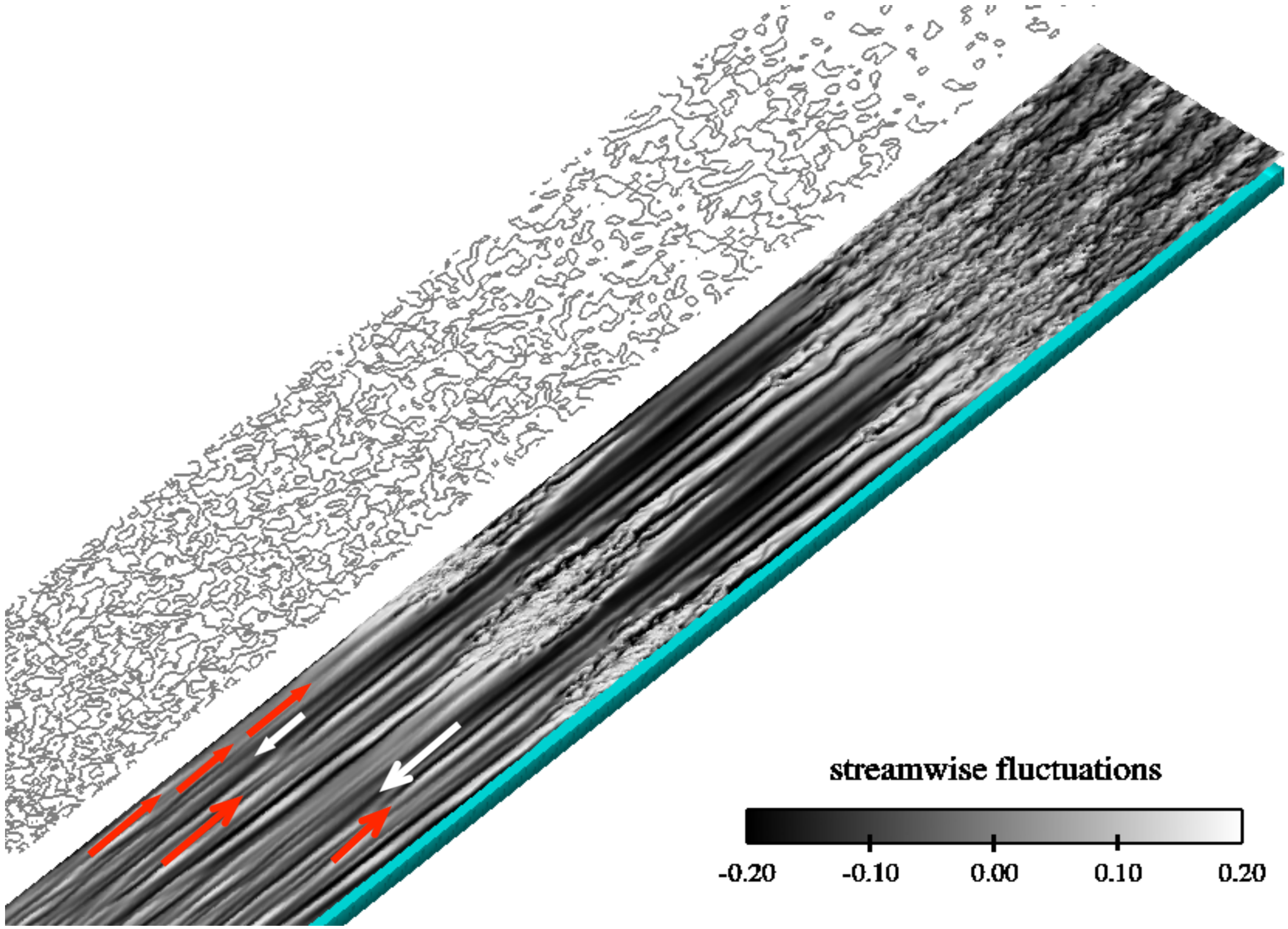
Evolution of continuous mode response ⟶ Klebanoff modes

Squire equation is forced by OS modes



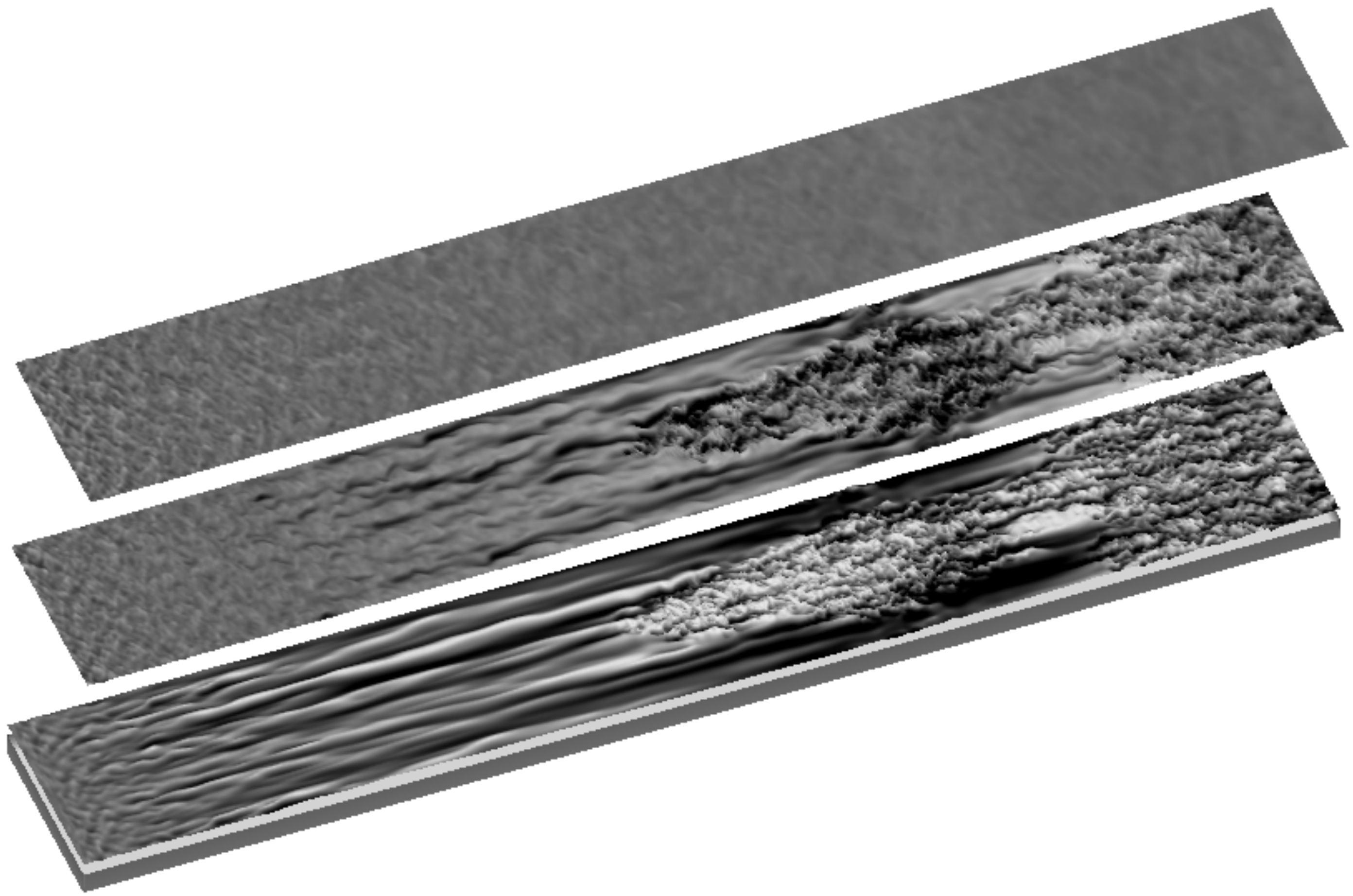
Superposition produces a `Klebanoff distortion'

3% f.s.t, Klebanoff modes

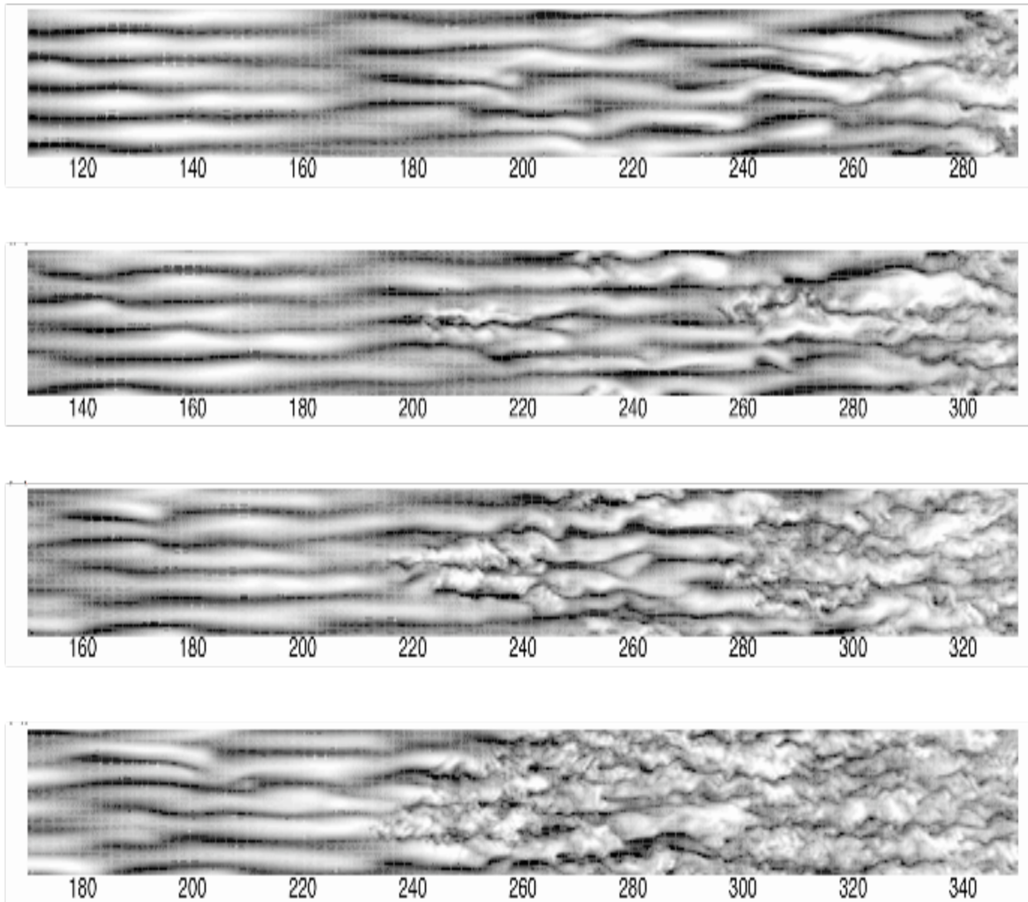


u' contours

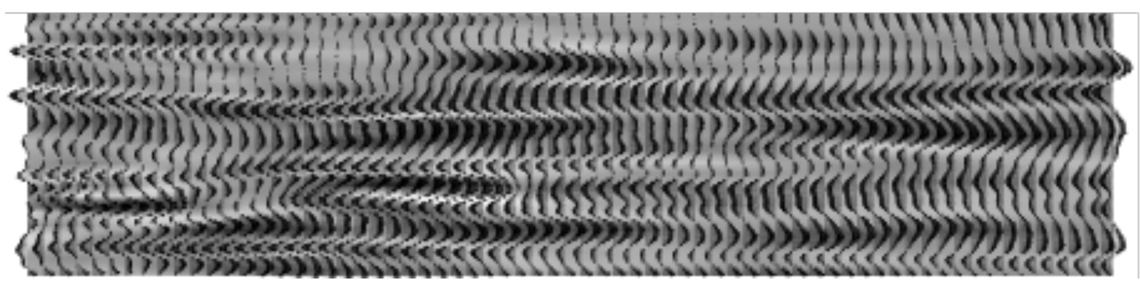
Three planes



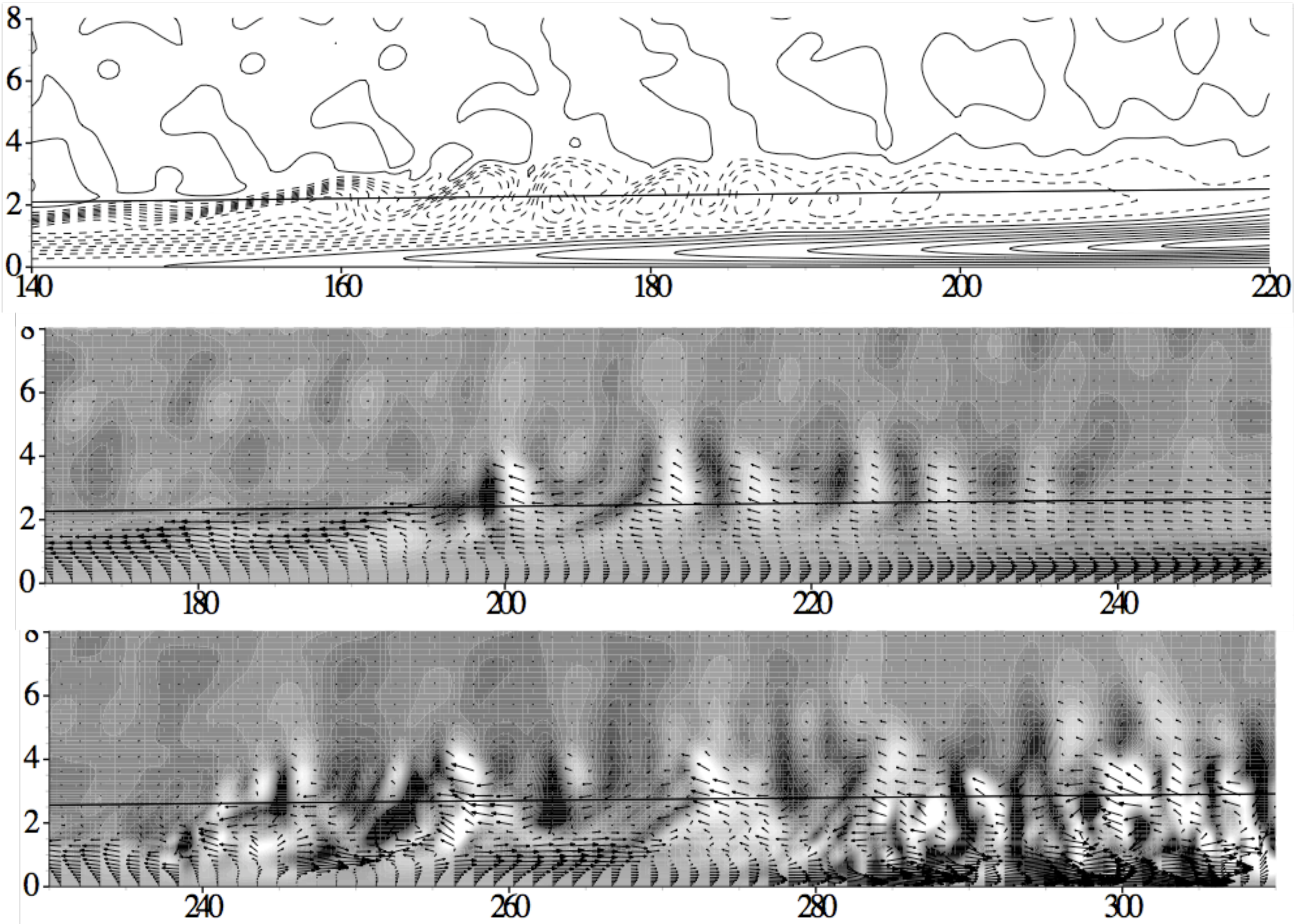
2 continuous modes, *u*-component jets (streaks)



