Turbulence As A Source of Sound

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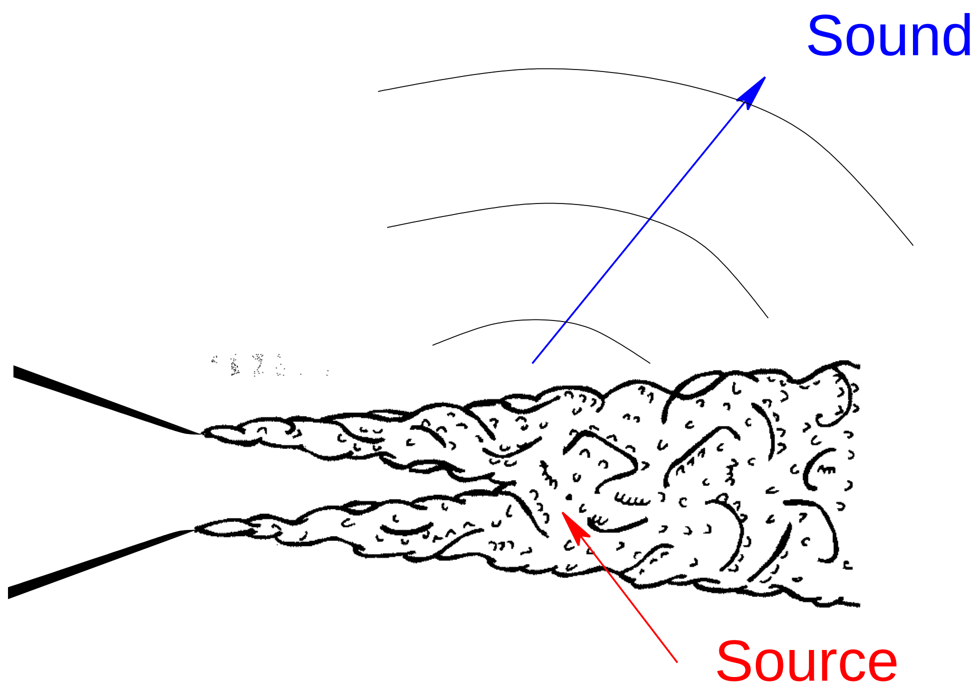
Turbulence Makes Sound



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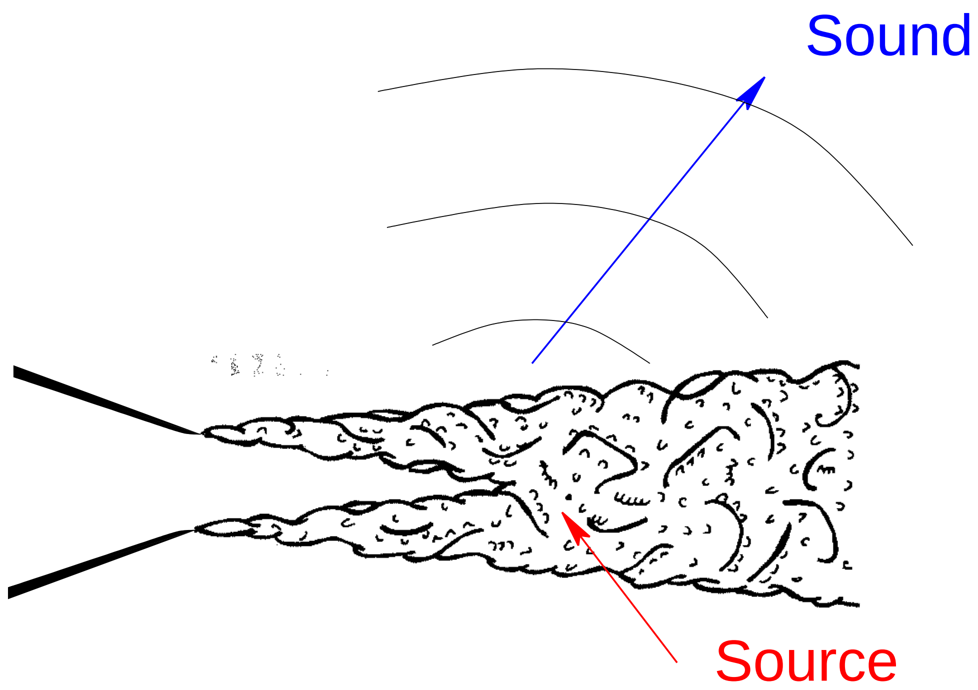

- Increasingly a concern...

Outline

- The acoustic limit
- Sources and sound
- Turbulence: the acoustic analogy
- Challenges in predicting sound from turbulence
  - Complex turbulence statistic
  - Phase velocity restriction
  - Coupled process: different source components, refraction,...
- Robustness as a criterion for formulation selection
- Outlook

Sound Energies Are Small

- Acoustic energy radiated from a jet at take-off insufficient to boil and egg
- Double exit velocity: ~ 250 times more acoustic power
- Typically neglected in conservation of energy analysis of mechanical systems

Acoustic Limit

What is sound?

- A solution of the compressible flow equations

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0$$

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} + \frac{\partial p}{\partial x_i} = \left[\text{viscous terms}\right]$$

- Approximately inviscid: interested in sound that propagates long distances, many wavelengths

$$f = 4\ \mathrm{kHz} \qquad \Rightarrow \qquad \lambda = a_o/f \approx 0.1\ \mathrm{m}$$

What is sound?

- Low energy $\rightarrow$ low amplitude $\rightarrow$ linearize

$$\rho(\mathbf{x},t) = \rho_o + \rho'(\mathbf{x},t) \qquad u_i(\mathbf{x},t) = 0 + u_i'(\mathbf{x},t) \qquad p(\mathbf{x},t) = p_0 + p'(\mathbf{x},t)$$

yielding

$$\frac{\partial \rho'}{\partial t} + \rho_o \frac{\partial u_i'}{\partial x_i} = 0$$
$$\rho_o \frac{\partial u_i'}{\partial t} + \frac{\partial p'}{\partial x_i} = 0$$

What is sound?

- Eliminate velocity:

$$\frac{\partial}{\partial t}\,[\text{mass}] \quad \Rightarrow \quad \frac{\partial^2 \rho'}{\partial t^2} + \boxed{\rho_o \frac{\partial^2 u_i'}{\partial t \partial x_i}} = 0$$

$$\frac{\partial}{\partial x_i}\,[\text{momentum}] \quad \Rightarrow \quad \boxed{\rho_o \frac{\partial^2 u_i'}{\partial t \partial x_i}} + \frac{\partial^2 p'}{\partial x_i \partial x_i} = 0$$

and subtract

$$\frac{\partial^2 \rho'}{\partial t^2} - \frac{\partial^2 p'}{\partial x_i \partial x_i} = 0$$

What is sound?

- Speed of sound

$$a_o = \left(\frac{\partial p}{\partial \rho}\right)_s \approx \frac{p'}{\rho'} \qquad \Rightarrow \qquad p' = a_o^2 \rho' \; + \; \text{h.o.t.}$$

yielding the linear, scalar wave equation for ρ'

$$\frac{\partial^2 \rho'}{\partial t^2} - a_o^2 \frac{\partial^2 \rho'}{\partial x_i \partial x_i} = 0$$

What is sound?

- Speed of sound

$$a_o = \left(\frac{\partial p}{\partial \rho}\right)_s \approx \frac{p'}{\rho'} \qquad \Rightarrow \qquad p' = a_o^2 \rho' \ + \text{ h.o.t.}$$

yielding the linear, scalar wave equation for ρ'

$$\frac{\partial^2 \rho'}{\partial t^2} - a_o^2 \frac{\partial^2 \rho'}{\partial x_i \partial x_i} = 0$$

or for p'

$$\frac{\partial^2 p'}{\partial t^2} - a_o^2 \frac{\partial^2 p'}{\partial x_i \partial x_i} = 0$$

Solution Forms

- Plane waves: $\omega^2 = a_o^2 k^2$

$$\rho' \sim \exp\left[i(\mathbf{k}\cdot\mathbf{x} + \omega t)\right] = \exp\left[ik(\hat{\mathbf{k}}\cdot\mathbf{x} \pm a_o t)\right]$$

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- Cylindrical waves (*e.g.* $r^2 = x_1^2 + x_2^2$)

$$\rho' \sim H_0^{(1 \text{ or } 2)}(kr)\exp\left[ika_o t\right] \sim \left[\frac{2}{\pi k r}\right]^{1/2} \exp\left[\mp ik(r - a_o t) \mp i\frac{\pi}{4}\right] \sim \frac{1}{r^{1/2}}$$

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- Spherical waves ($r = |\mathbf{x}|$)

$$\rho' \sim \frac{1}{r}\exp[ik(r \pm a_o t)] \sim \frac{1}{r}$$

Acoustic Perturbations Are Related

- Plane wave traveling in $+x_1$:

$$\rho' = \Re\left[Ae^{ikx-i\omega t}\right] \qquad u_1' = \Re\left[\frac{a_o}{\rho_o}Ae^{ikx-i\omega t}\right] \qquad p' = \Re\left[Aa_o^2e^{ikx-i\omega t}\right]$$

$$u_2' = u_3' = 0$$

so

$$p' = \rho_o a_o u_1' \qquad \rho' = \frac{\rho_o}{a_o}u_1'$$

Acoustic Intensity

- Acoustic intensity, mean power flux

$$I = \langle p' u' \rangle = \frac{a_o^3}{\rho_o} \left\langle (\rho')^2 \right\rangle$$

- Large r:

cylindrical: $I \sim \dfrac{1}{r}$ spherical: $I \sim \dfrac{1}{r^2}$

- Intensity usually metric of practical interest

Sources of Sound

Sources of Sound

- A mass source...