u' contours at $y/\delta_0 = 0.74$

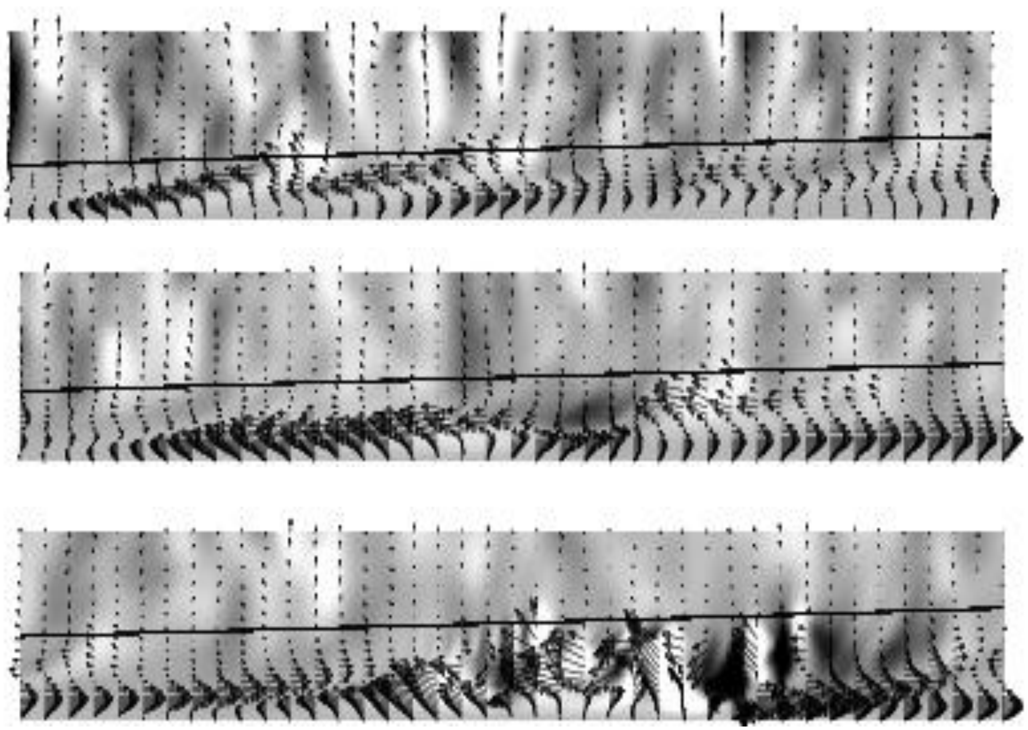


Breakdown of lifted jets

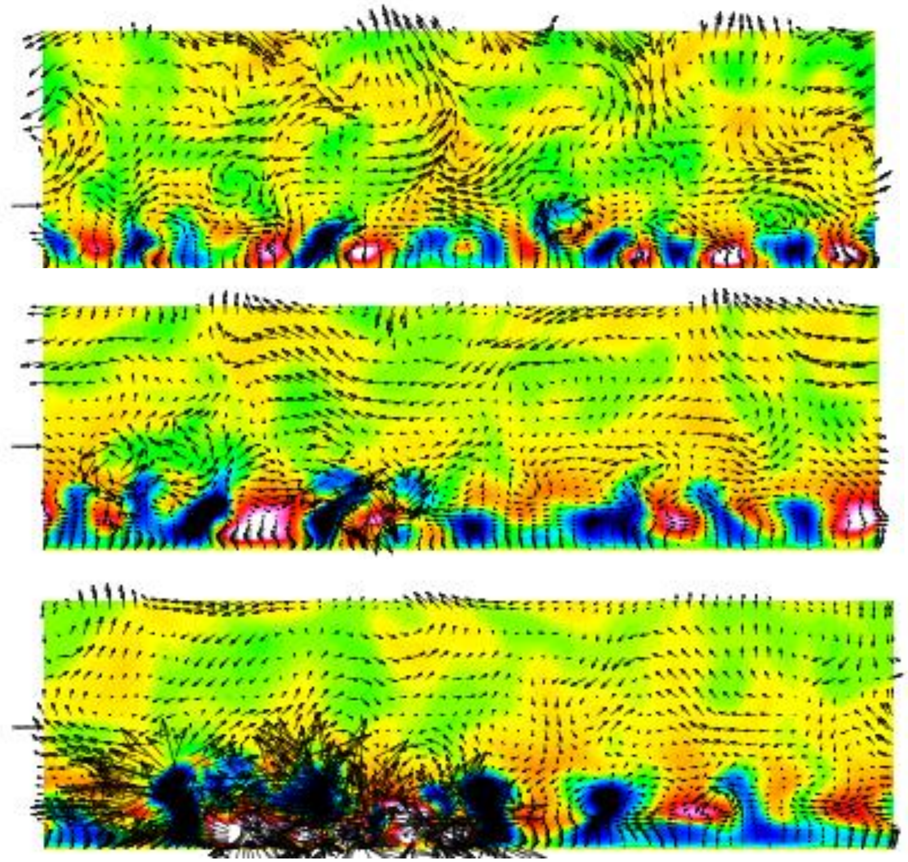


Side and end views of lifted jets

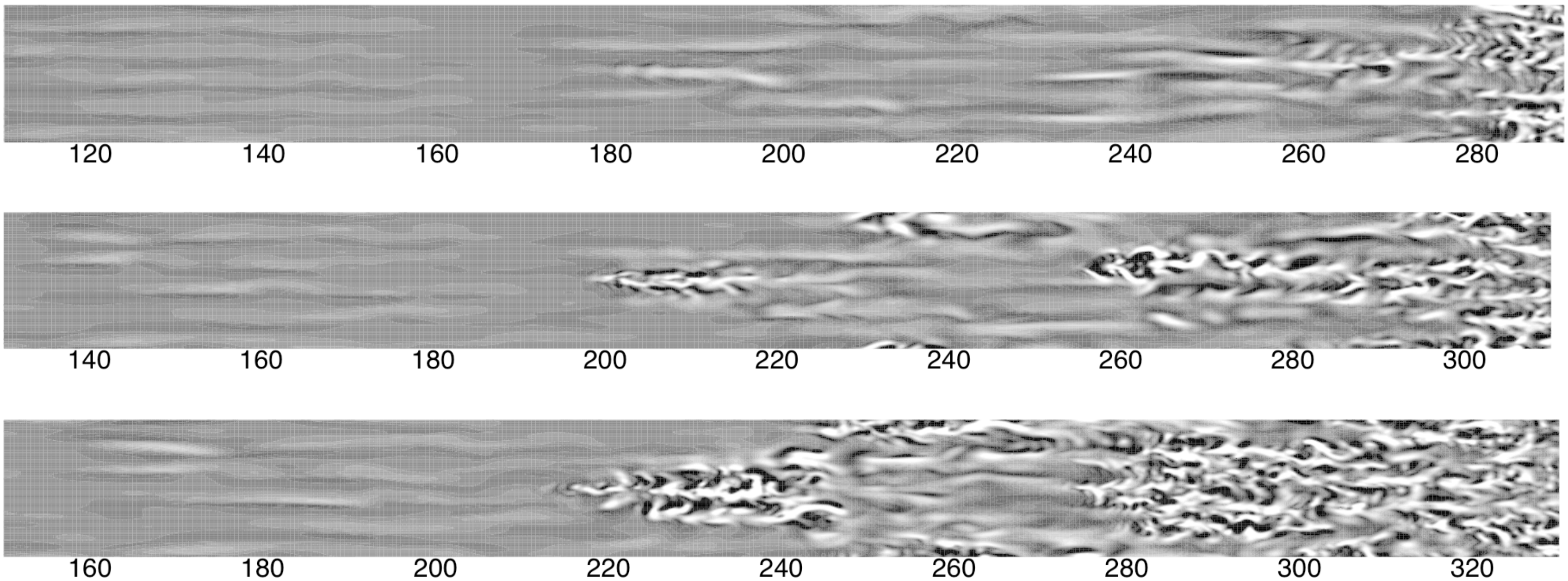
Side



End

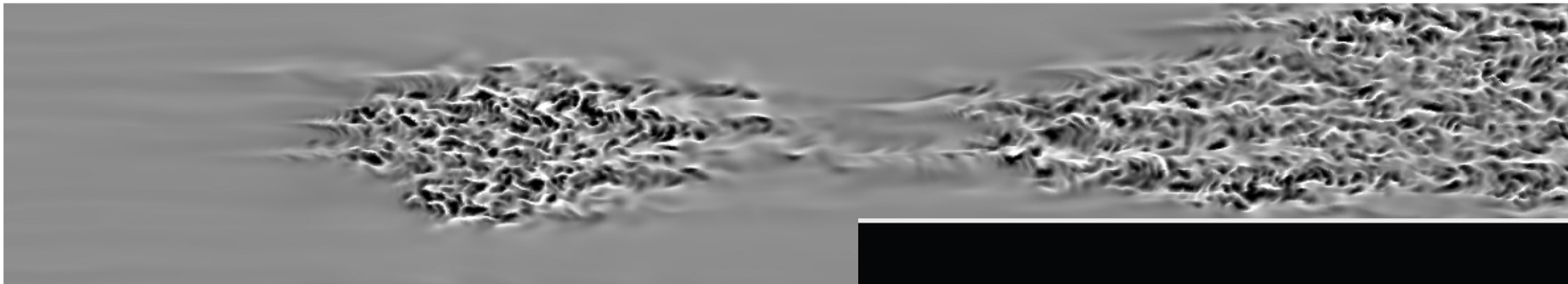


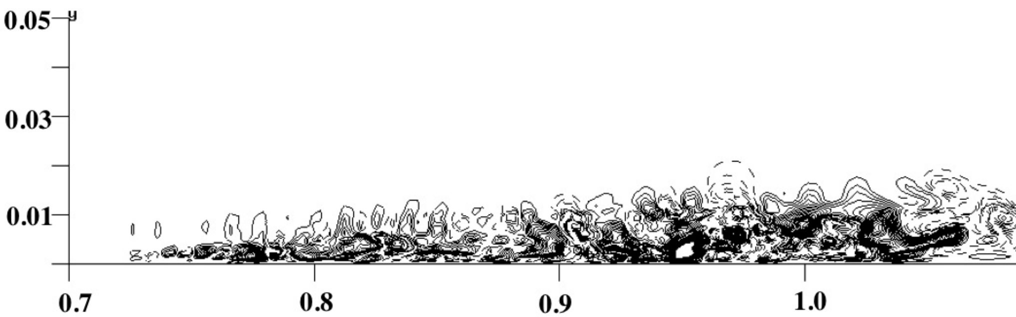
2 continuous modes, *v*-component
Spots

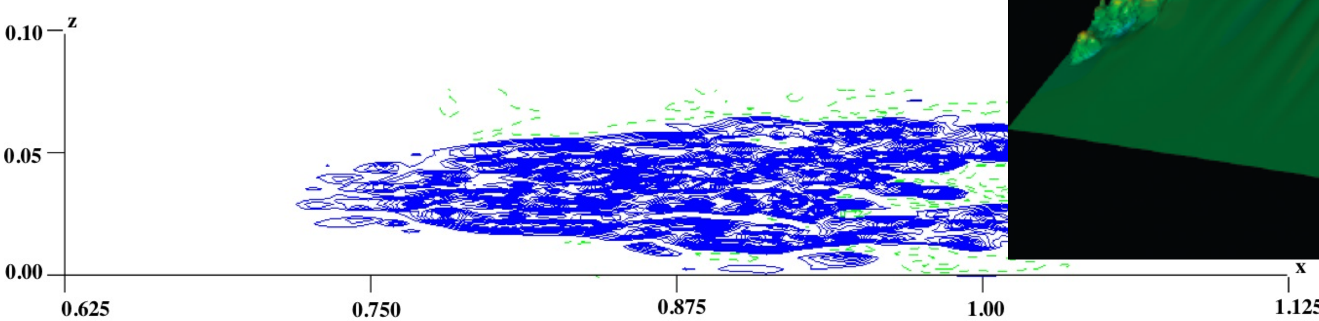


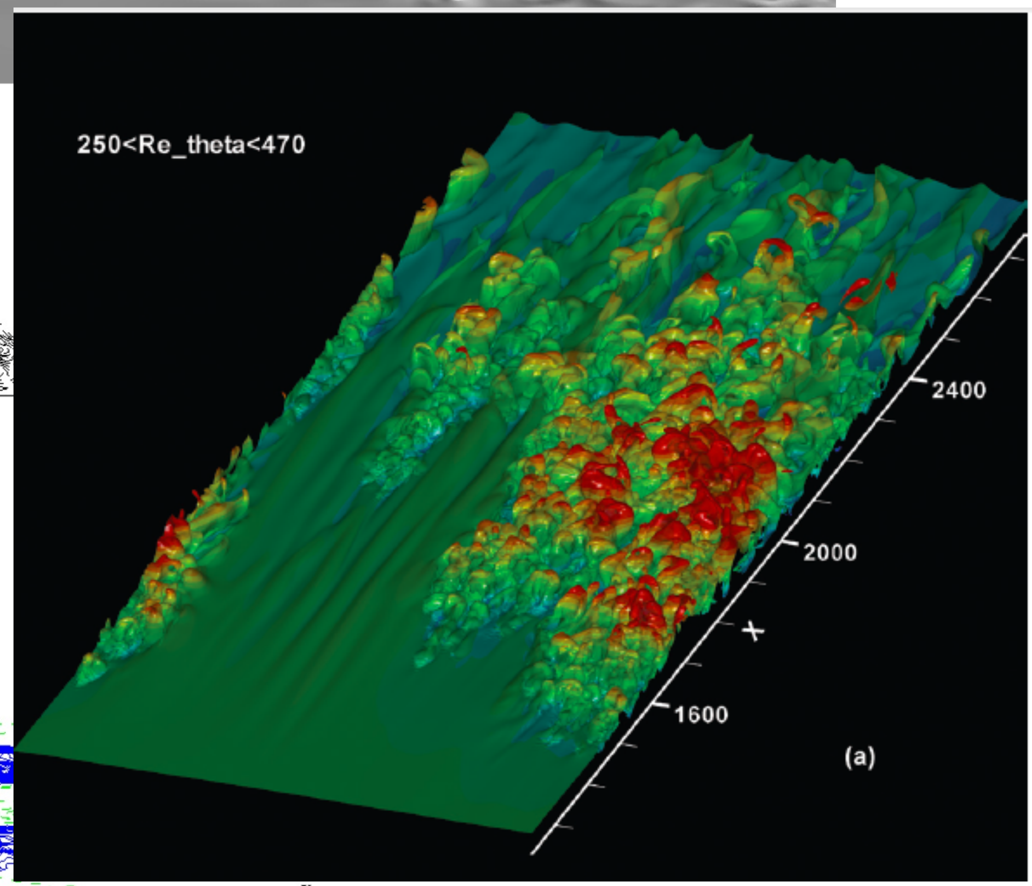
Turbulent spots

Contours of *v*



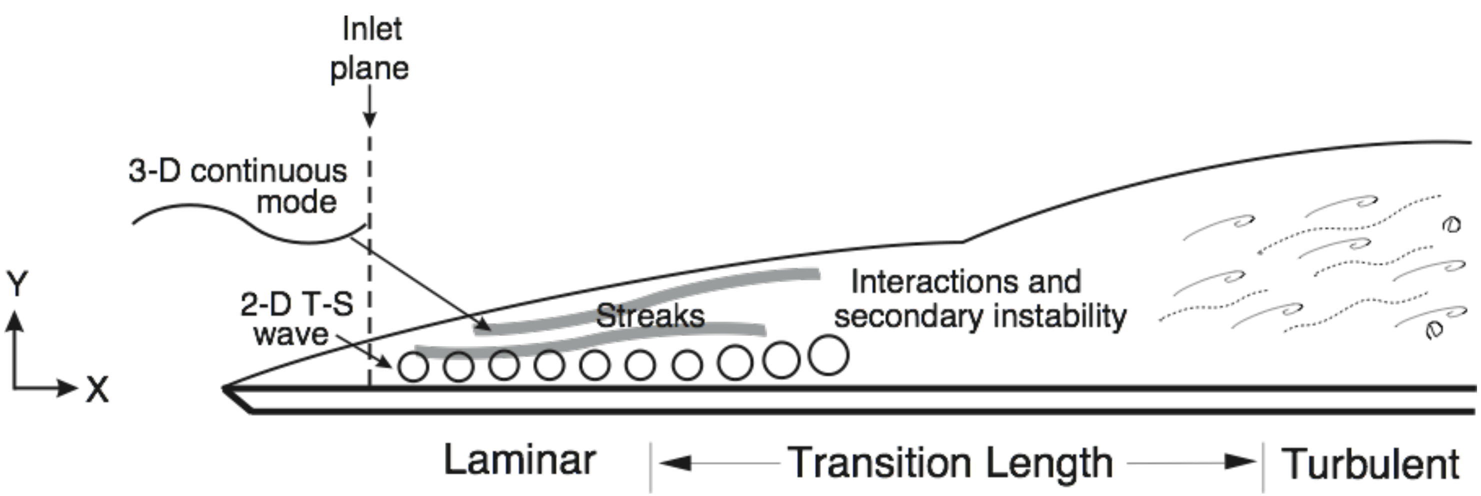






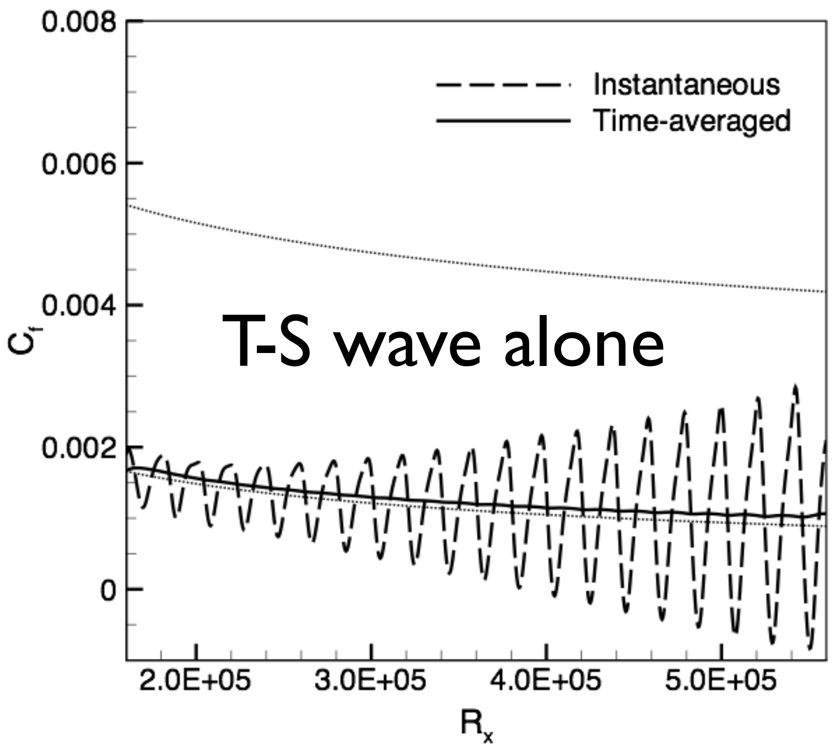
Discrete plus continuous modes