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = \boxed{Q(\mathbf{x}, t)}$$

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} + \frac{\partial p}{\partial x_i} = \text{viscous terms}$$

- Linearize, differentiate, form wave equation,....

$$\frac{\partial^2 \rho'}{\partial t^2} - a_o^2 \frac{\partial^2 \rho'}{\partial x_i \partial x_i} = \boxed{\frac{\partial Q}{\partial t} \equiv q(\mathbf{x}, t)}$$

Green's Function Solution

- Greens function:

$$\frac{\partial^2 G}{\partial t^2} - a_o^2 \frac{\partial^2 G}{\partial x_i \partial x_i} = \delta(\mathbf{x})\delta(t)$$

has solution

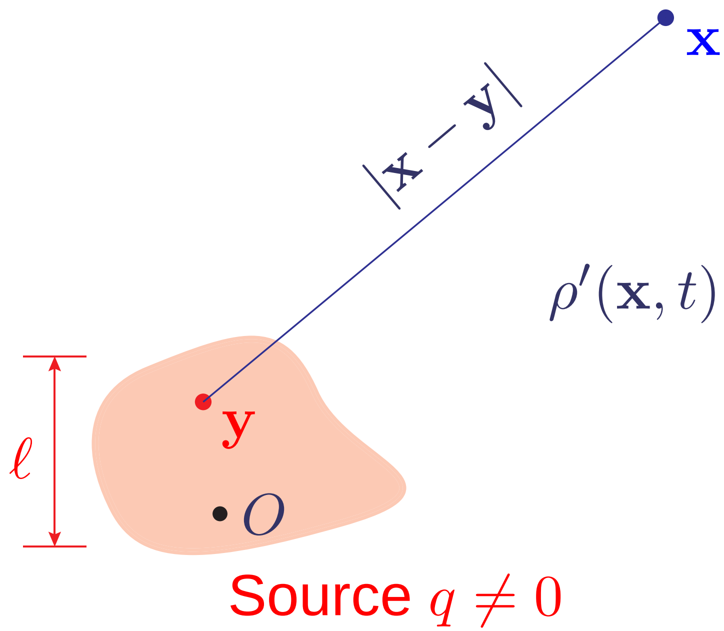
$$G(\mathbf{x}, t) = \frac{\delta(\mathbf{x} - a_o t)}{4\pi a_o |\mathbf{x}|} \sim \frac{1}{r}$$

- Solution of

$$\frac{\partial^2 \rho'}{\partial t^2} - a_o^2 \frac{\partial^2 \rho'}{\partial x_i \partial x_i} = q(\mathbf{x}, t)$$

is

$$\rho'(\mathbf{x}, t) = \frac{1}{4\pi a_o^2} \int \frac{q\left(\mathbf{y}, t - \frac{|\mathbf{x}-\mathbf{y}|}{a_o}\right)}{|\mathbf{x} - \mathbf{y}|} \, d\mathbf{y}$$



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- Source scales: ℓ, u, ρ_o
- $q \sim \rho_o u^2/\ell^2$

Compact Source Approximation

$$\rho'(\mathbf{x},t) = \frac{1}{4\pi a_o^2} \int \frac{q\left(\mathbf{y}, t - \frac{|\mathbf{x}-\mathbf{y}|}{a_o}\right)}{|\mathbf{x}-\mathbf{y}|}\, d\mathbf{y}$$

- Source scales: ℓ, u, ρ_o
- $q \sim \rho_o u^2/\ell^2$
- Difference in emission times across source $\tau_{\text{emission}} = \ell/a_o$
- Source changes on time scale $\tau_{\text{source}} = \ell/u$
- Consider $\tau_{\text{emission}} \ll \tau_{\text{source}}$:
  - $\tau_{\text{emission}}/\tau_{\text{source}} = u/a_o \equiv m \ll 1$ — low Mach number
  - integrand: $q\left(\mathbf{y}, t - \frac{|\mathbf{x}-\mathbf{y}|}{a_o}\right) \approx q\left(\mathbf{y}, t - \frac{|\mathbf{x}|}{a_o}\right)$

Far-field Intensity

- Consider far field $|\mathbf{x}| \gg \ell$, so

$$\frac{1}{|\mathbf{x}-\mathbf{y}|} \approx \frac{1}{|\mathbf{x}|}$$

- Compact-source and far-field approximations

$$\rho'(\mathbf{x},t) = \underbrace{\frac{1}{4\pi a_o^2 |\mathbf{x}|}}_{\sim 1/r a_o^2} \int \underbrace{q\left(\mathbf{y}, t - \frac{|\mathbf{x}|}{a_o}\right)}_{\sim \rho_o u^2/\ell^2} \underbrace{d\mathbf{y}}_{\sim \ell^3}$$

thus: $\rho' \sim \rho_o \frac{\ell}{r} m^2$

- Intensity

$$I \sim (\rho')^2 \sim m^4$$

Sources of Sound: Force

- A momentum source...

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0$$

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} + \frac{\partial p}{\partial x_i} = \boxed{F_i(\mathbf{x},t)} + [\text{ vis. terms }]$$

- Linearize, differentiate, inviscid, form acoustic equations,....

$$\frac{\partial^2 \rho'}{\partial t^2} - a_o^2 \frac{\partial^2 \rho'}{\partial x_i \partial x_i} = \boxed{\frac{\partial F_i}{\partial y_i}}$$

Far-field, Compact

- Same Green's function solution

$$\rho'(\mathbf{x},t) = \frac{1}{4\pi a_o^2} \int \frac{\partial F_i\left(\mathbf{y}, t - \frac{|\mathbf{x}-\mathbf{y}|}{a_o}\right)}{\partial y_i} \frac{1}{|\mathbf{x}-\mathbf{y}|}\, d\mathbf{y}$$

- Far-field, compact ($m \ll 1$):

$$\rho'(\mathbf{x},t) = \frac{1}{4\pi a_o^2 |\mathbf{x}|} \int \frac{\partial F_i\left(\mathbf{y}, t - \frac{|\mathbf{x}|}{a_o}\right)}{\partial y_i}\, d\mathbf{y}$$

- Same source scaling ($F \sim \rho_o u^2/\ell$) also yields

$$\rho' \sim \rho_o \frac{\ell}{r} m^2 \qquad \text{and} \qquad I \sim \rho^2 \sim \rho_o^2 \left(\frac{\ell}{r}\right)^2 m^4$$

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but this is very wrong....

A Missed Cancellation

$$\rho'(\mathbf{x},t) = \frac{1}{4\pi a_o^2|\mathbf{x}|}\int \frac{\partial F_i\left(\mathbf{y}, t - \frac{|\mathbf{x}|}{a_o}\right)}{\partial y_i}\, d\mathbf{y}$$

- Divergence theorem for compact region of finite F_i

$$\int \frac{\partial F_i\left(\mathbf{y}, t - \frac{|\mathbf{x}|}{a_o}\right)}{\partial y_i}\, d\mathbf{y} = 0$$

Build Near-Cancellation into Formulation

$$\rho'(\mathbf{x},t) = \frac{1}{4\pi a_o^2}\int \frac{\partial F\left(\mathbf{y}, t - \frac{|\mathbf{x}-\mathbf{y}|}{a_o}\right)}{\partial y_i}\frac{1}{|\mathbf{x}-\mathbf{y}|}\,d\mathbf{y}$$

$$= \frac{1}{4\pi a_o^2}\frac{\partial}{\partial x_i}\int \frac{F\left(\mathbf{y}, t - \frac{|\mathbf{x}-\mathbf{y}|}{a_o}\right)}{|\mathbf{x}-\mathbf{y}|}\,d\mathbf{y}$$

via

$$\frac{\partial}{\partial x_i}\int f(\mathbf{y})g(\mathbf{x}-\mathbf{y})\,d\mathbf{y} = \int f(\mathbf{y})\frac{\partial}{\partial x_i}g(\mathbf{x}-\mathbf{y})\,d\mathbf{y}$$

$$= -\int f(\mathbf{y})\frac{\partial}{\partial y_i}g(\mathbf{x}-\mathbf{y})\,d\mathbf{y}$$

$$= +\int \frac{\partial f(\mathbf{y})}{\partial y_i}g(\mathbf{x}-\mathbf{y})\,d\mathbf{y} - \cancelto{0}{\int \frac{\partial}{\partial y_i}\left[f(\mathbf{y})g(\mathbf{x}-\mathbf{y})\right]\,d\mathbf{y}}$$

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Far-field, Compact (again)

- Far-field, compact

$$
\begin{aligned}
\rho'(\mathbf{x},t) &= \frac{1}{4\pi a_o^2}\frac{\partial}{\partial x_i}\int \frac{F\left(\mathbf{y}, t-\frac{|\mathbf{x}-\mathbf{y}|}{a_o}\right)}{|\mathbf{x}-\mathbf{y}|}\,d\mathbf{y} \\
&\approx \frac{1}{4\pi a_o^2}\frac{\partial}{\partial x_i}\int \frac{F\left(\mathbf{y}, t-\frac{|\mathbf{x}|}{a_o}\right)}{|\mathbf{x}|}\,d\mathbf{y} \\
&\approx \frac{1}{4\pi a_o^2}\int \left[\frac{\partial F\left(\mathbf{y}, t-\frac{|\mathbf{x}|}{a_o}\right)}{\partial x_i}\frac{1}{|\mathbf{x}|} + F\left(\mathbf{y}, t-\frac{|\mathbf{x}|}{a_o}\right)\frac{\partial}{\partial x_i}\left(\frac{1}{|\mathbf{x}|}\right)\right] d\mathbf{y} \\
&= -\frac{1}{4\pi a_o^2|\mathbf{x}|}\int \frac{\partial F}{\partial t}\left(\mathbf{y}, t-\frac{|\mathbf{x}|}{a_o}\right)\frac{1}{a_o}\frac{\partial|\mathbf{x}|}{\partial \mathbf{x}_i}\,d\mathbf{y} + O\left(\frac{1}{|\mathbf{x}|^2}\right)
\end{aligned}
$$

Far-field, Compact (again)

- Far-field, compact

$$\begin{aligned}
\rho'(\mathbf{x},t) &= \frac{1}{4\pi a_o^2}\frac{\partial}{\partial x_i}\int \frac{F\left(\mathbf{y}, t-\frac{|\mathbf{x}-\mathbf{y}|}{a_o}\right)}{|\mathbf{x}-\mathbf{y}|}\,d\mathbf{y} \\
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\end{aligned}$$

- $\frac{\partial|\mathbf{x}|}{\partial \mathbf{x}_i} =?$ ...

Far-field, Compact (again)

$$\rho'(\mathbf{x},t) = \frac{1}{4\pi a_o^2|\mathbf{x}|}\int \frac{\partial F}{\partial t}\left(\mathbf{y}, t - \frac{|\mathbf{x}|}{a_o}\right)\frac{1}{a_o}\frac{\partial|\mathbf{x}|}{\partial \mathbf{x}_i}\, d\mathbf{y}$$

- Noting that

$$\frac{\partial|\mathbf{x}|}{\partial x_i} = \frac{\partial}{\partial x_i}\sqrt{x_1^2 + x_2^2 + x_3^2} = \frac{x_i}{|\mathbf{x}|}$$

yields

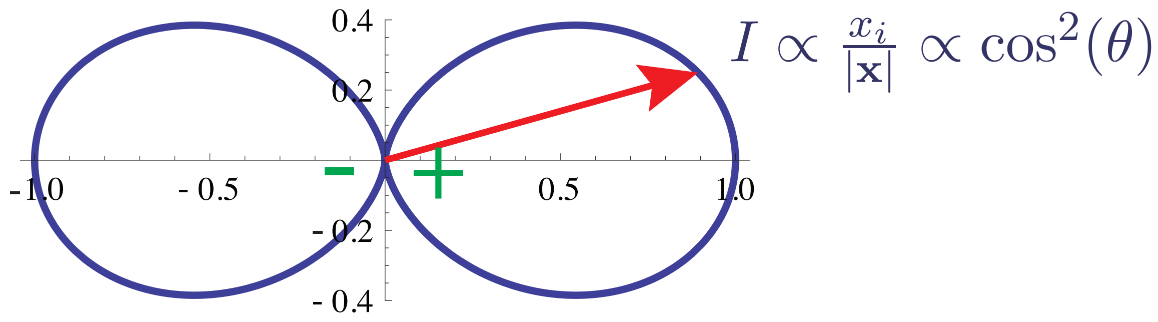
$$\rho'(\mathbf{x},t) = \frac{1}{4\pi a_o^3|\mathbf{x}|}\frac{x_i}{|\mathbf{x}|}\int \underbrace{\frac{\partial F}{\partial t}\left(\mathbf{y}, t - \frac{|\mathbf{x}|}{a_o}\right)}_{F\sim\rho_o u^2/\ell;t\sim\ell/u}\underbrace{d\mathbf{y}}_{\ell^3}$$

so

$$\rho' \sim \rho_o\left(\frac{u}{a_o}\right)^3\frac{l}{r} \sim m^3 \qquad \text{and} \qquad I \sim m^6$$

Dipole Character

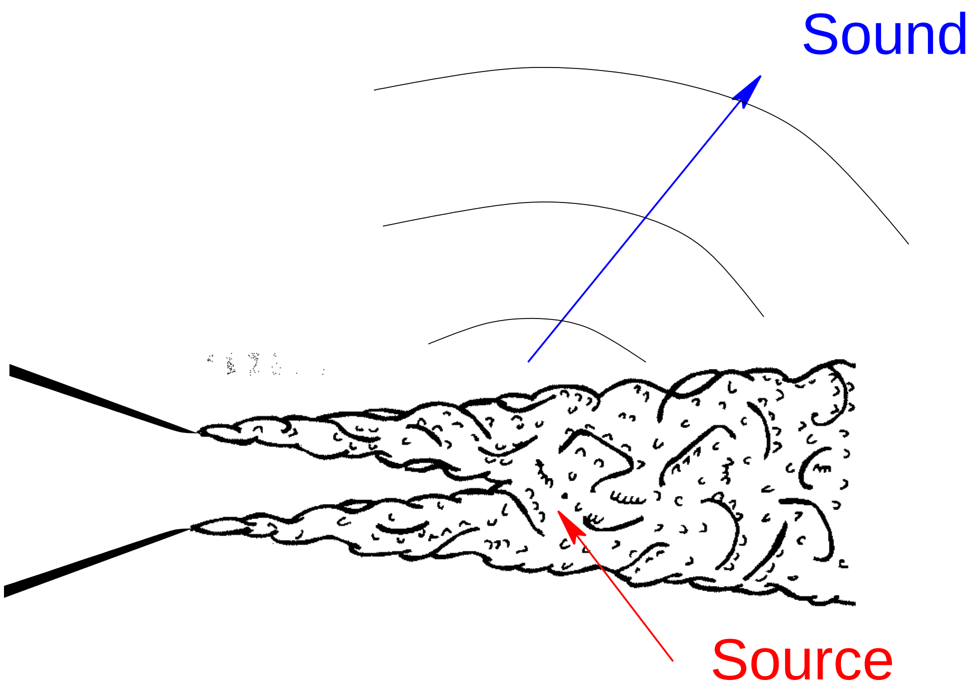
- Dipole — equivalent to nearly canceling equal and opposite q's



- Initial wrong approach missed cancellation (or got zero)
- Space derivative ∂_{y_i} of source was key factor
- This also affects how turbulence makes sound...