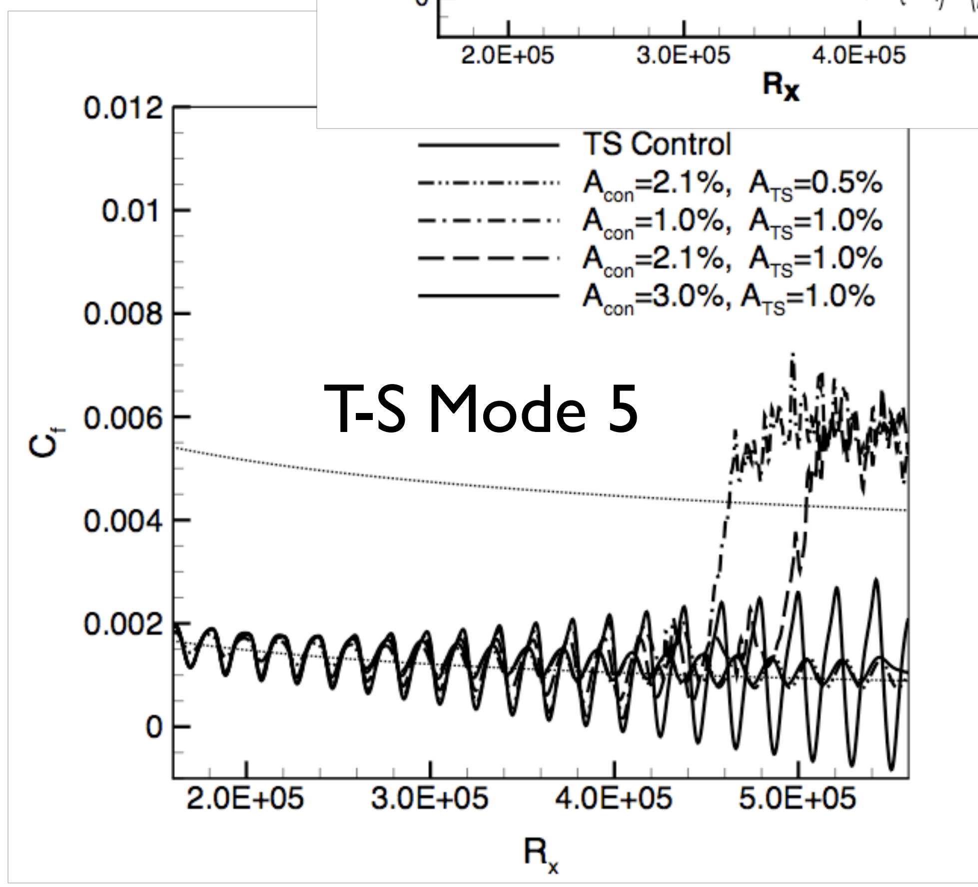
To illustrate natural vs. bypass

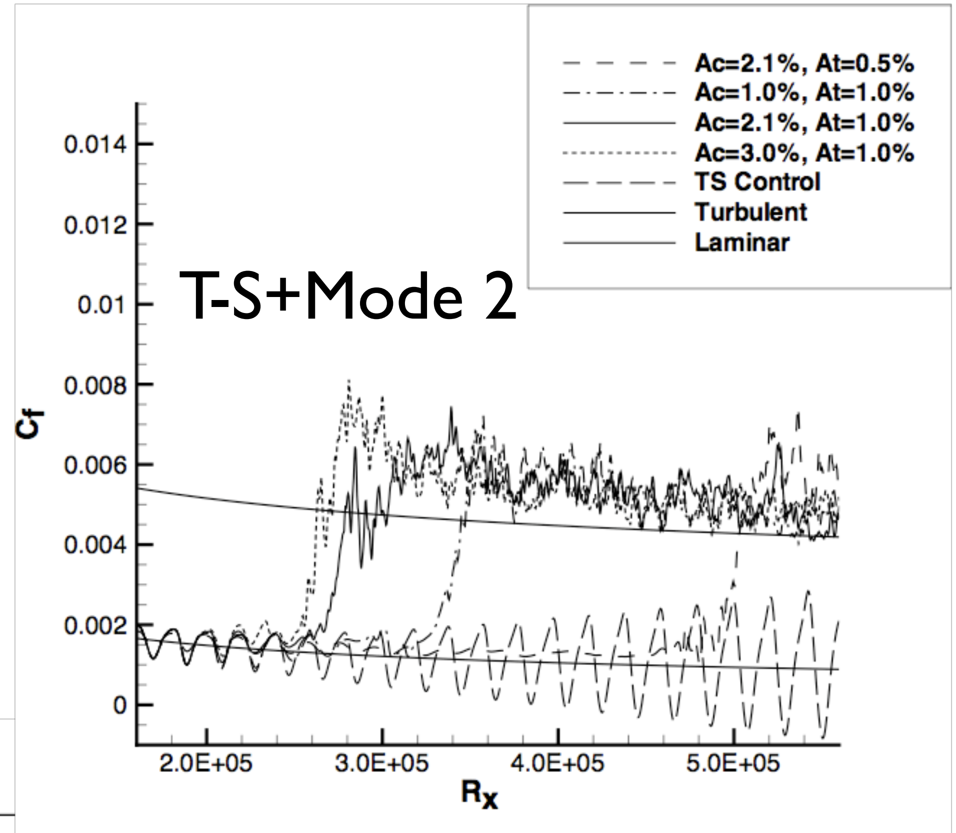


Boundary layer response

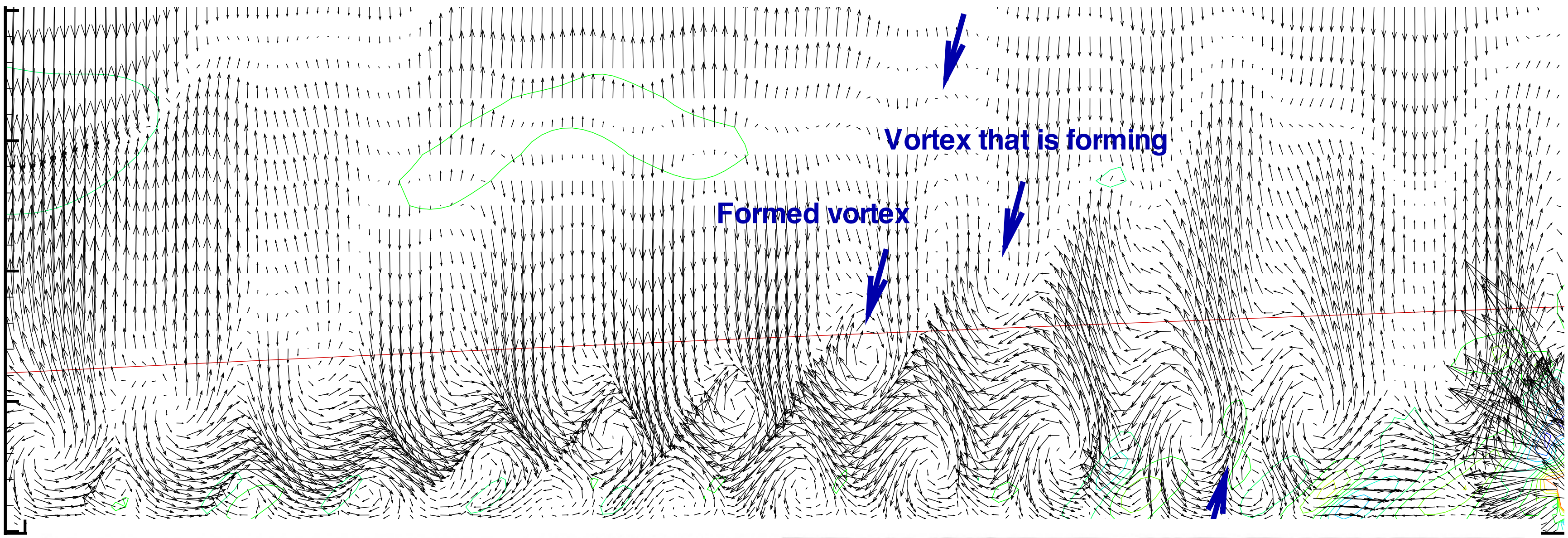
'natural' transition

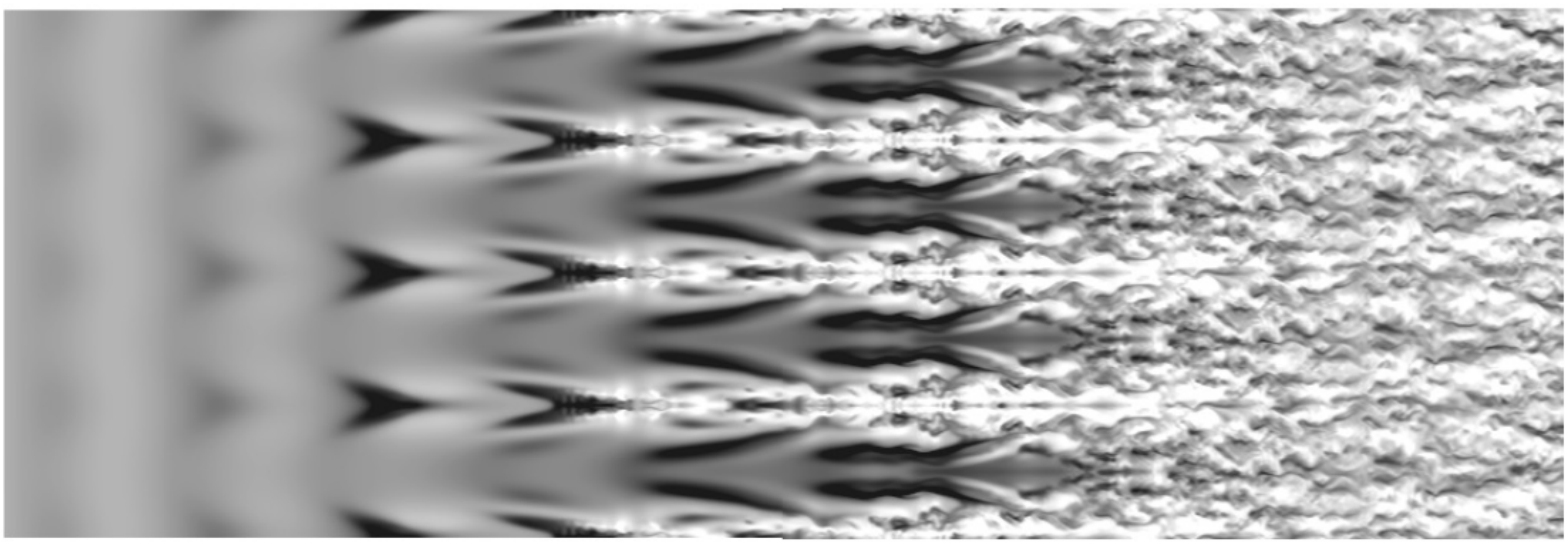






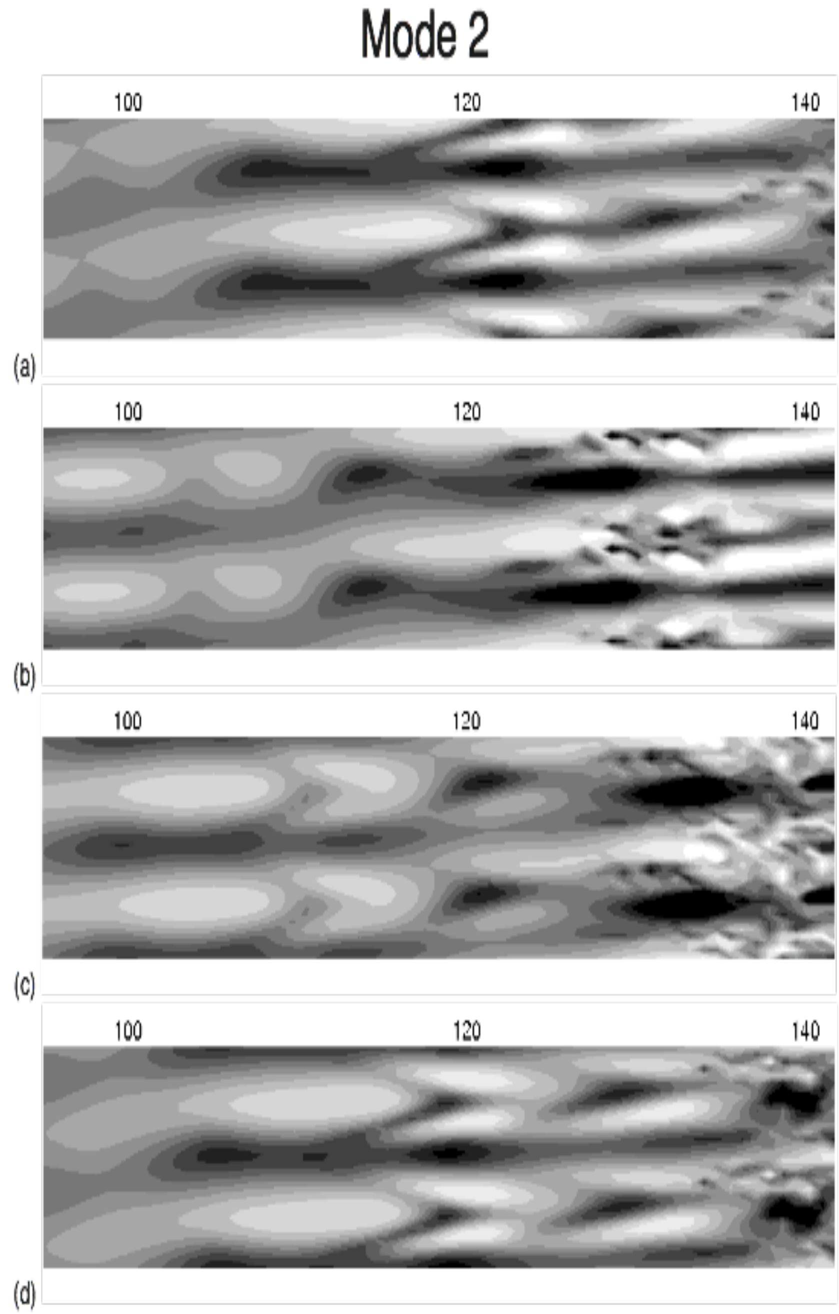
Growth and breakdown of TS waves



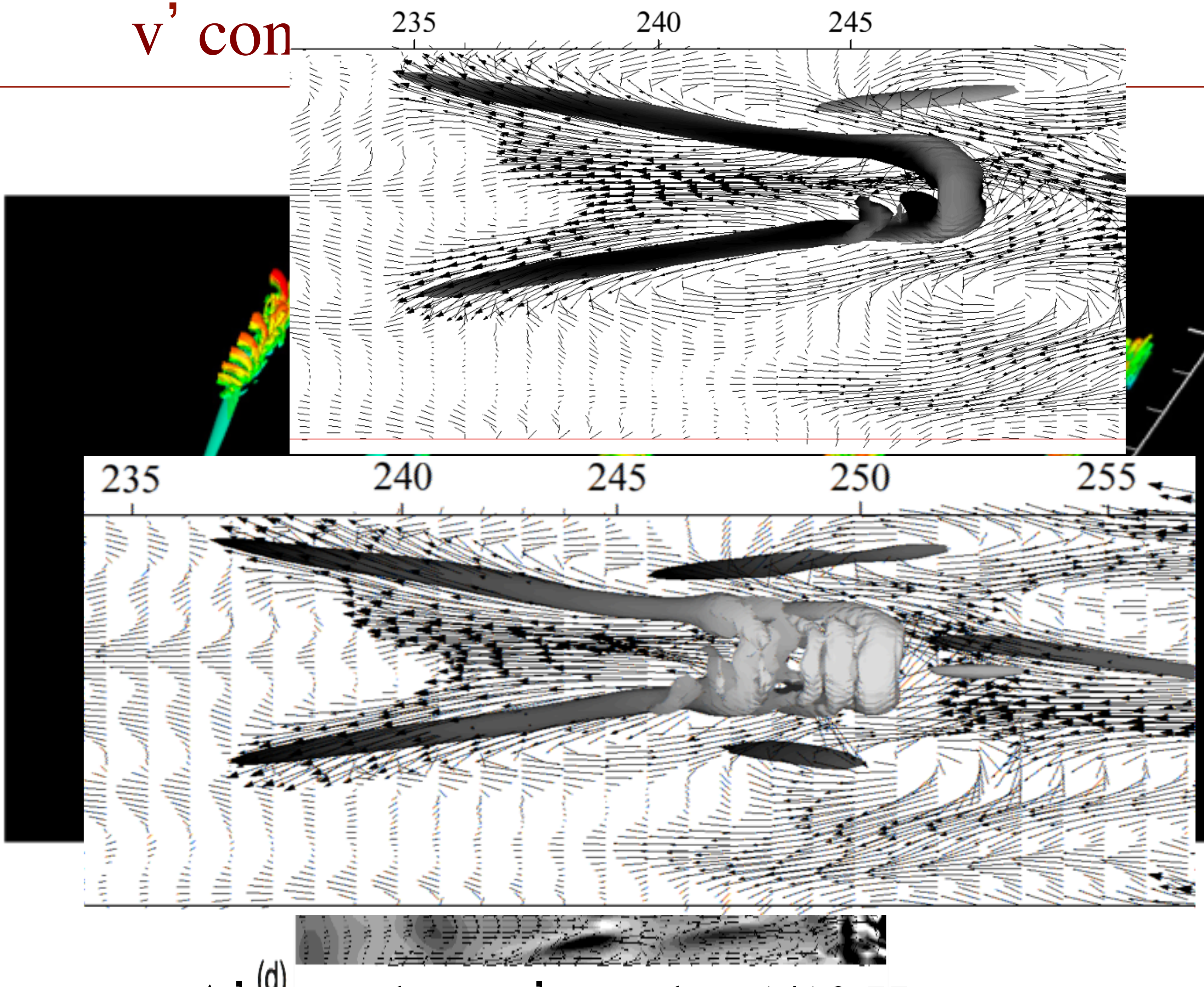


Secondary instability: Λ vortices

u' contours in x-z plane at y=0.5δ_{99}

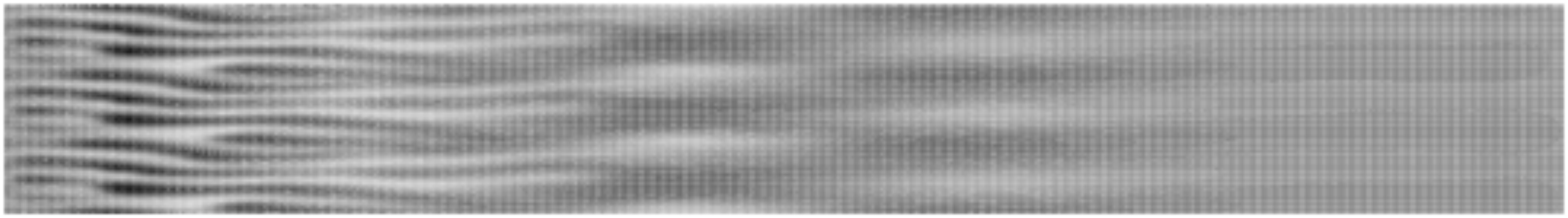


v' con

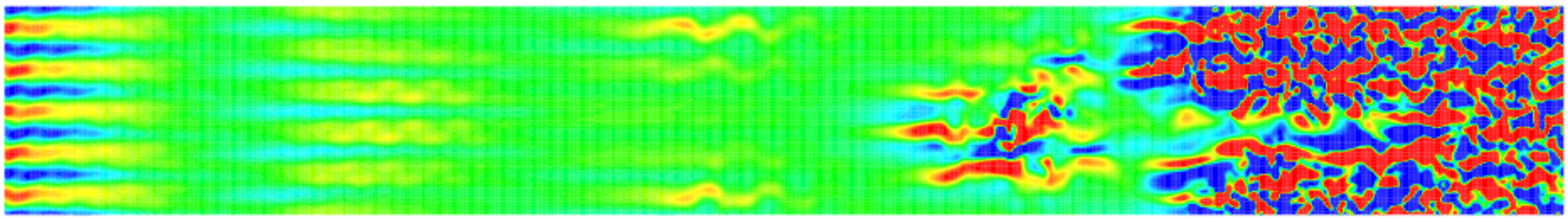


Λ's match cont's mode; 1/10 H-type

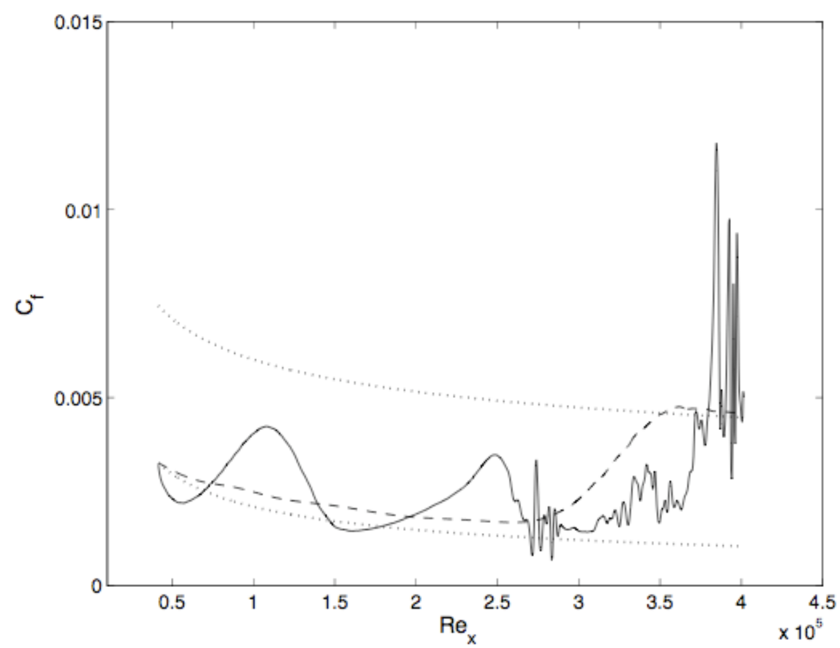