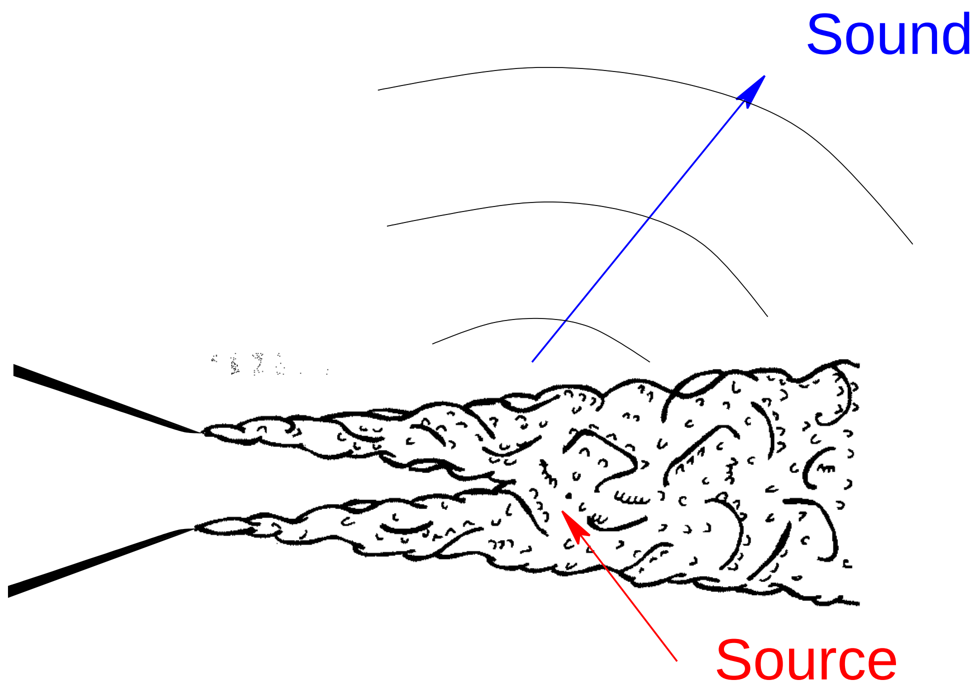
Turbulence As A Source of Sound

A Turbulent Jet



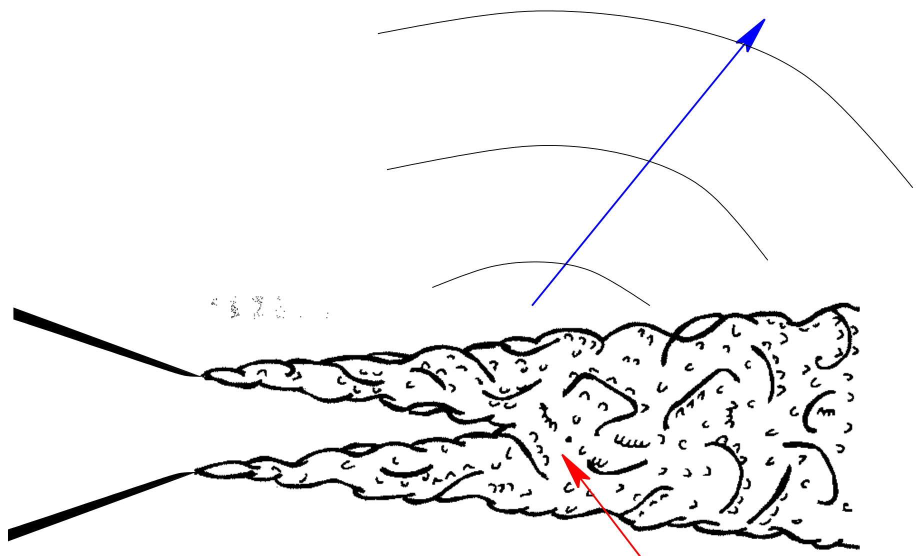
- Source and sound are intuitively obvious

A Turbulent Jet



- No obvious length/time-scale separation to clarify distinction
- $\overline{u'u'}/U^2 = O(1)$ — beyond weakly nonlinear
- Simplifications of standard acoustics do not apply

A Turbulent Jet



- So what do we know for sure...? A short list:

Our Only Truth

$$\mathcal{N}(\vec{q}) = 0$$

Our Only Truth

$$\mathcal{N}(\vec{q}) = 0$$

- The flow equations $\mathcal{N}$ govern the flow variables $\vec{q}$

Our Only Truth

$$\mathcal{N}(\vec{q}) = 0$$

- So what can we do...?

Give $\vec{q}$ a dual role

- Seems that we must use $\vec{q}$ in two ways *simultaneously*
- Rearrange $\mathcal{N}(\vec{q}) = 0$ into $\mathcal{L}\vec{q} = S(\vec{q})$
  - $\mathcal{N}$ — compressible flow equations
  - $\mathcal{L}$ — wave propagation operator (usually linear)
  - $\mathcal{S}$ — nominal noise source (usually nonlinear)

$$\mathcal{L} = \partial_{tt} - a_o^2 \nabla^2$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0$$

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} + \frac{\partial p}{\partial x_i} = \text{viscous terms}$$

- Turbulence fluctuations are not small... can't just linearize
- $\partial_t [\text{mass}] - \partial_{x_i} [\text{momentum}]$ as before:

$$\frac{\partial^2 \rho}{\partial t^2} - a_o^2 \frac{\partial^2 \rho}{\partial x_i \partial x_i} = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}$$

the Lighthill equation, where the Lighthill stress is

$$T_{ij} = \rho u_i u_j + (p - a_o^2 \rho) + \tau_{\text{viscous}}$$

- Note: this is an *exact* re-arrangement of the flow equations

Acoustic Analogy

$$\frac{\partial^2 \rho}{\partial t^2} - a_o^2 \frac{\partial^2 \rho}{\partial x_i \partial x_i} = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}$$

- Treat T_{ij} as analogous to externally applied stress
- Same solution procedure....

$$\begin{aligned} \rho(\mathbf{x}, t) &= \frac{1}{4\pi a_o^2} \int \frac{\partial^2 T_{ij}\left(\mathbf{y}, t - \frac{|\mathbf{x}-\mathbf{y}|}{a_o}\right)}{\partial y_i \partial y_j} \frac{1}{|\mathbf{x}-\mathbf{y}|}\, d\mathbf{y} \\ &= \frac{1}{4\pi a_o^2} \frac{\partial^2}{\partial x_i \partial x_j} \int T_{ij}\left(\mathbf{y}, t - \frac{|\mathbf{x}-\mathbf{y}|}{a_o}\right) \frac{1}{|\mathbf{x}-\mathbf{y}|}\, d\mathbf{y} \end{aligned}$$

Acoustic Analogy

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- Compact source, far field....

Acoustic Analogy

$$\frac{\partial^2 \rho}{\partial t^2} - a_o^2 \frac{\partial^2 \rho}{\partial x_i \partial x_i} = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}$$

- Compact-source, far-field assumptions...

$$\rho(\mathbf{x}, t) = \frac{1}{4\pi a_o^2} \frac{\partial^2}{\partial x_i \partial x_j} \int T_{ij}\left(\mathbf{y}, t - \frac{|\mathbf{x}|}{a_o}\right) \frac{1}{|\mathbf{x}|}\, d\mathbf{y}$$

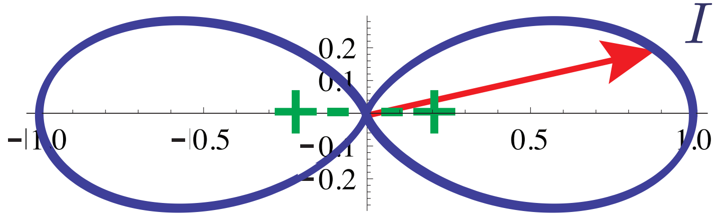
$$= \frac{1}{4\pi a_o^4 |\mathbf{x}|} \frac{x_i x_j}{|\mathbf{x}|^2} \int \frac{\partial^2 T_{ij}\left(\mathbf{y}, t - \frac{|\mathbf{x}|}{a_o}\right)}{\partial t^2}\, d\mathbf{y} + O\left(\frac{1}{|\mathbf{x}|^2}\right)$$

- Scaling: $T_{ij} \approx \rho u_i u_j \sim \rho_o u^2$...