Boundary layer response



2 low freq. modes

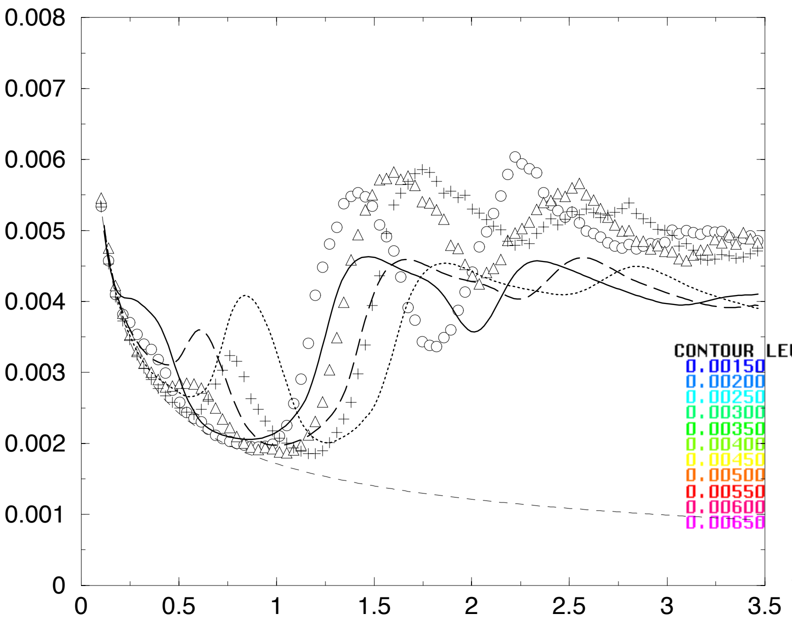


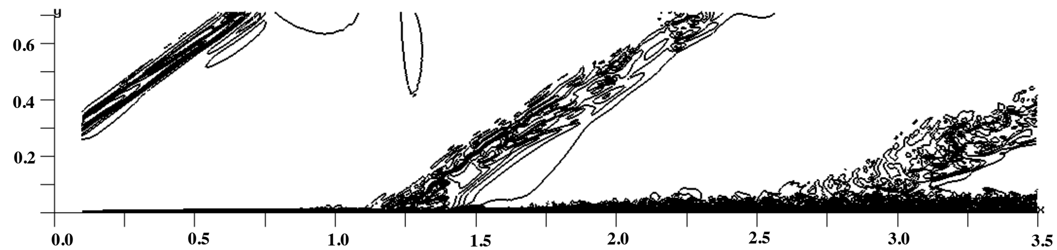
1 low + 1 high

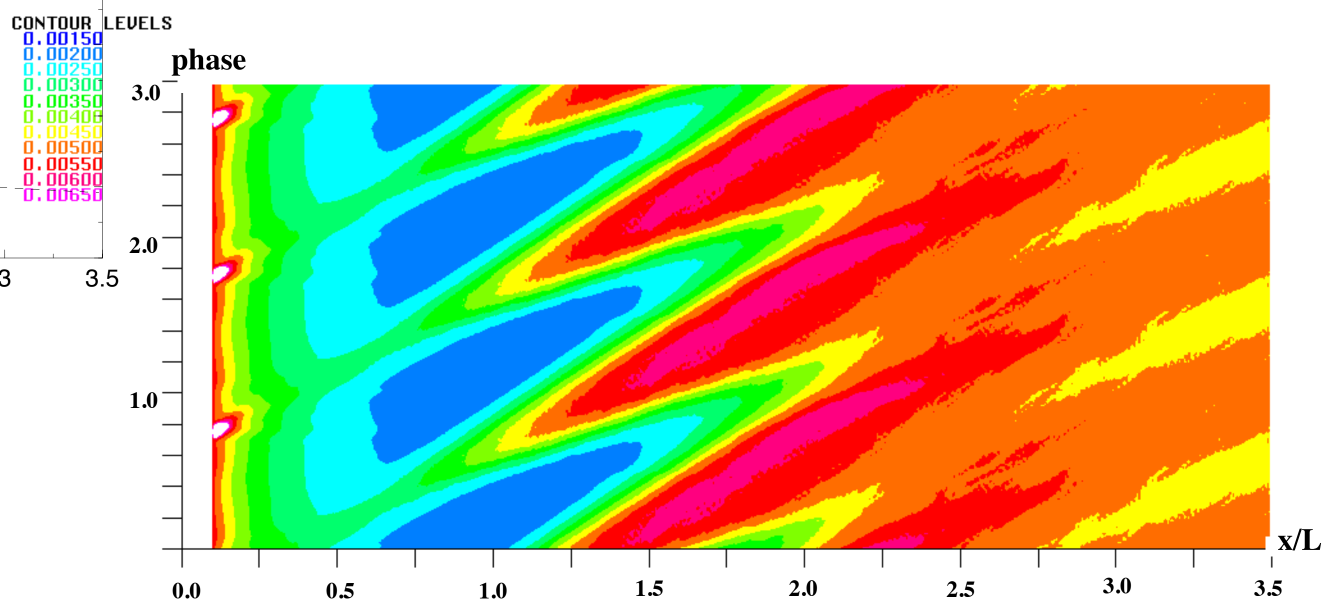


Turbine and compressor blade DNS

Passing wakes

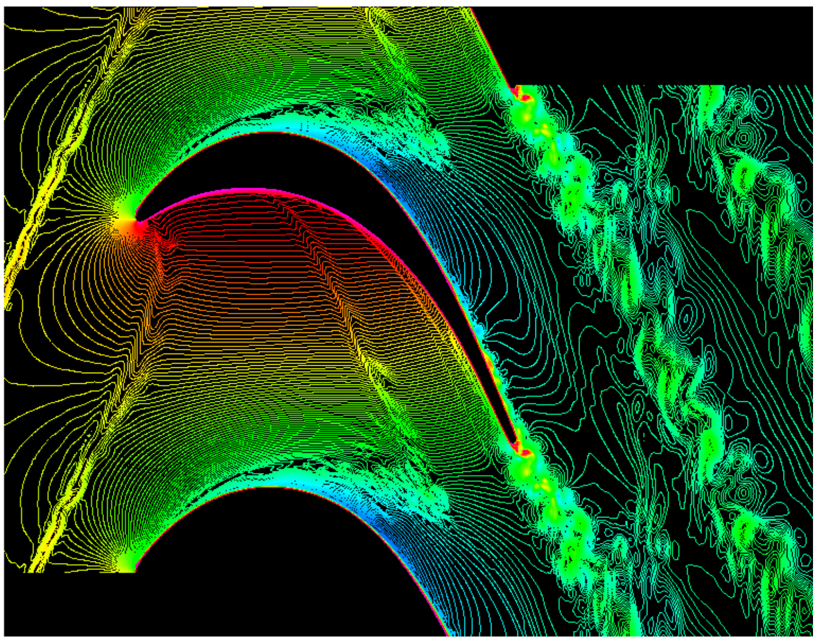


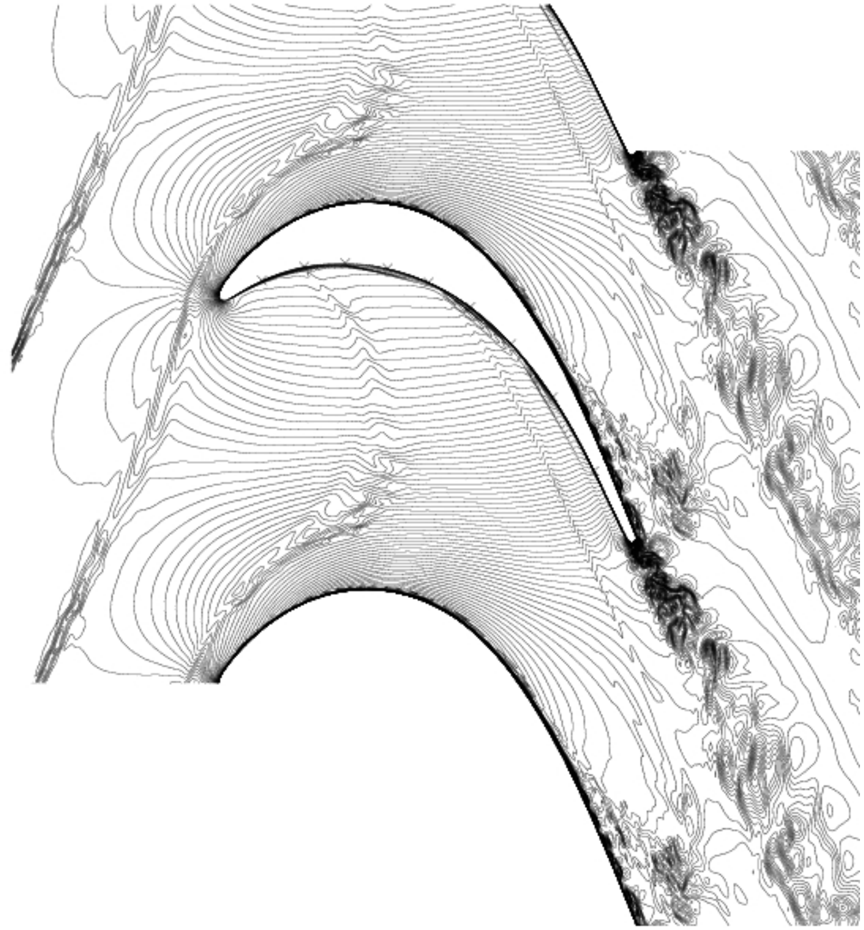




LP turbine

Direct Numerical Simulation
(DNS)





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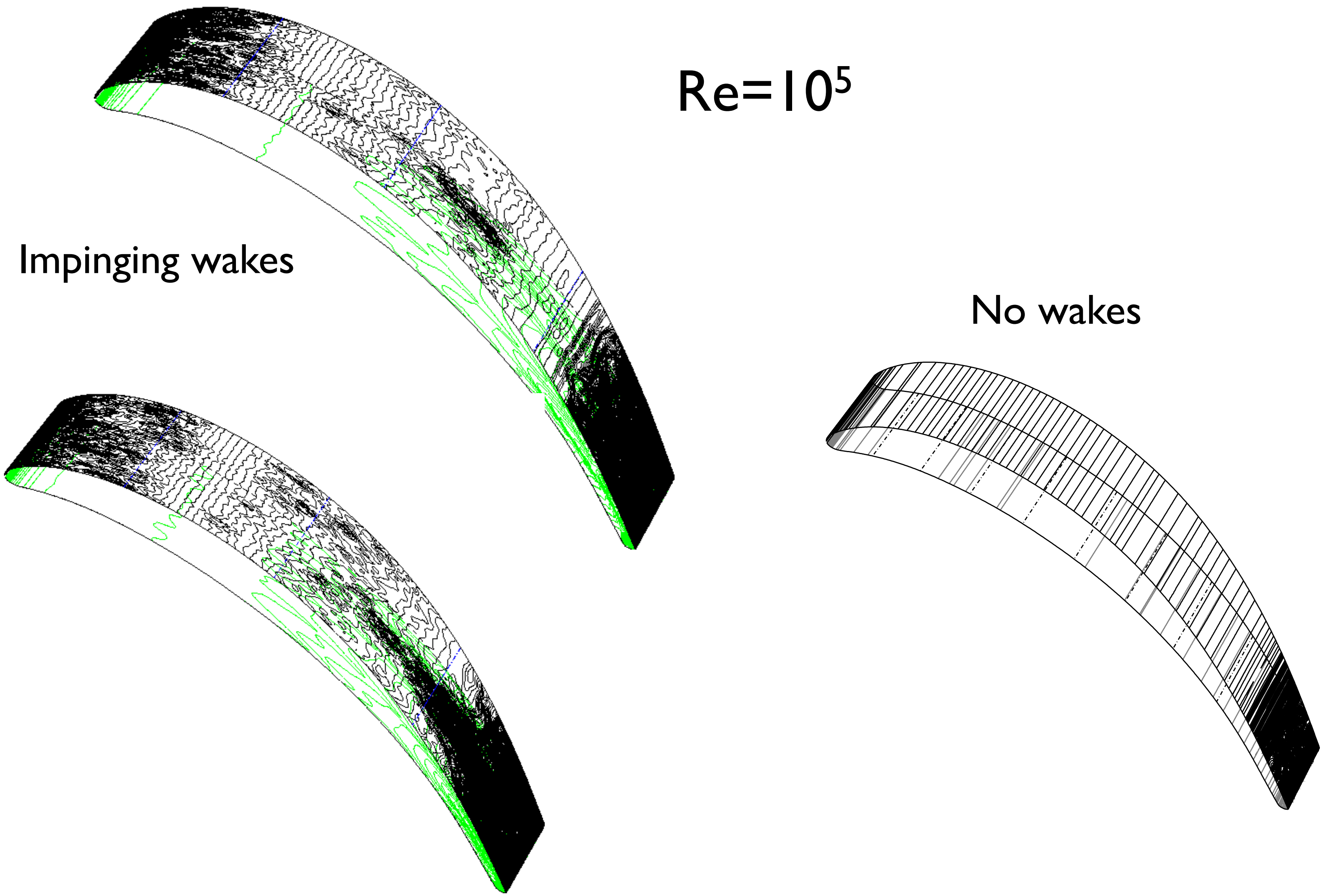
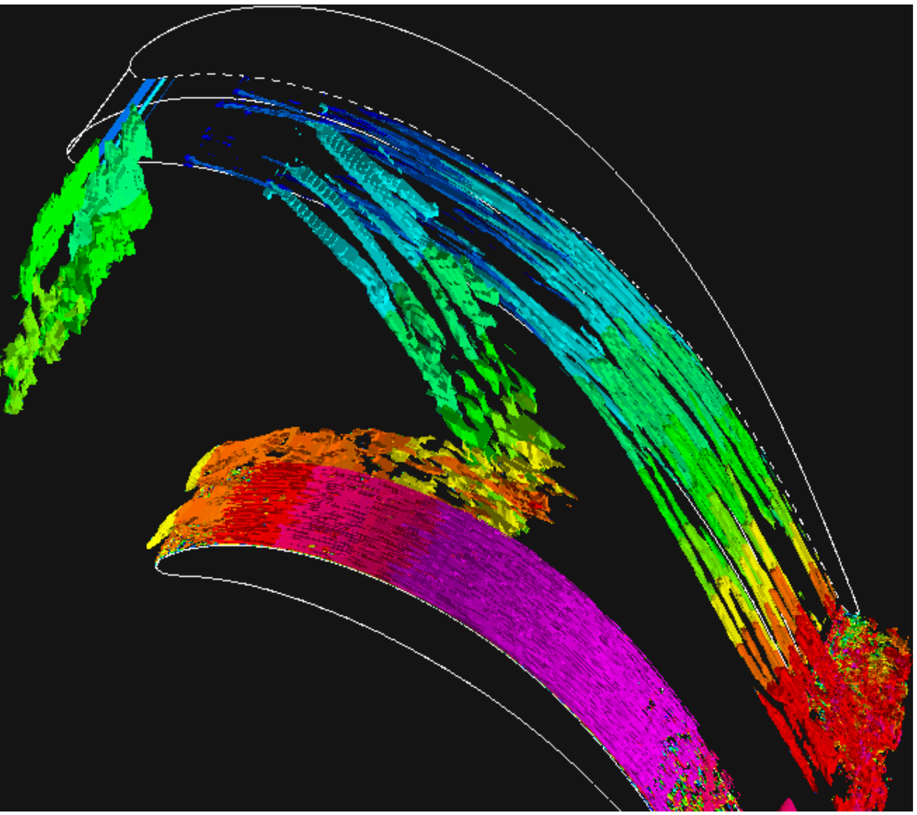
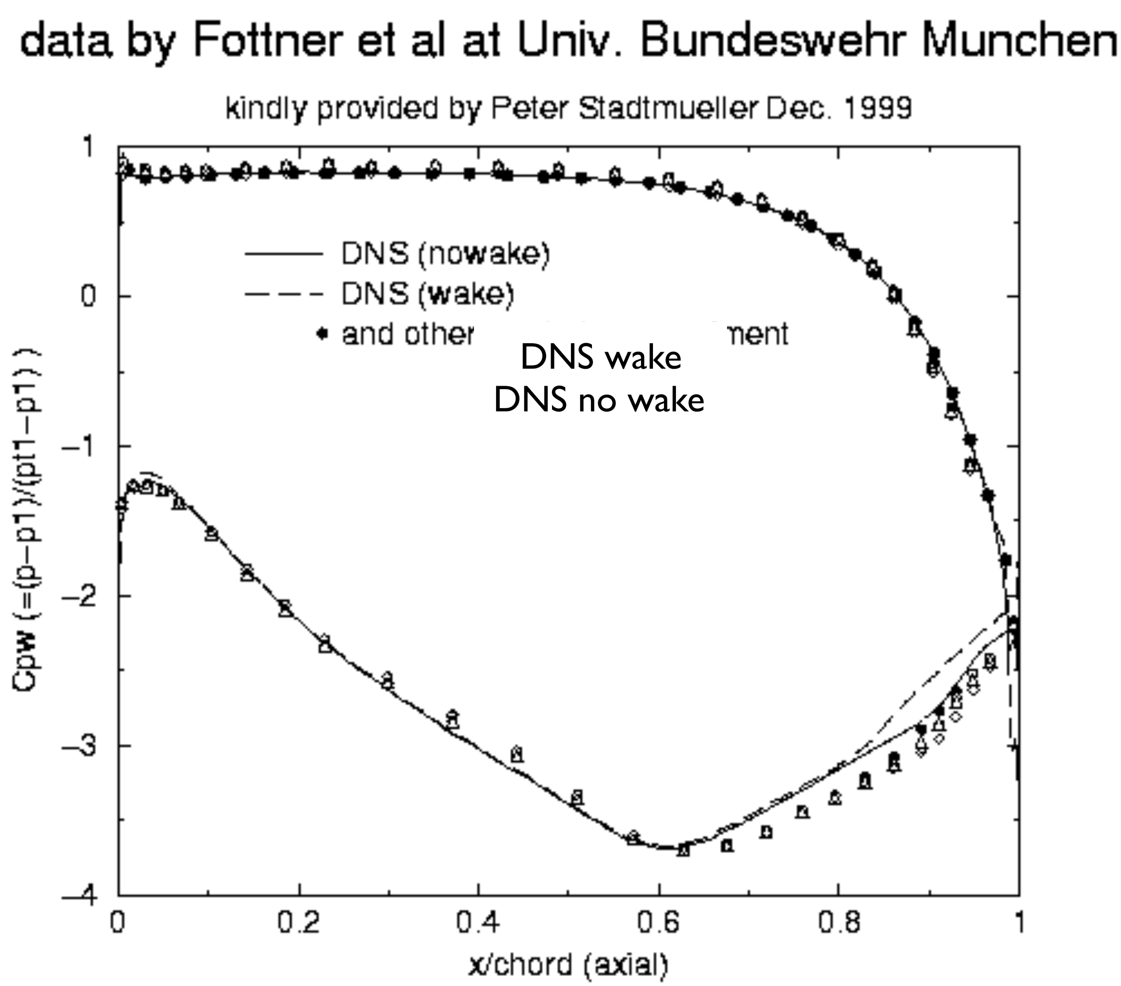
Transition on LP Turbine blade