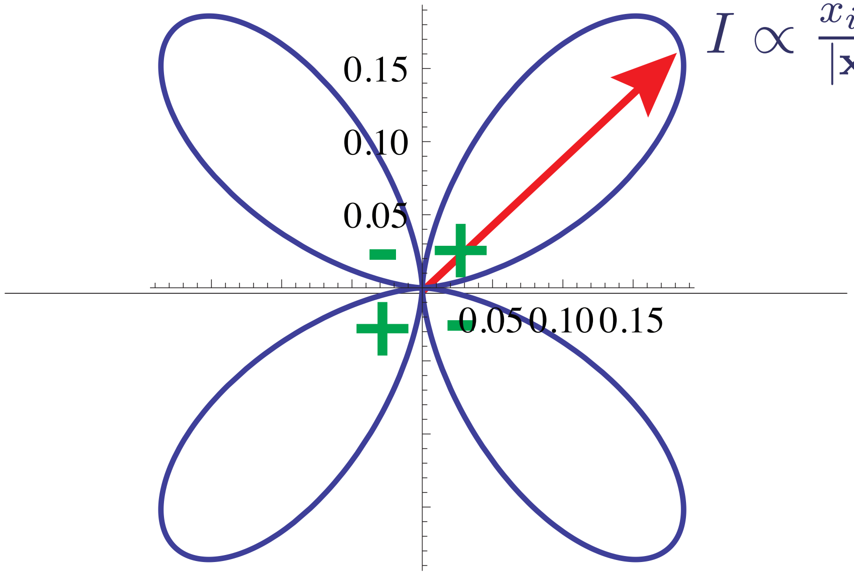
$$\rho \sim \rho_o \frac{\ell}{r} m^4 \qquad \text{and} \qquad I \sim m^8$$

Quadrupole Character

- Quadrupole — equivalent to nearly canceling equal and opposite q's



$$I \propto \left.\frac{x_i x_j}{|\mathbf{x}|^2}\right|_{i=j} \propto \cos^4(\theta)$$



$$I \propto \left.\frac{x_i x_j}{|\mathbf{x}|^2}\right|_{i\neq j} \propto \cos(\theta)^2 \sin^2(\theta)$$

- Far-field exact in the Mach number $M \to 0$ limit...

Consequence: U^8

- Predicts that jet-noise power should scale as U^8

Consequence: U^8

- Predicts that jet-noise power should scale as U^8

UIUC Jet Noise Facility

Consequence: U^8

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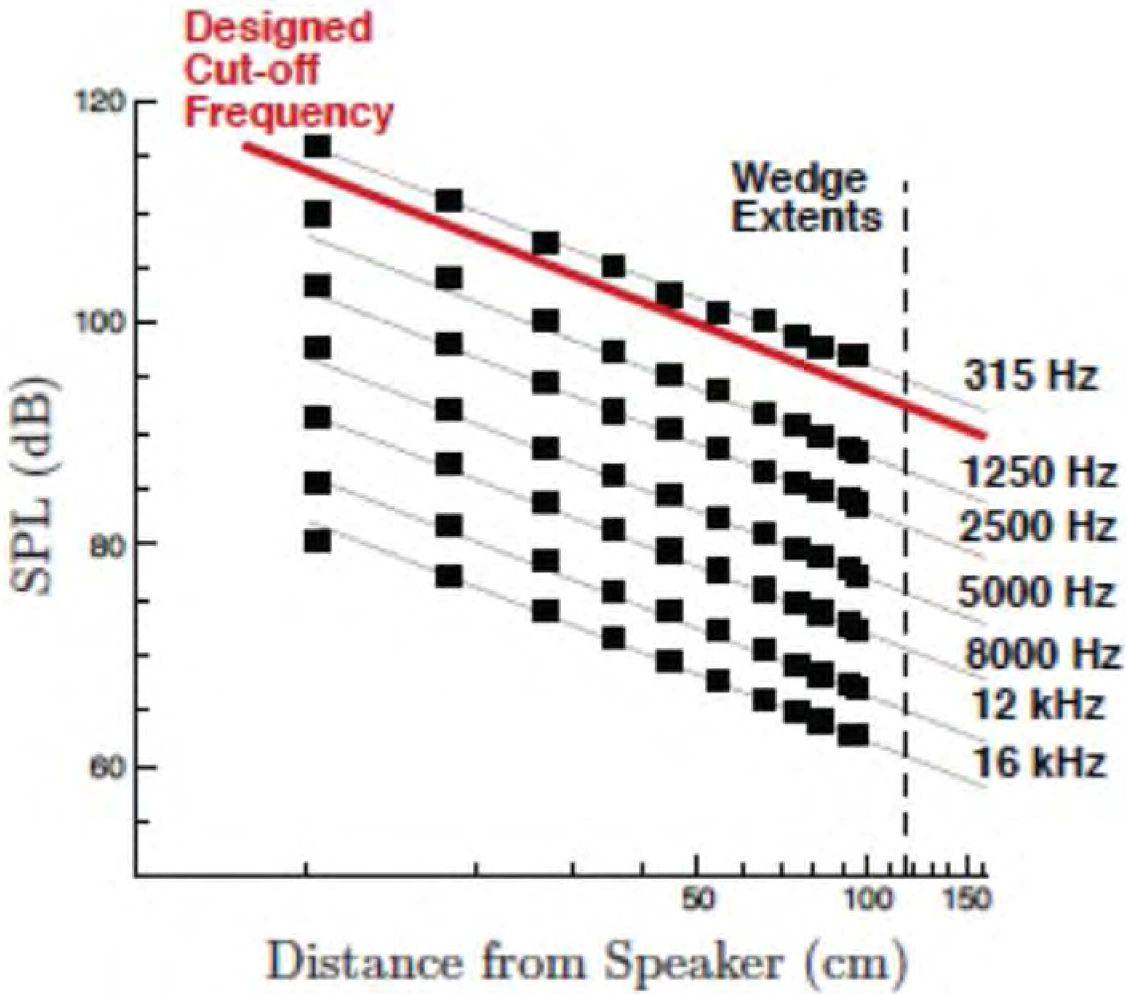
Anechoic



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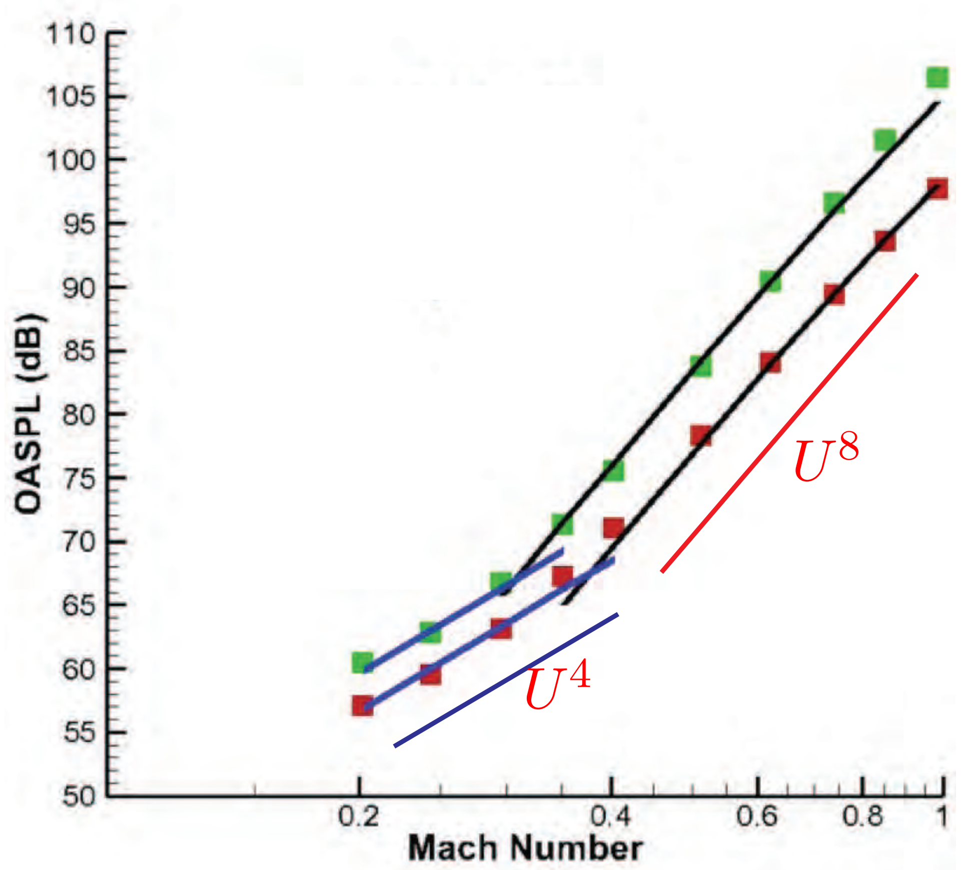
Sound Power versus Exit Velocity



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    - most of T_{ij} is non-radiating

Complex Turbulence Statistic

- Can predict sound via prediction of $\frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}$...
- Mean intensity: ($I = \langle \rho^2 \rangle$)
- Compact source, far field

$$I(\mathbf{x}) = \frac{x_i x_j x_k x_l}{16\pi^2 a_\infty^5 |\mathbf{x}|^5} \int_\infty \int_\infty \frac{\partial^4}{\partial \tau^4} \overline{T_{ij}(\mathbf{y}, t) T_{kl}(\mathbf{y} + \boldsymbol{\xi}, t + \tau)} \, d\boldsymbol{\xi} \, d\mathbf{y},$$

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fourth-order space retarded-time covariance tensor....