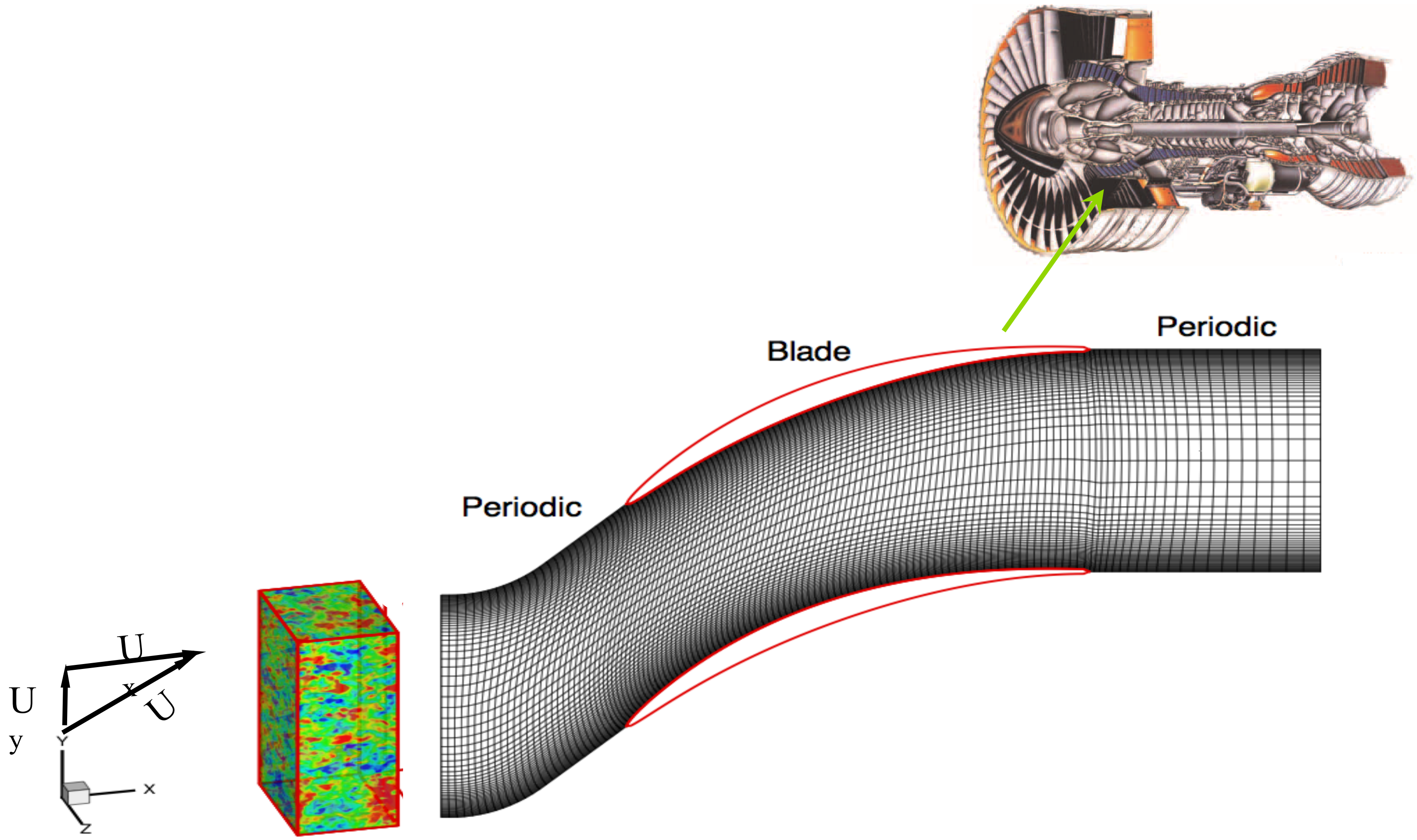


Distorted wakes

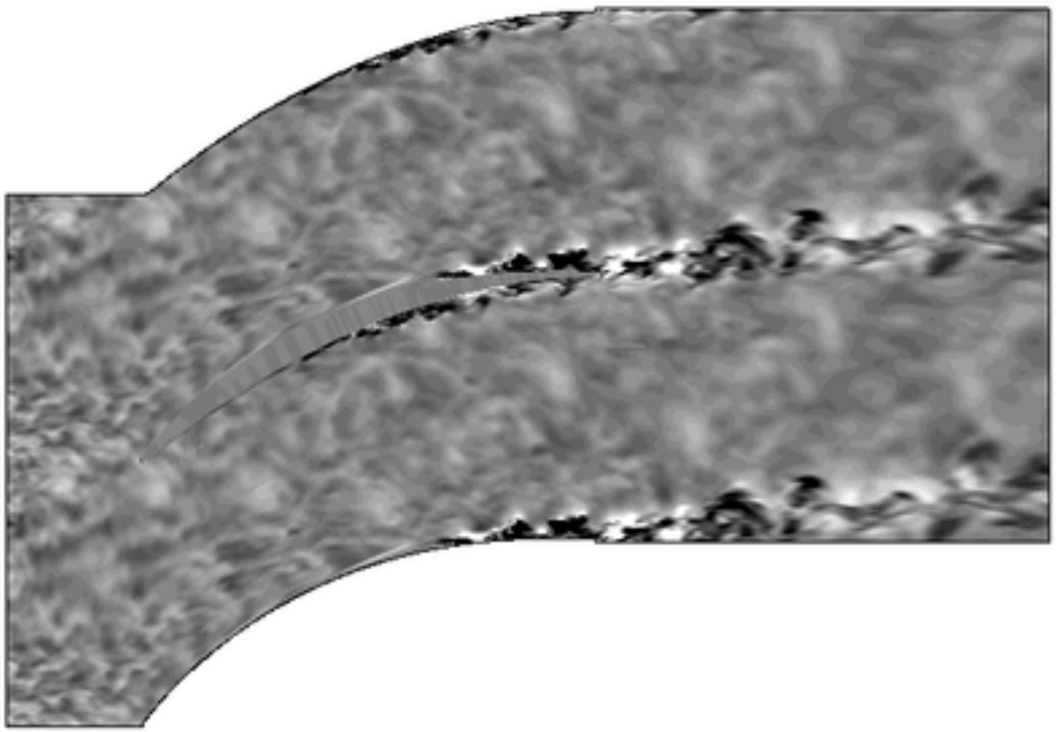




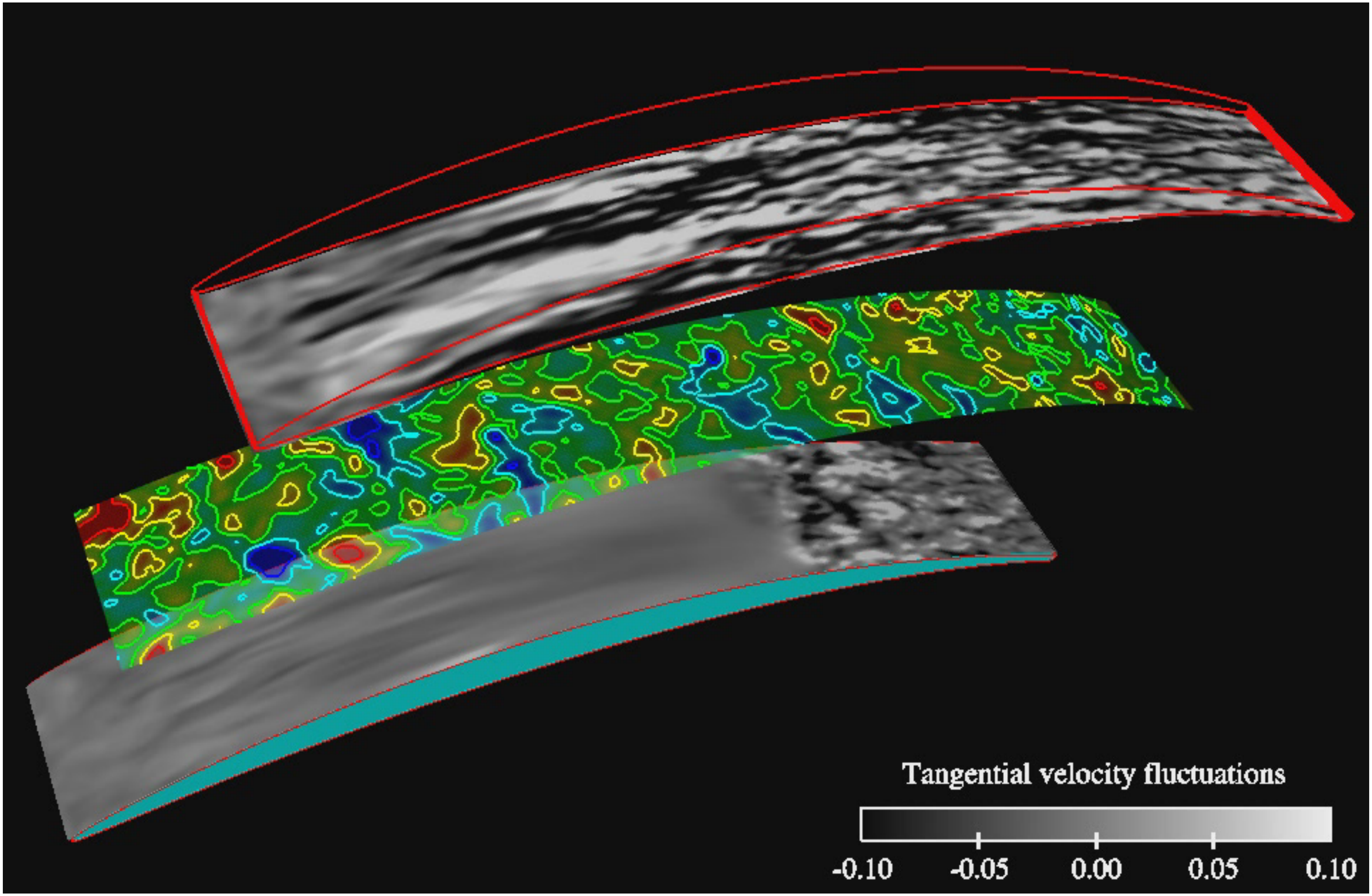
Compressor passage



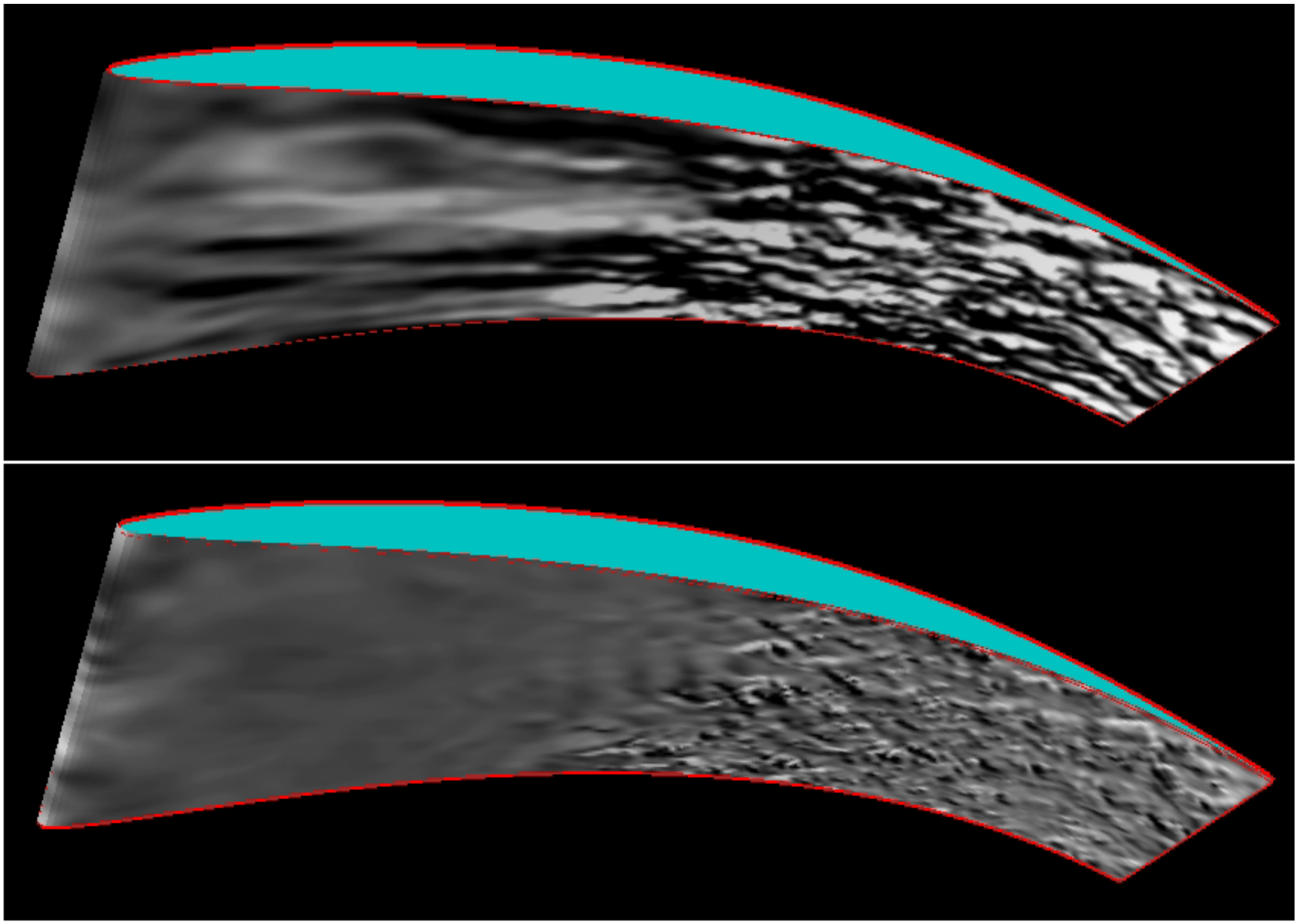
Compressor DNS sideview



Pressure, suction surfaces and f.s.t.