- Computed in DNS (Freund, *Phys. Fluids* 2003)
- Components have been measured
- No universal character

Refraction

- Appears as a source in:

$$\frac{\partial^2 \rho}{\partial t^2} - a_o^2 \frac{\partial^2 \rho}{\partial x_i \partial x_i} = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}$$

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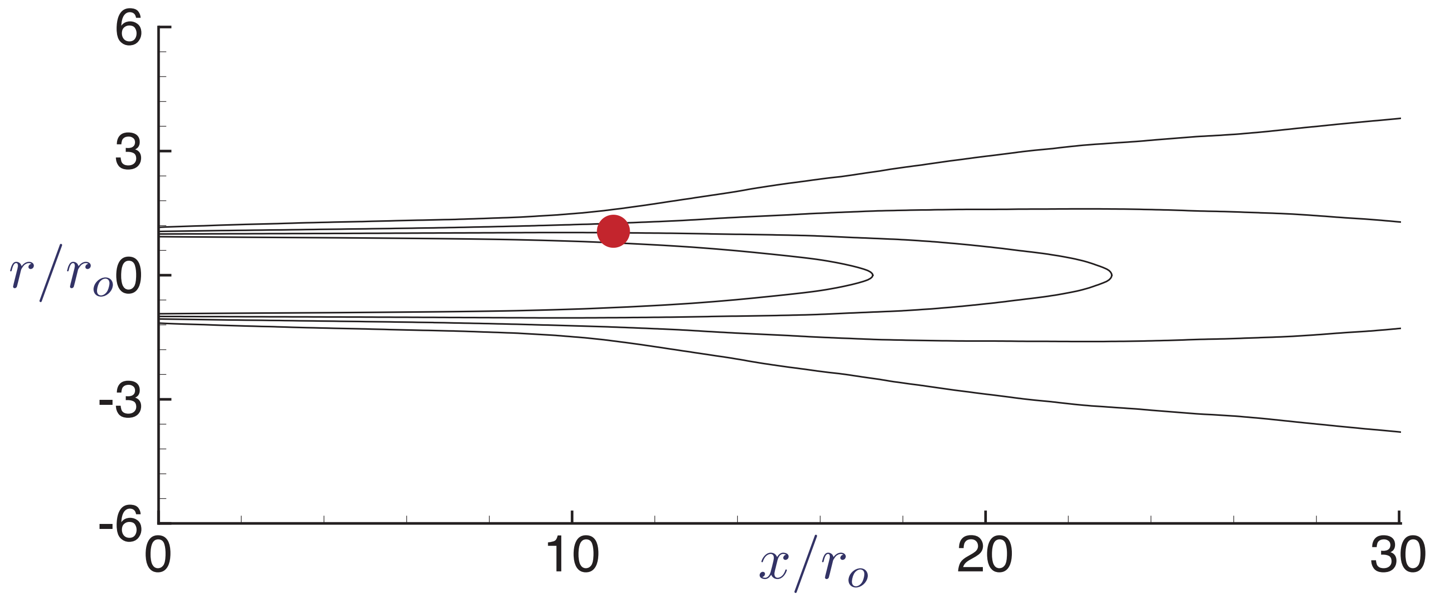

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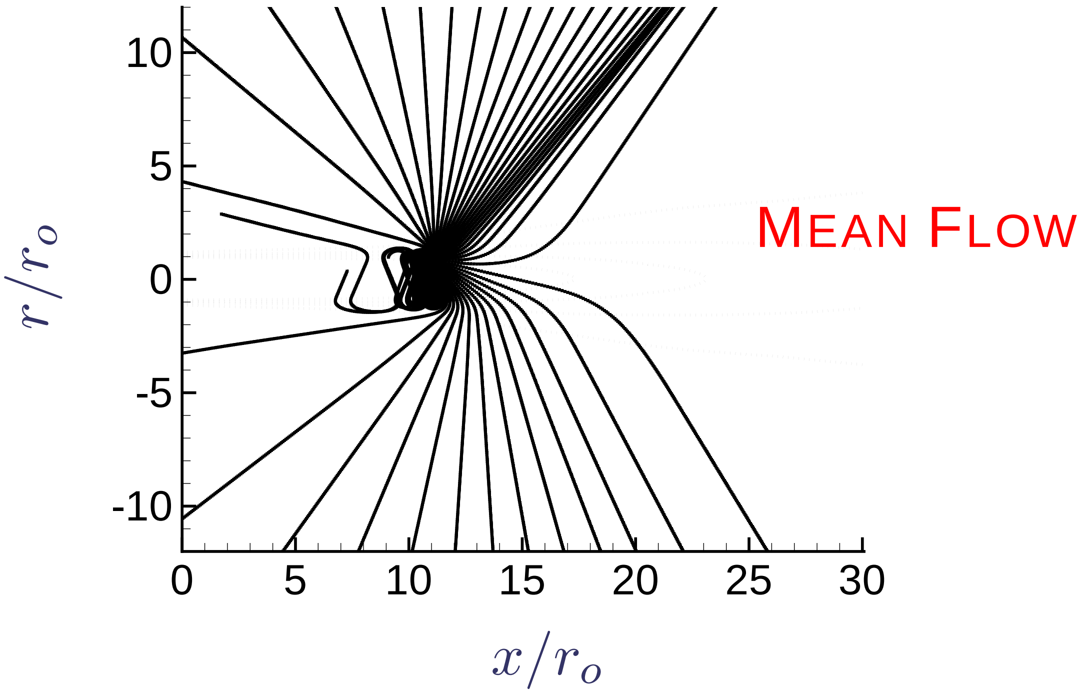