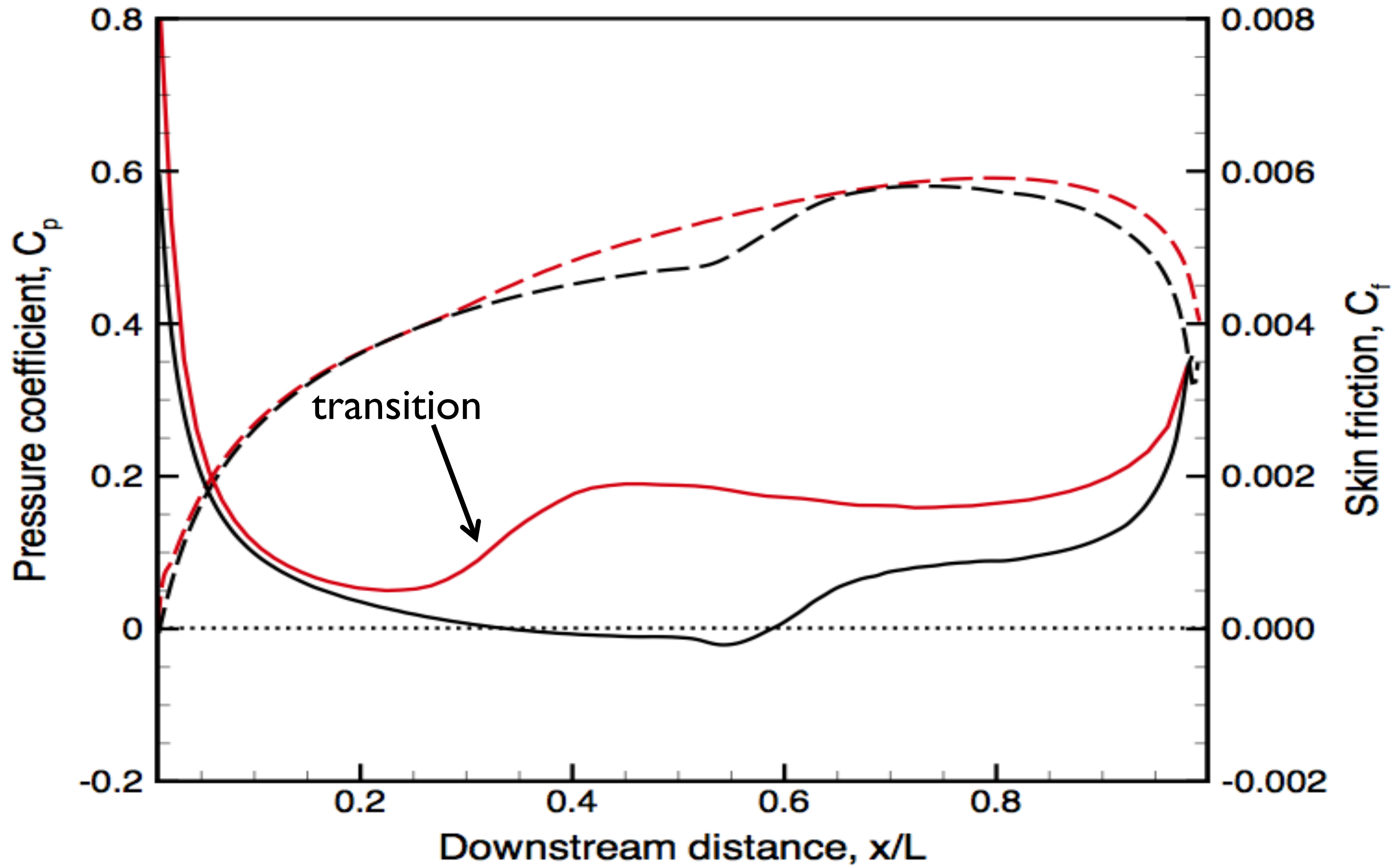
Instantaneous *velocity* contours



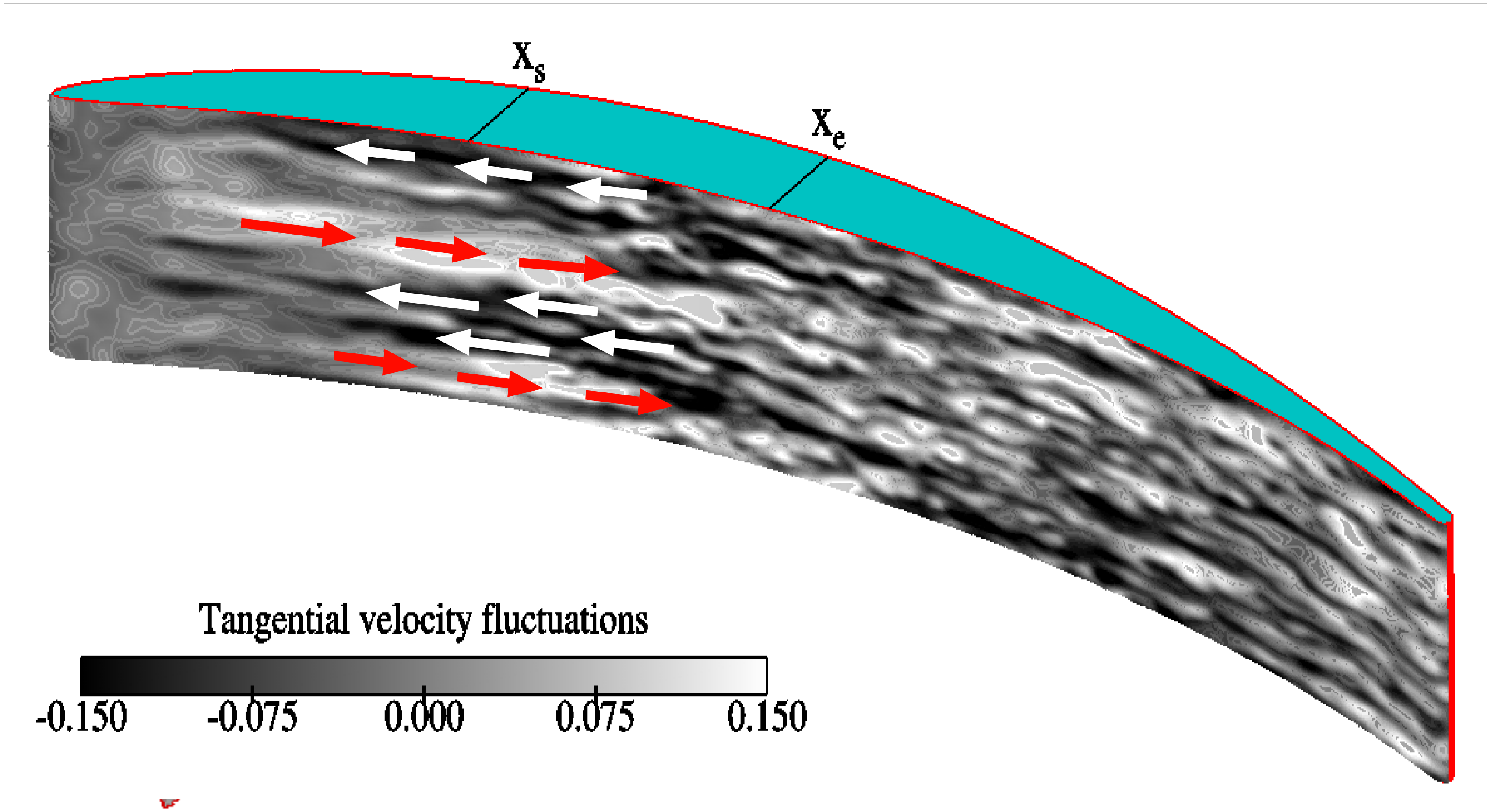
u

v

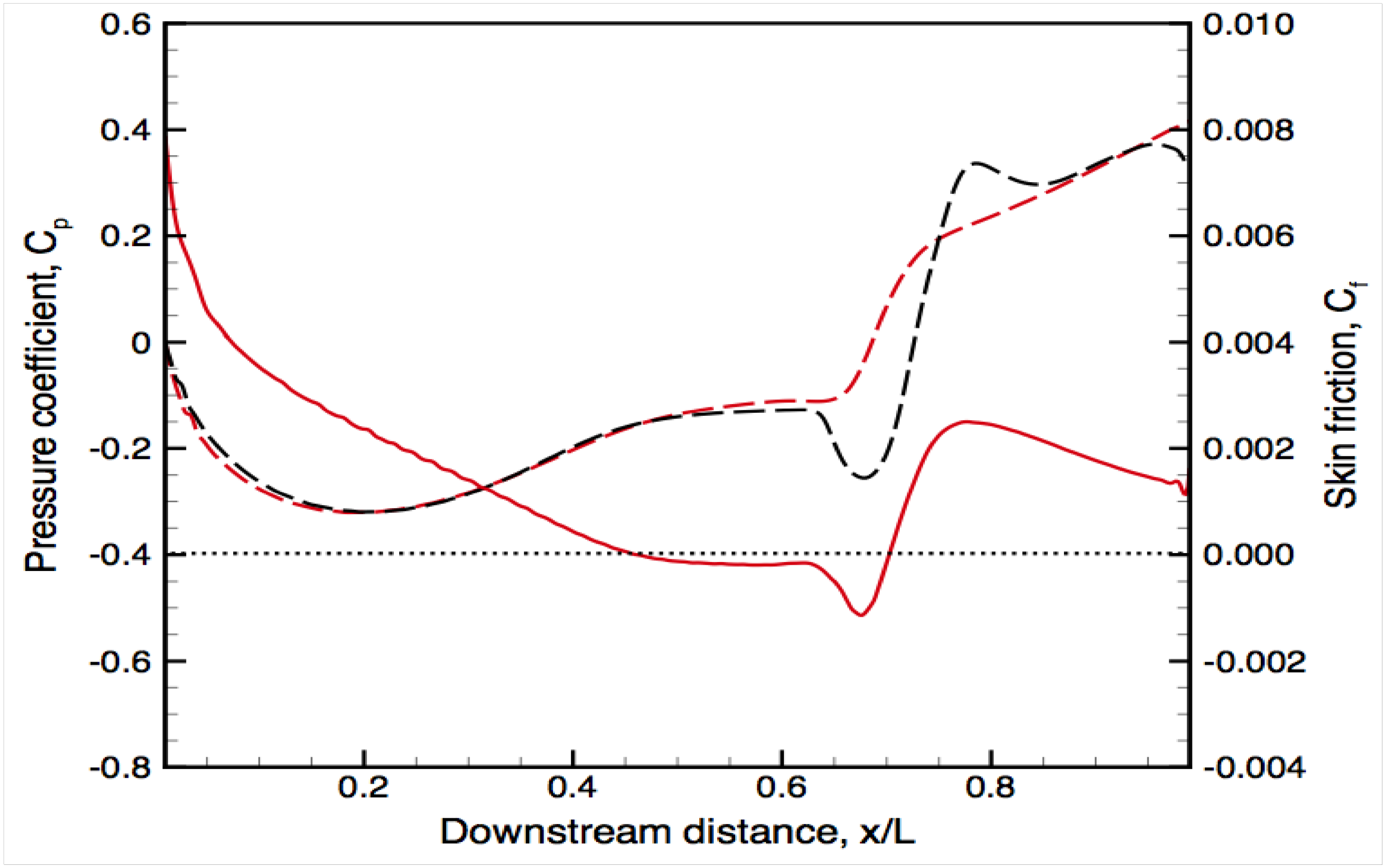
Pressure side



Jets and spots

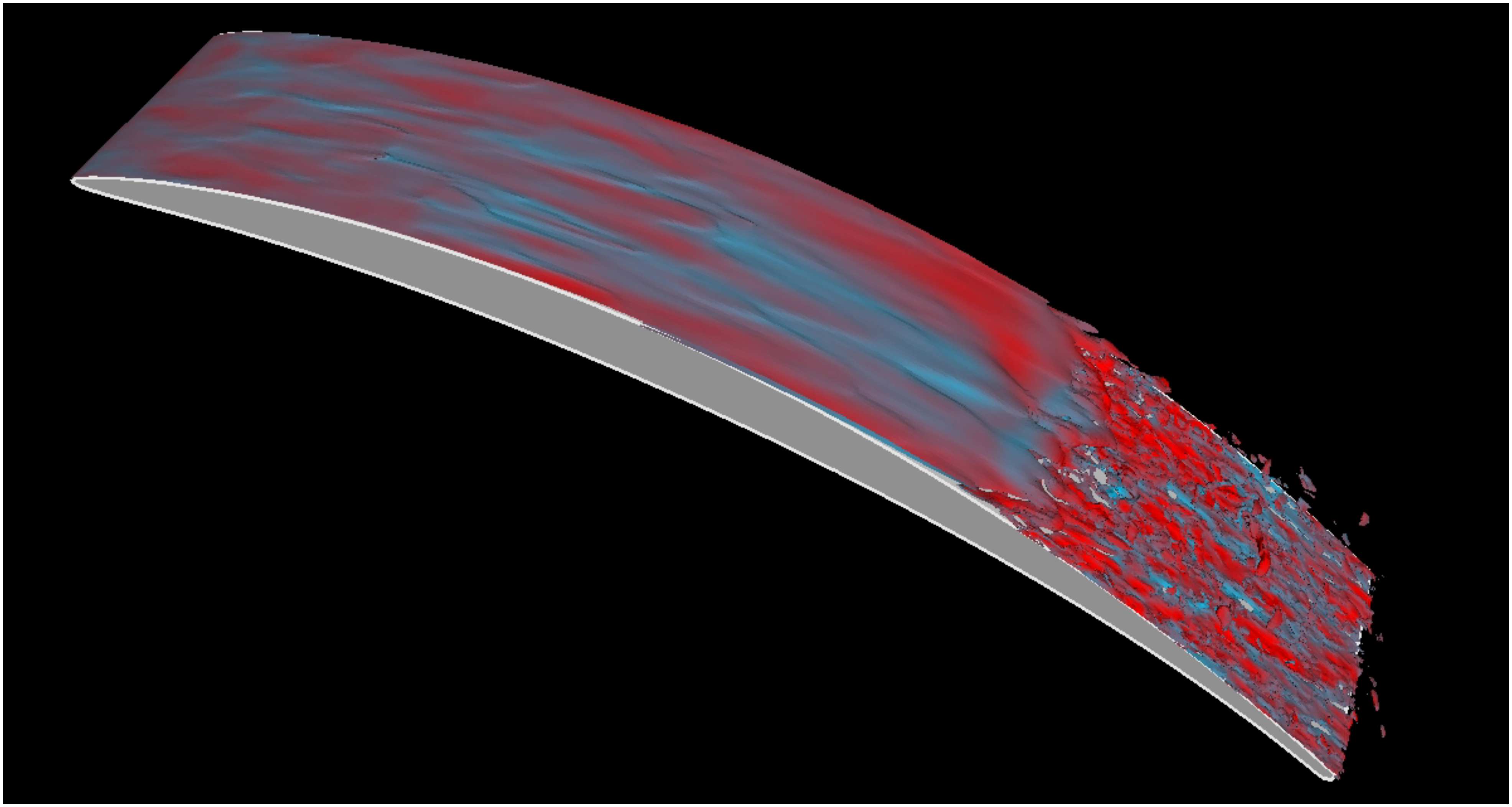


Suction side

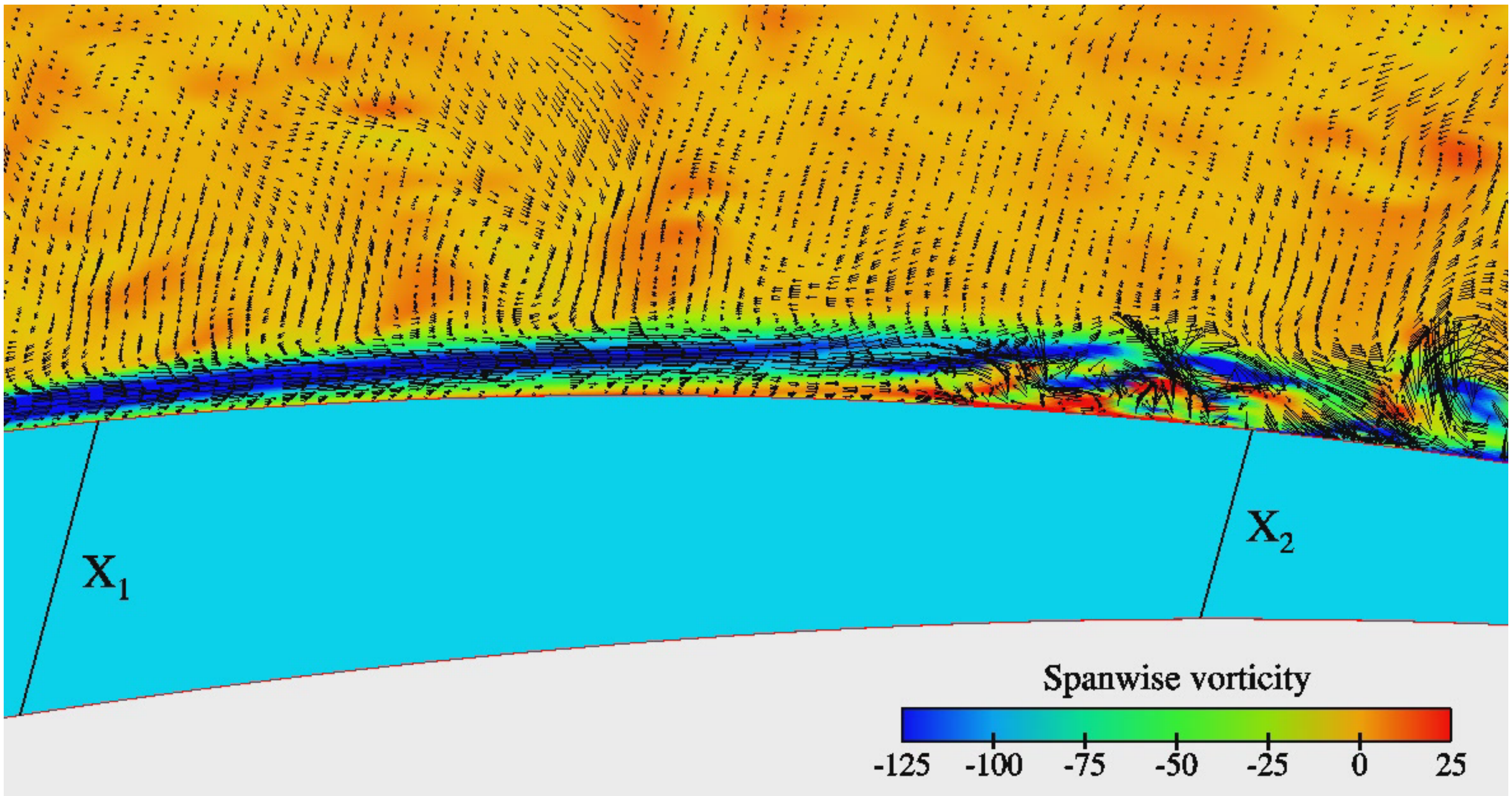


Suction side: mixed mode transition

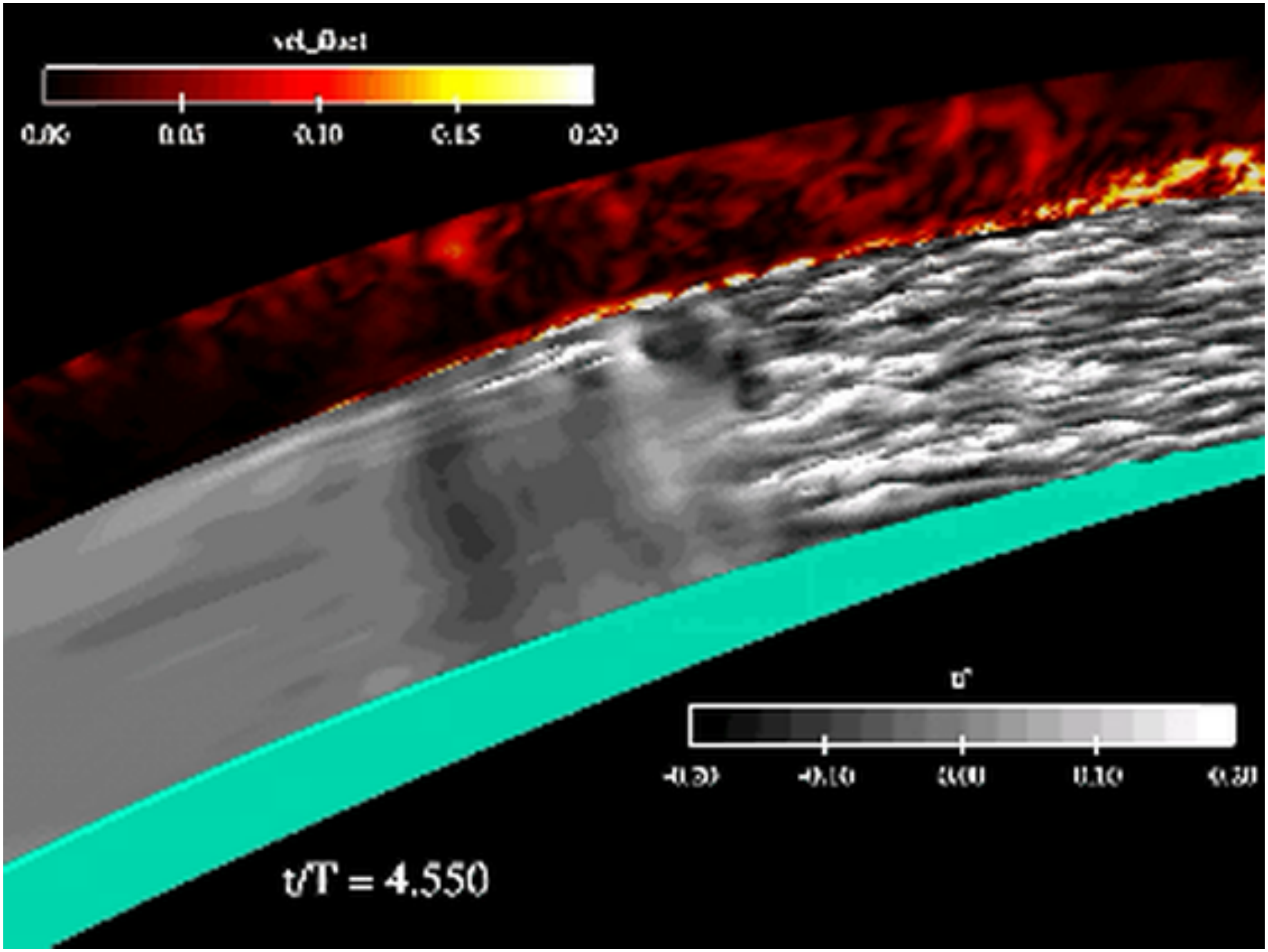
Iso-vorticity contours



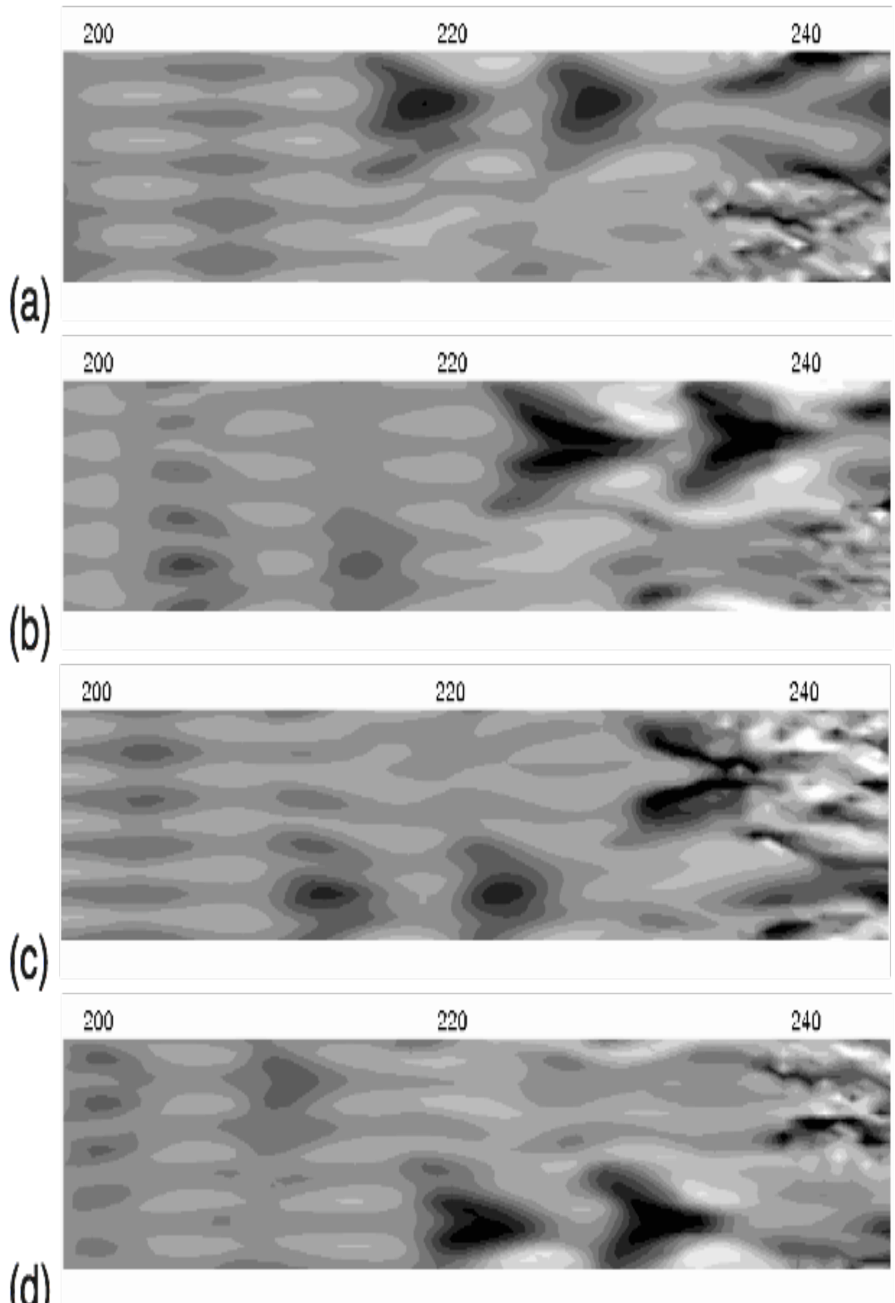
Instability on the suction side



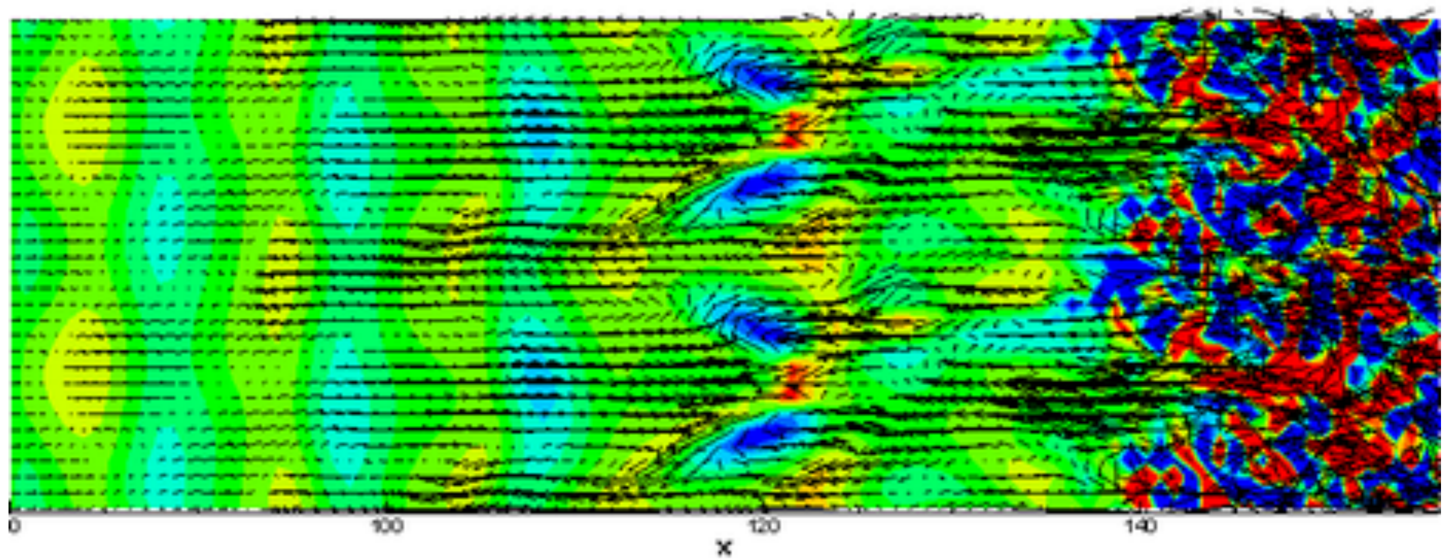
Impinging wakes

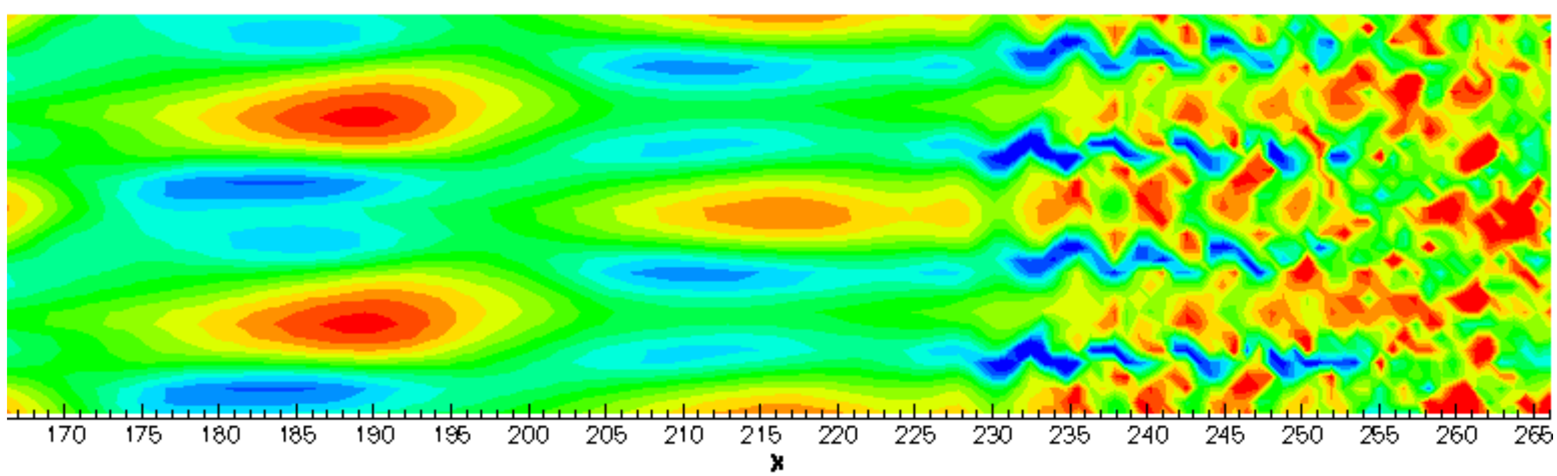


u' mode 5



Mode 2 visualiztions



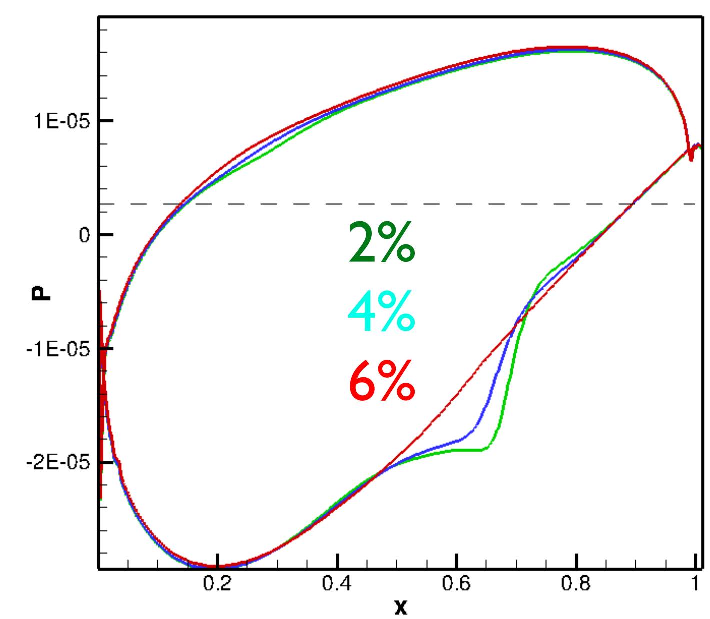


Compressor:

Continuous mode transition is seen on the pressure side

Suction side has three-dimensional instability after separation.

Suction side depends on %f.s.t. Impinging wakes intermittently reattach the boundary layer



Modeling for CFD

Two recent approaches to model transition for general purpose CFD: Laminar fluctuation (*Walters & Cokljat*) and Intermittency (*Langtry & Menter*)

Laminar fluctuation energy k_L

- Klebanoff modes?
- k_L feeds k_T

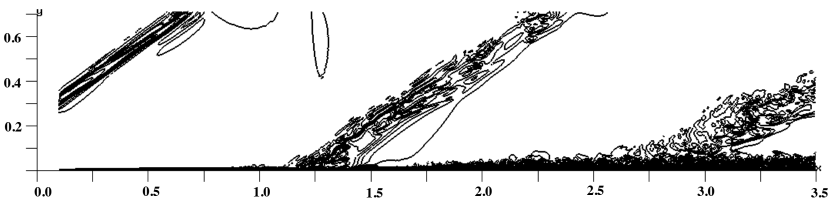
Intermittency function (*Narishima*) $0 \leq \gamma \leq 1$

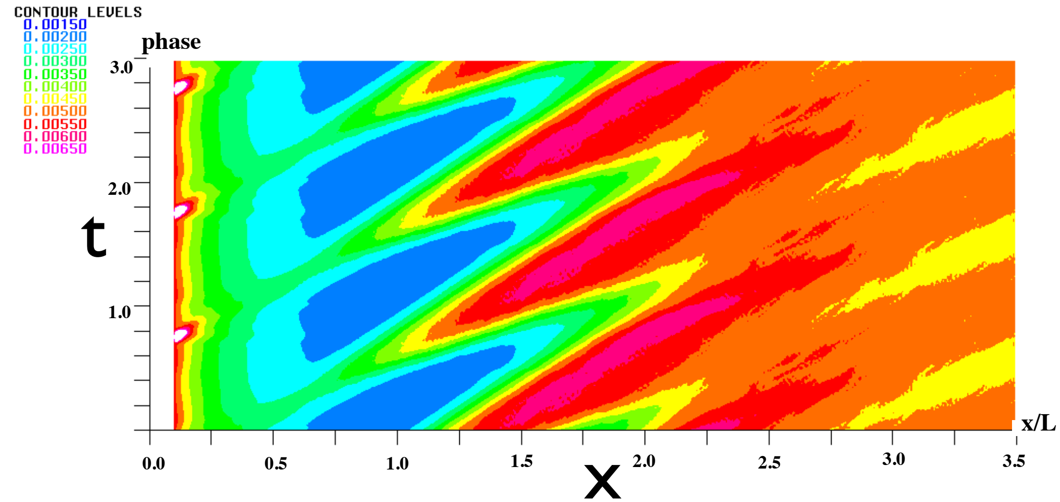
- nominally the fraction of time the flow is turbulent
- practically it is a switch that ramps up turbulent production: $\gamma = P_{turbulent}/(P_{turbulent} + P_{laminar})$

- Actually, the third approach: rely on turbulence model
  - Generally questionable: models not calibrated for transition
  - Not viable with k-ω, S-A: transition way too early

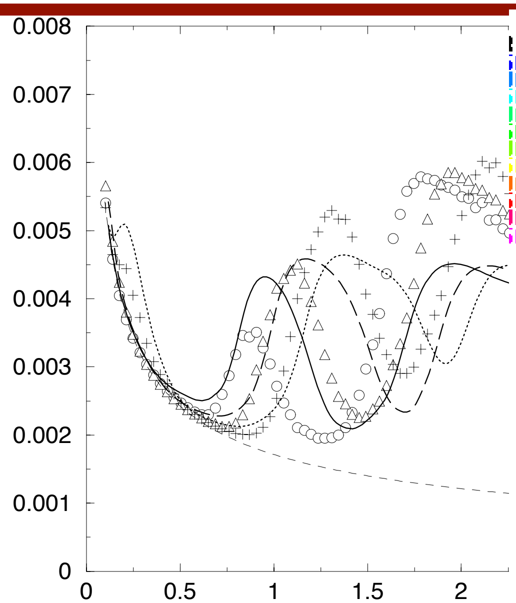
Rely on turbulence model ??

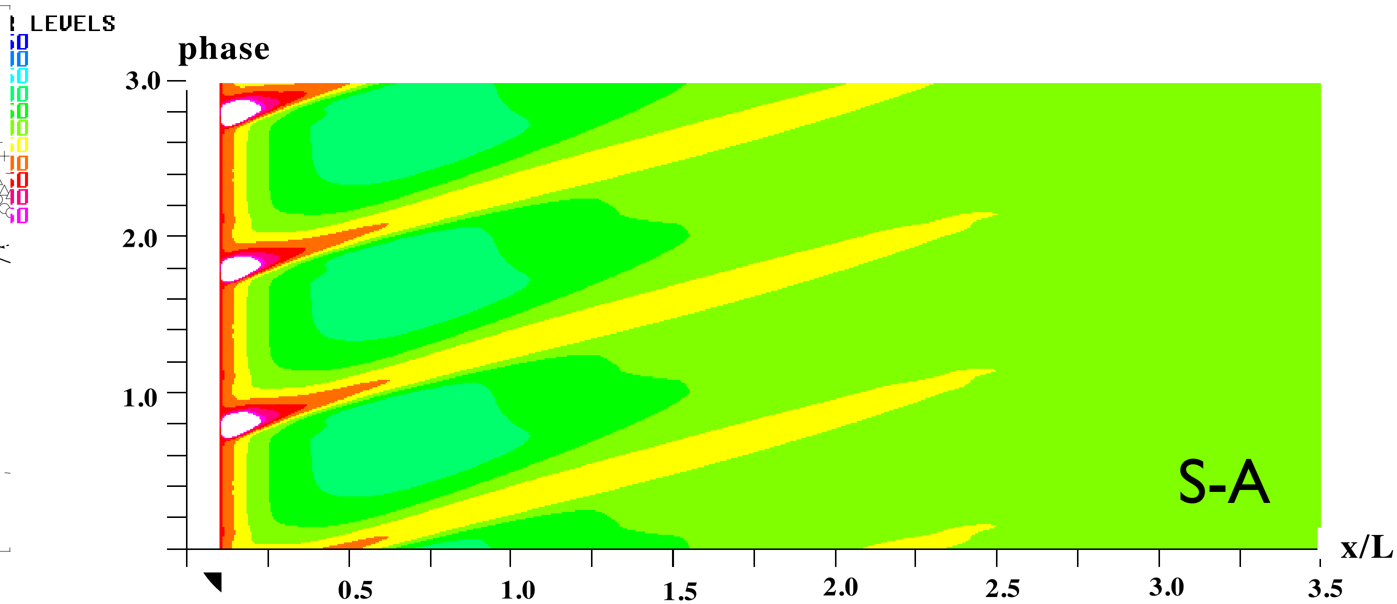
DNS data





C_f contours





Laminar fluctuation model

Walters & Cokljat 2009

Turbulent k.e. and laminar k.e.

$$\frac{Dk_T}{Dt} = P_{k_T} + R_{\mathrm{BP}} + R_{\mathrm{NAT}} - \omega k_T - D_T + \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\alpha_T}{\sigma_k}\right)\frac{\partial k_T}{\partial x_j}\right]$$

K-modes
or
TS waves?

$$\frac{Dk_L}{Dt} = P_{k_L} - R_{\mathrm{BP}} - R_{\mathrm{NAT}} - D_L + \frac{\partial}{\partial x_j}\left[\nu\frac{\partial k_L}{\partial x_j}\right]$$

$$\frac{D\omega}{Dt} = C_{\omega 1}\frac{\omega}{k_T}P_{k_T} + \left(\frac{C_{\omega R}}{f_W} - 1\right)\frac{\omega}{k_T}(R_{\mathrm{BP}} + R_{\mathrm{NAT}}) - C_{\omega 2}\omega^2$$
$$+ C_{\omega 3} f_\omega \alpha_T f_W^2 \frac{\sqrt{k_T}}{d^3} + \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\alpha_T}{\sigma_\omega}\right)\frac{\partial \omega}{\partial x_j}\right]$$

Intermittency model

Langtry-Menter 2004

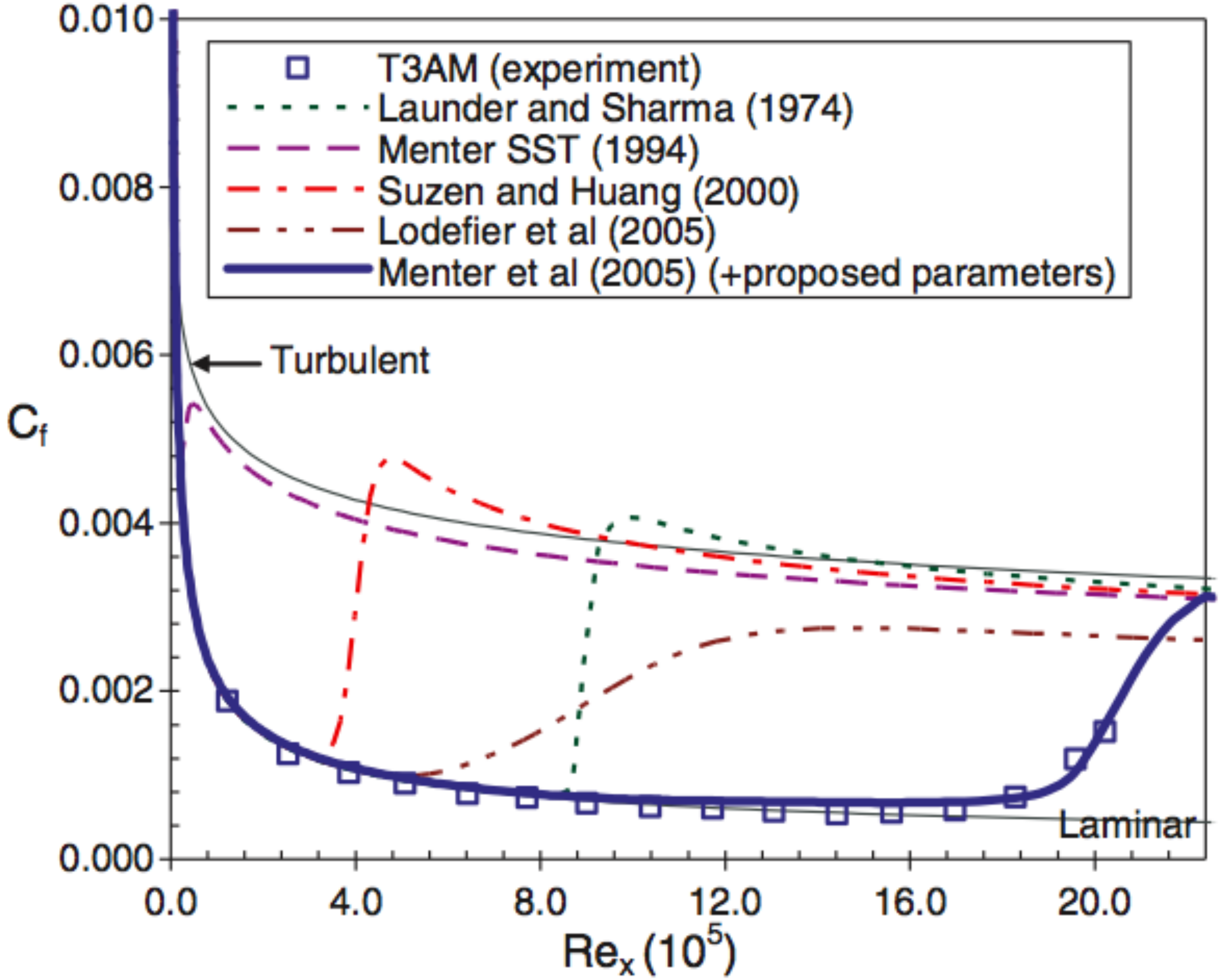
$$\frac{\partial(\rho\gamma)}{\partial t} + \frac{\partial\left(\rho U_j \gamma\right)}{\partial x_j} = P_\gamma - E_\gamma + \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_f}\right)\frac{\partial\gamma}{\partial x_j}\right]$$

source

$$P_{\gamma 1} = F_{length} c_{a1} \rho S \left[\gamma F_{onset}\right]^{0.5} (1 - \gamma)$$

Sink to ensure laminar region

$$E_\gamma = c_{a2} \rho \Omega \gamma F_{turb} (c_{e2}\gamma - 1)$$



T3AM (experiment)
Launder and Sharma (1974)
Menter SST (1994)
Suzen and Huang (2000)
Lodefier et al (2005)
Menter et al (2005) (+proposed parameters)
Turbulent
Laminar
C_f
$Re_x (10^5)$
0.010
0.008
0.006
0.004
0.002
0.000
0.0
4.0
8.0
12.0
16.0
20.0

Intermittency model

The transport equation for the intermittency, γ, reads:

$$\frac{\partial(\rho\gamma)}{\partial t}+\frac{\partial(\rho U_j\gamma)}{\partial x_j}=P_{\gamma 1}-E_{\gamma 1}+P_{\gamma 2}-E_{\gamma 2}+\frac{\partial}{\partial x_j}\left[\left(\mu+\frac{\mu_t}{\sigma_\gamma}\right)\frac{\partial\gamma}{\partial x_j}\right] \quad (3)$$

The transition sources are defined as follows:

$$P_{\gamma 1}=F_{length}\,c_{a1}\,\rho S\left[\gamma F_{onset}\right]^{c_a};\quad E_{\gamma 1}=c_{e1}P_{\gamma 1}\gamma \quad (4)$$

where S is the strain rate magnitude. F_{length} is an empirical correlation that controls the length of the transition region. The destruction/relaminarization sources are defined as follows:

$$P_{\gamma 2}=c_{a2}\,\rho\Omega\,\gamma F_{turb};\quad E_{\gamma 2}=c_{e2}P_{\gamma 2}\gamma \quad (5)$$

where Ω is the vorticity magnitude. The transition onset is controlled by the following functions:

$$\mathrm{Re}_V=\frac{\rho y^2 S}{\mu};\quad R_T=\frac{\rho k}{\mu\omega} \quad (6)$$

$$F_{onset1}=\frac{\mathrm{Re}_v}{2.193\cdot\mathrm{Re}_{\theta c}};\quad F_{onset2}=\min\left(\max\left(F_{onset1},F_{onset1}^{4}\right),2.0\right)$$

$$F_{onset3}=\max\left(1-\left(\frac{R_T}{2.5}\right)^3,0\right);\quad F_{onset}=\max\left(F_{onset2}-F_{onset3},0\right);\quad F_{turb}=e^{-\left(\frac{R_T}{4}\right)^4}$$

$\mathrm{Re}_{\theta c}$ is the critical Reynolds number where the intermittency first starts to increase in the boundary layer. Thi occurs upstream of the transition Reynolds number, $\tilde{\mathrm{Re}}_{\theta t}$, and the difference between the two must be obtaine from an empirical correlation. Both the F_{length} and $\mathrm{Re}_{\theta c}$ correlations are functions of $\tilde{\mathrm{Re}}_{\theta t}$.

The constants for the intermittency equation are:

$$c_{e1}=1.0;\quad c_{a1}=2.0;\quad c_a=0.5;\quad c_{e2}=50;\quad c_{a2}=0.06;\quad \sigma_\gamma=1.0;$$

The modification for separation-induced transition is:

$$\gamma_{sep}=\min\left(2\cdot\max\left[\left(\frac{\mathrm{Re}_v}{3.235\,\mathrm{Re}_{\theta c}}\right)-1,0\right]F_{reattach},2\right)F_{\theta t};\ F_{reattach}=e^{-\left(\frac{R_T}{20}\right)^4};\ \gamma_{eff}=\max\left(\gamma,\gamma_{sep}\right)$$

The model constants in Equ. 10 have been adjusted from those of Menter et al (2004) in order to improve th predictions of separated flow transition. The main difference is the constant that controls the relation between Re and $\mathrm{Re}_{\theta c}$ was changed from 2.193, it's value for a Blasius boundary layer, to 3.235, the value at a separation poi where the shape factor is 3.5 (see for example Figure 2 in Menter et al, 2004). The boundary condition for γ at wall is zero flux while ... γ is equal to 1.0.

The transport equation for the transition momentum thickness Reynolds number, $\tilde{\mathrm{Re}}_{\theta t}$, reads:

$$\frac{\partial(\rho\tilde{\mathrm{Re}}_{\theta t})}{\partial t}+\frac{\partial(\rho U_j\tilde{\mathrm{Re}}_{\theta t})}{\partial x_j}=P_{\theta t}+\frac{\partial}{\partial x_j}\left[\sigma_{\theta t}(\mu+\mu_t)\frac{\partial\tilde{\mathrm{Re}}_{\theta t}}{\partial x_j}\right] \quad (11)$$

The source term is defined as follows:

$$P_{\theta t}=c_{\theta t}\frac{\rho}{t}\left(\mathrm{Re}_{\theta t}-\tilde{\mathrm{Re}}_{\theta t}\right)\left(1.0-F_{\theta t}\right);\quad t=\frac{500\,\mu}{\rho U^2} \quad (12)$$

$$F_{\theta t}=\min\left(\max\left(F_{wake}\cdot e^{-\left(\frac{y}{\delta}\right)^4},1.0-\left(\frac{\gamma-1/c_{e2}}{1.0-1/c_{e2}}\right)^2\right),1.0\right) \quad (13)$$

$$\theta_{BL}=\frac{\tilde{\mathrm{Re}}_{\theta t}\,\mu}{\rho U};\quad \delta_{BL}=\frac{15}{2}\theta_{BL};\quad \delta=\frac{50\Omega y}{U}\cdot\delta_{BL} \quad (14)$$

$$\mathrm{Re}_\omega=\frac{\rho\omega y^2}{\mu};\quad F_{wake}=e^{-\left(\frac{\mathrm{Re}_\omega}{1E+5}\right)^2} \quad (15)$$

The model constants for the $\tilde{\mathrm{Re}}_{\theta t}$ equation are:

$$c_{\theta t}=0.03;\quad \sigma_{\theta t}=2.0 \quad (16)$$

The boundary condition for $\tilde{\mathrm{Re}}_{\theta t}$ at a wall is zero flux. The boundary condition for $\tilde{\mathrm{Re}}_{\theta t}$ at an inlet should be calculated from the empirical correlation based on the inlet turbulence intensity.