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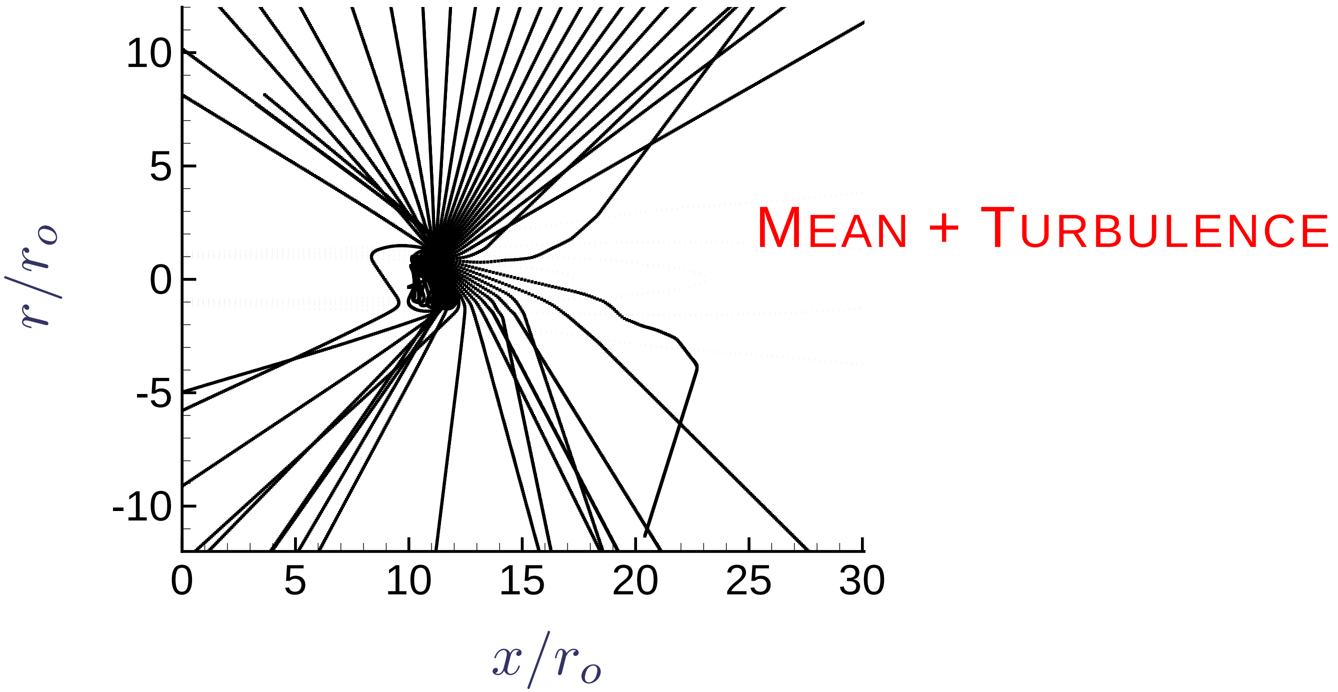
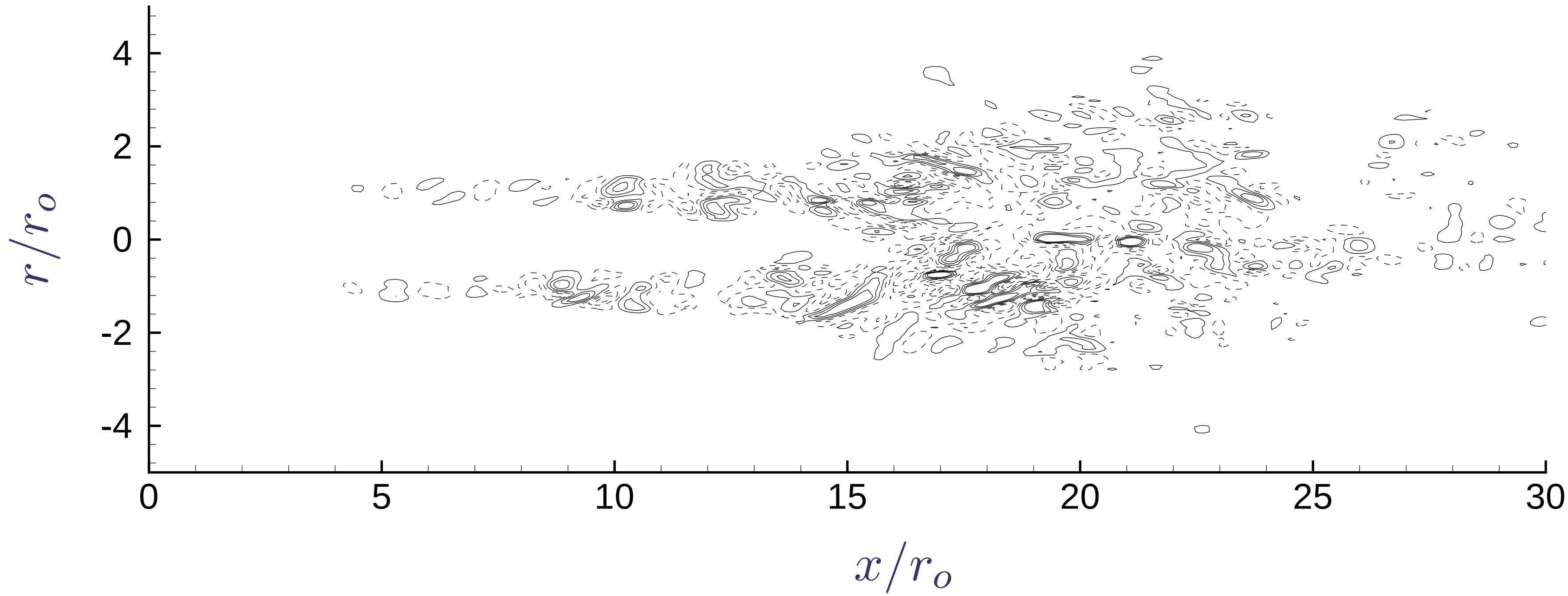


Lighthill Source

- $T_{ij,ij}$ for $M = 0.9$, $Re = 3600$ DNS (Freund, 2001)



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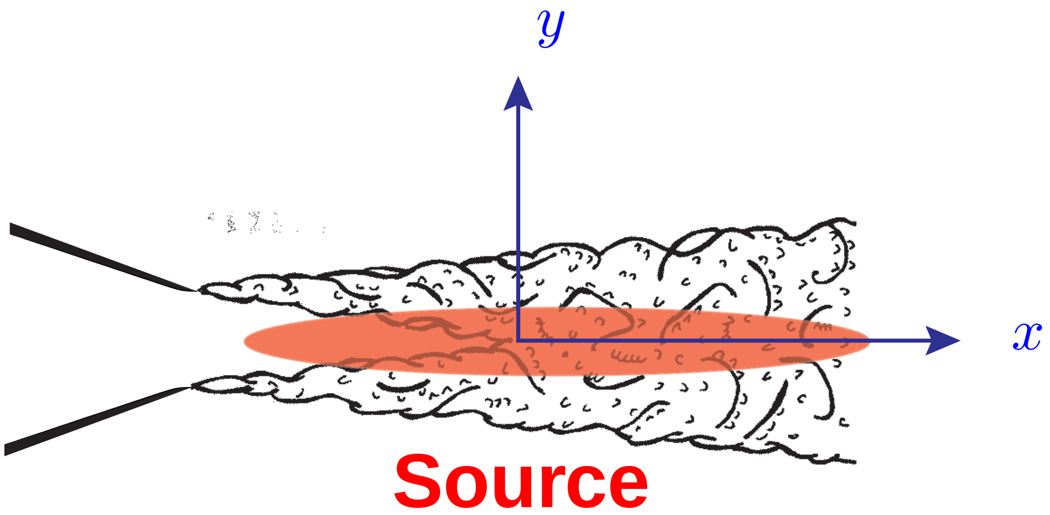

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$$\frac{d^2 \hat{\rho}}{dy^2} + \left(\omega^2 - a_o^2 k^2 \right) \hat{\rho} = -\hat{S}$$

Mostly Non-radiating

- Solutions

$$\hat{\rho}(k, y, \omega) = [\cdots]\, e^{\pm y\sqrt{a_o^2 k^2 - \omega^2}}$$

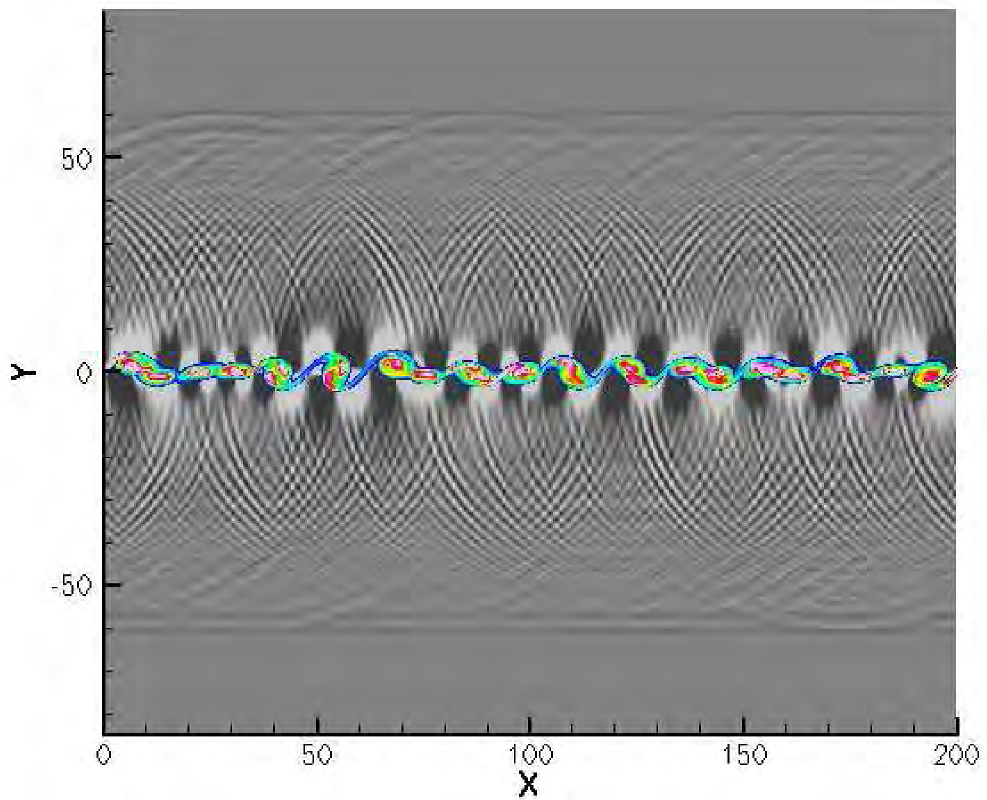
decay (not waves) in $\pm y$ unless $\omega^2 > a_o^2 k^2$