The model contains three empirical correlations. $\mathrm{Re}_{\Theta t}$ is the transition onset as observed in experiments. This has been modified from Menter et al. (2004) in order to improve the predictions for natural transition. It is used in Eq.12. F_{length} is the length of the transition zone and goes into Eq. 4. $\mathrm{Re}_{\Theta c}$ is the point where the model is activated in order to match both, $\mathrm{Re}_{\Theta t}$ and F_{length}, it goes into Eq. 7. At present these empirical correlations are proprietary and are not given in the paper.

$$\mathrm{Re}_{\Theta t}=f(Tu,\lambda);\quad F_{length}=f\left(\tilde{\mathrm{Re}}_{\Theta t}\right);\quad \mathrm{Re}_{\Theta c}=f\left(\tilde{\mathrm{Re}}_{\Theta t}\right) \quad (17)$$

The first empirical correlation is a function of the local turbulence intensity, Tu, and the Thwaites' pressure gradient coefficient λ_θ defined as:

$$\lambda_\theta=(\theta^2/\nu)dU/ds \quad (18)$$

Motive: Can formulation be simpler, more comprehensible?

Physics: free-stream disturbance diffuses into boundary layer

Diffusion

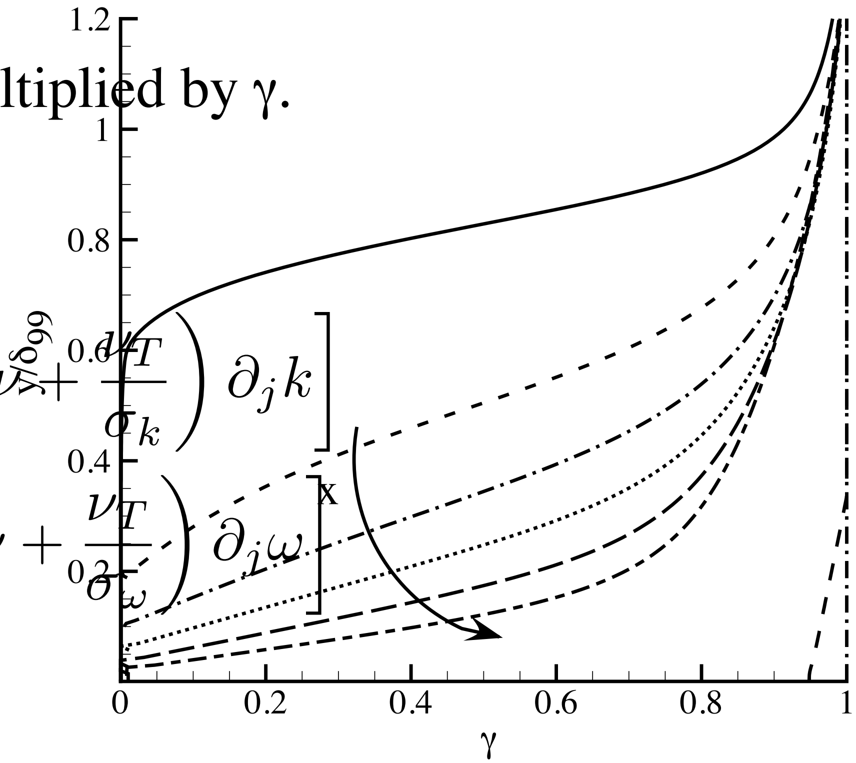
$$\frac{D\gamma}{Dt} = \partial_j\left[\left(\frac{\nu}{\sigma_l}+\frac{\nu_T}{\sigma_\gamma}\right)\partial_j\gamma\right] + F_\gamma|\Omega|(1-\gamma)\sqrt{\gamma}$$

$\sigma_l = 5$ and $\sigma_\gamma = 0.23$ $\gamma_\infty = 1,\ \partial_y\gamma|_0 = 0$

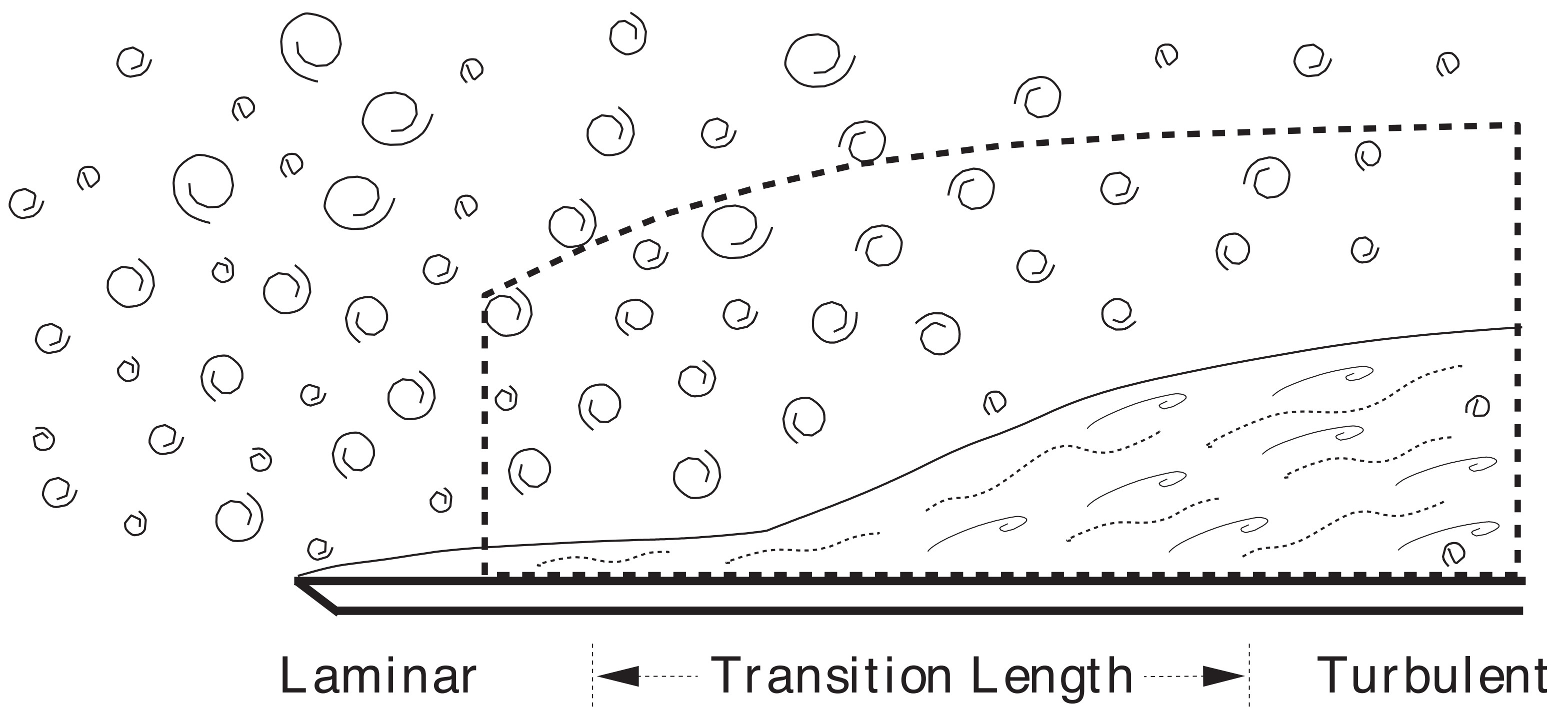
In energy equation production is multiplied by γ.
Applied to *k*-*ω* model

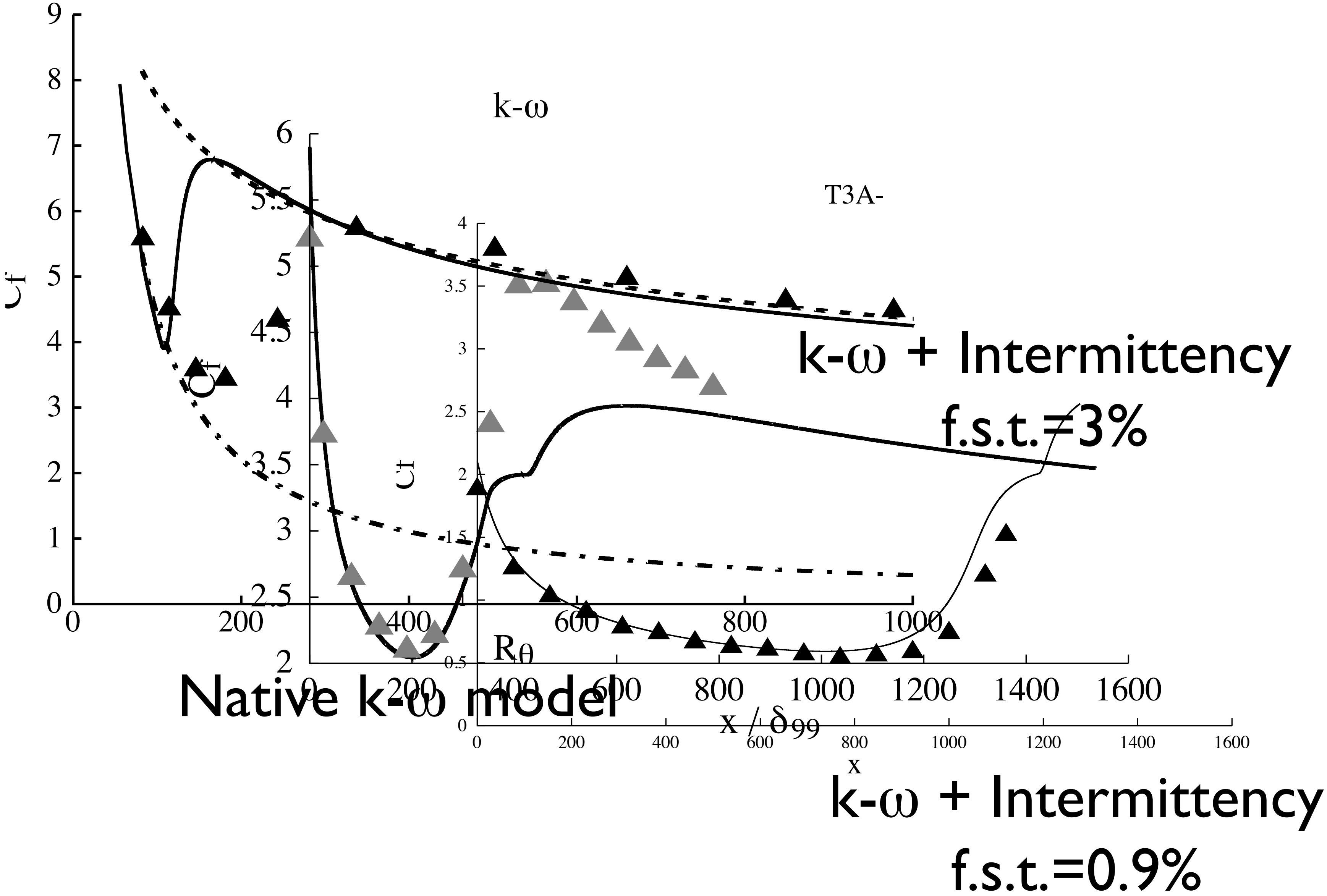
$$\frac{Dk}{Dt} = 2\,\gamma\,\nu_T|S|^2 - C_\mu k\omega + \partial_j\left[\left(\nu+\frac{\nu_T}{\sigma_k}\right)\partial_j k\right]$$

$$\frac{D\omega}{Dt} = 2C_{\omega 1}|S|^2 - C_{\omega 2}\omega^2 + \partial_j\left[\left(\nu+\frac{\nu_T}{\sigma_\omega}\right)\partial_j \omega\right]$$



Flat plate boundary layer

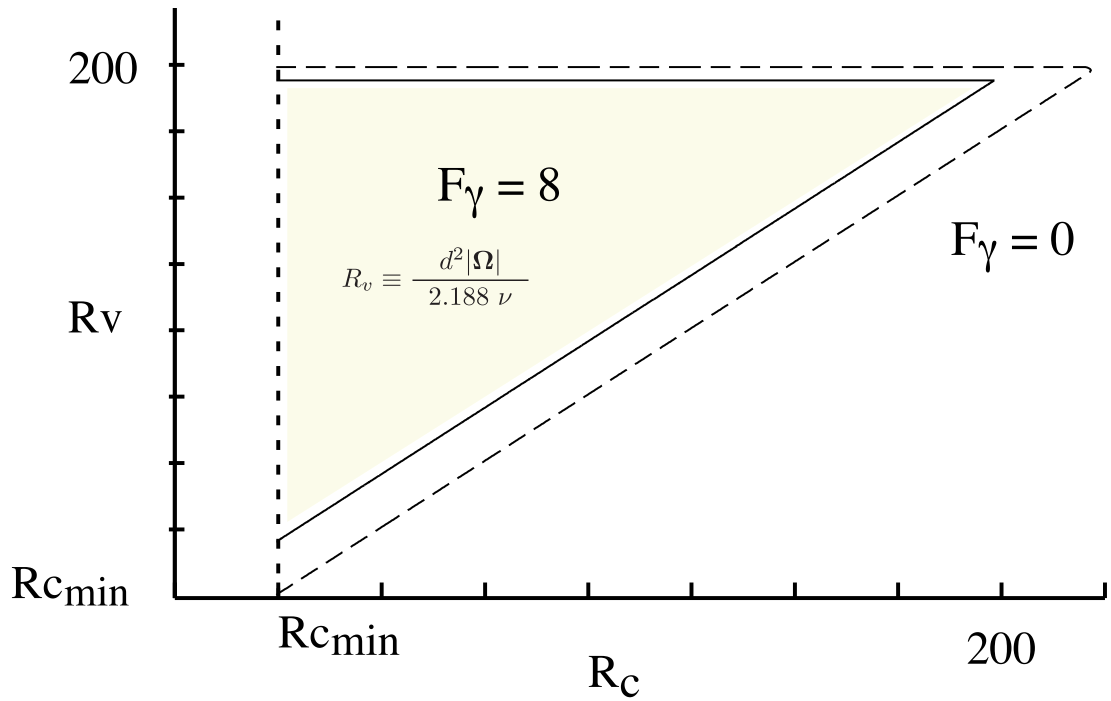



k-ω
T3A-
C_f
R_θ
x / δ_{99}
x
k-ω + Intermittency
f.s.t.=3%
Native k-ω model
k-ω + Intermittency
f.s.t.=0.9%

Source: Fγ

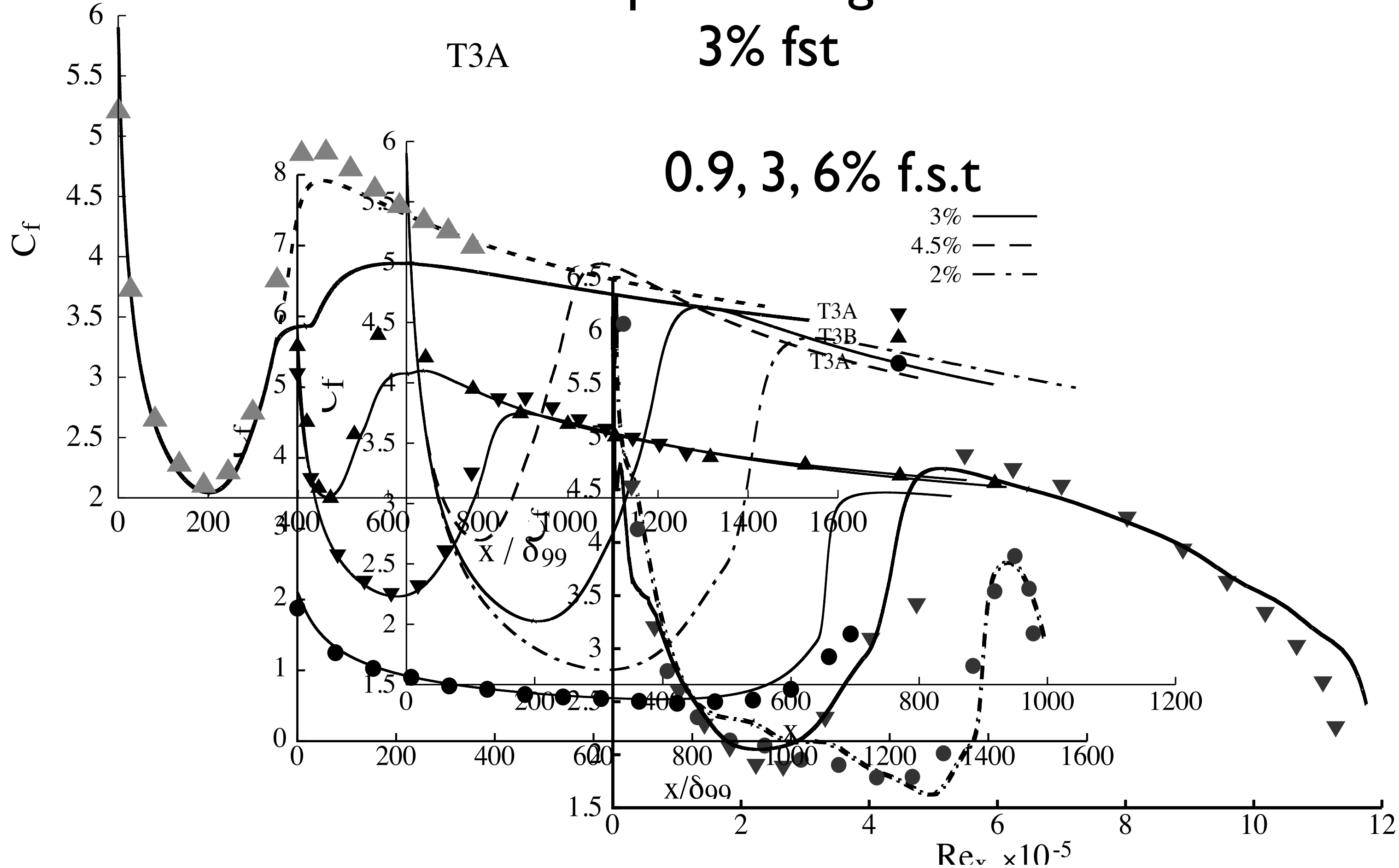
$$\frac{D\gamma}{Dt} = \partial_j \left[\left(\frac{\nu}{\sigma_l} + \frac{\nu_T}{\sigma_\gamma} \right) \partial_j \gamma \right] + F_\gamma |\Omega| (1-\gamma) \sqrt{\gamma}$$

$$R_v \equiv \frac{d^2 |\mathbf{\Omega}|}{2.188\ \nu}$$



$$R_c = 400 - 260 \min\left(\frac{T_\omega}{2}, 1\right)$$

Zero pressure gradient
3% fst



FPG-APG at 2 Reynolds #s (T3C2, 5)

Summary

Theory:

Continuous mode transition is a theoretical framework for bypass beneath vortical disturbances. Disturbances diffuse into the boundary layer, moderated by shear filtering

DNS:

Transition is at *low Reynolds number*: DNS on realistic geometries is quite feasible

Modeling:

Intermittency/RANS models probably can be simplified. That will make it easier to apply this approach to other RANS closures, and to modify models