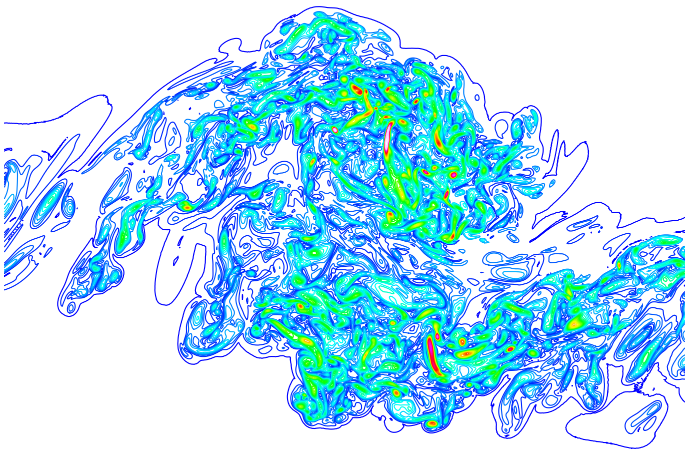
- $\omega^2 > a_o^2 k^2$ corresponds to supersonic phase velocity

$$\left|\frac{\omega}{k}\right| > a_o$$

- Most of turbulence in a $M \approx 1$ jet is moving with convection velocity (phase velocity) $U_c \lesssim a_o$:
  - subsonic phase velocity

$M = 0.9$





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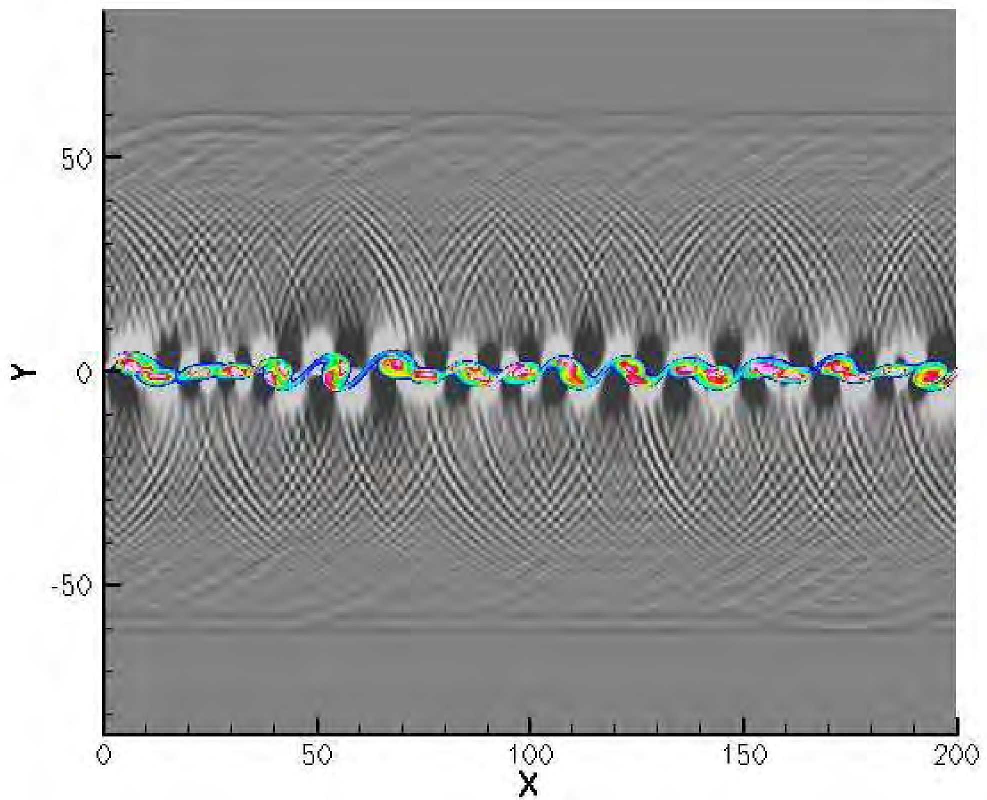
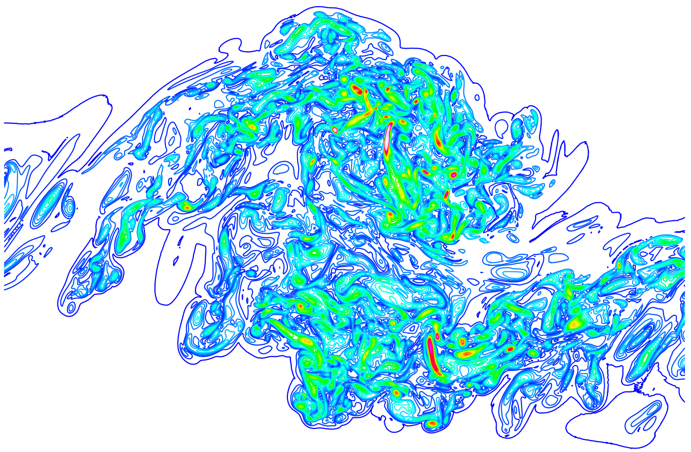
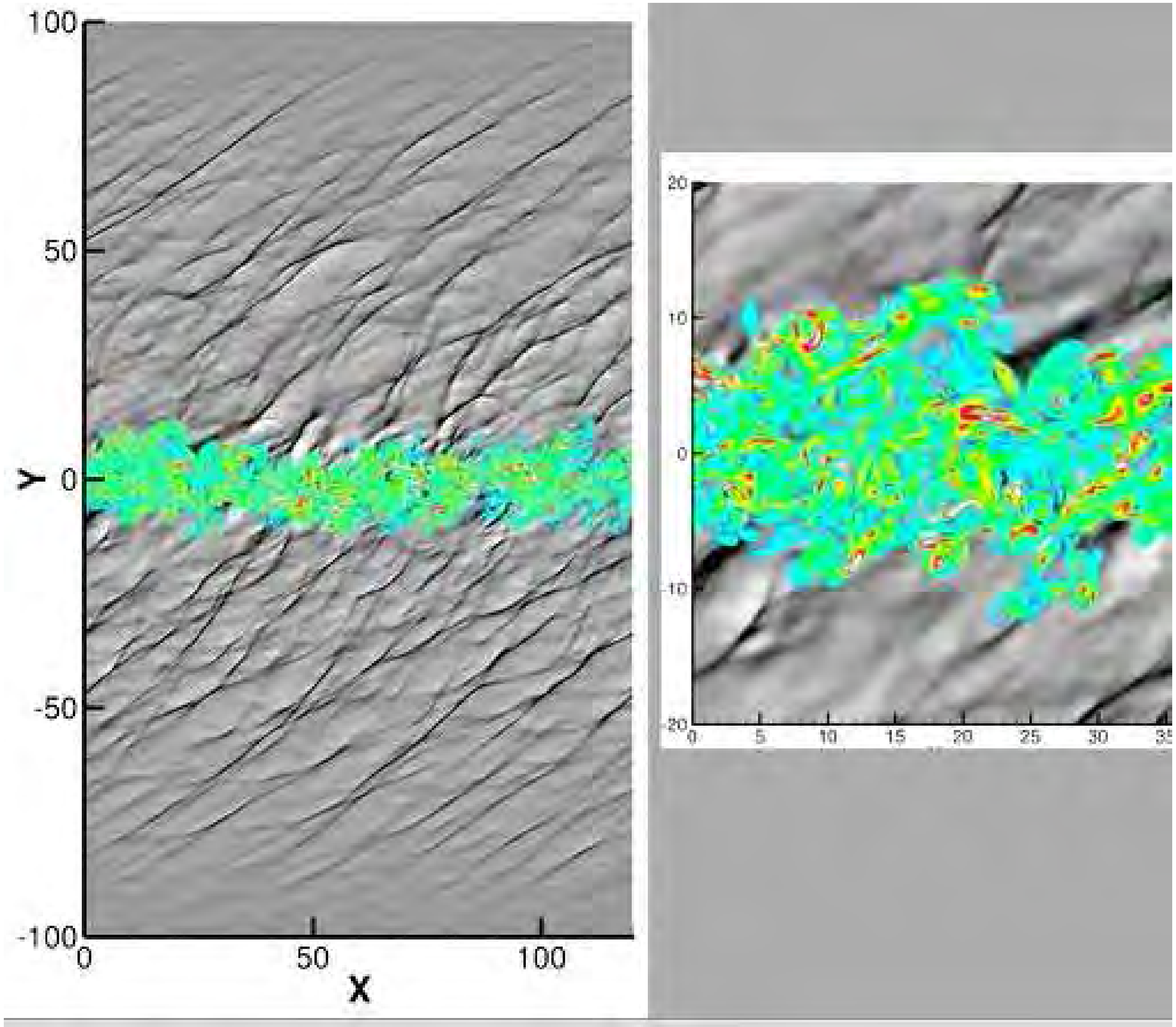




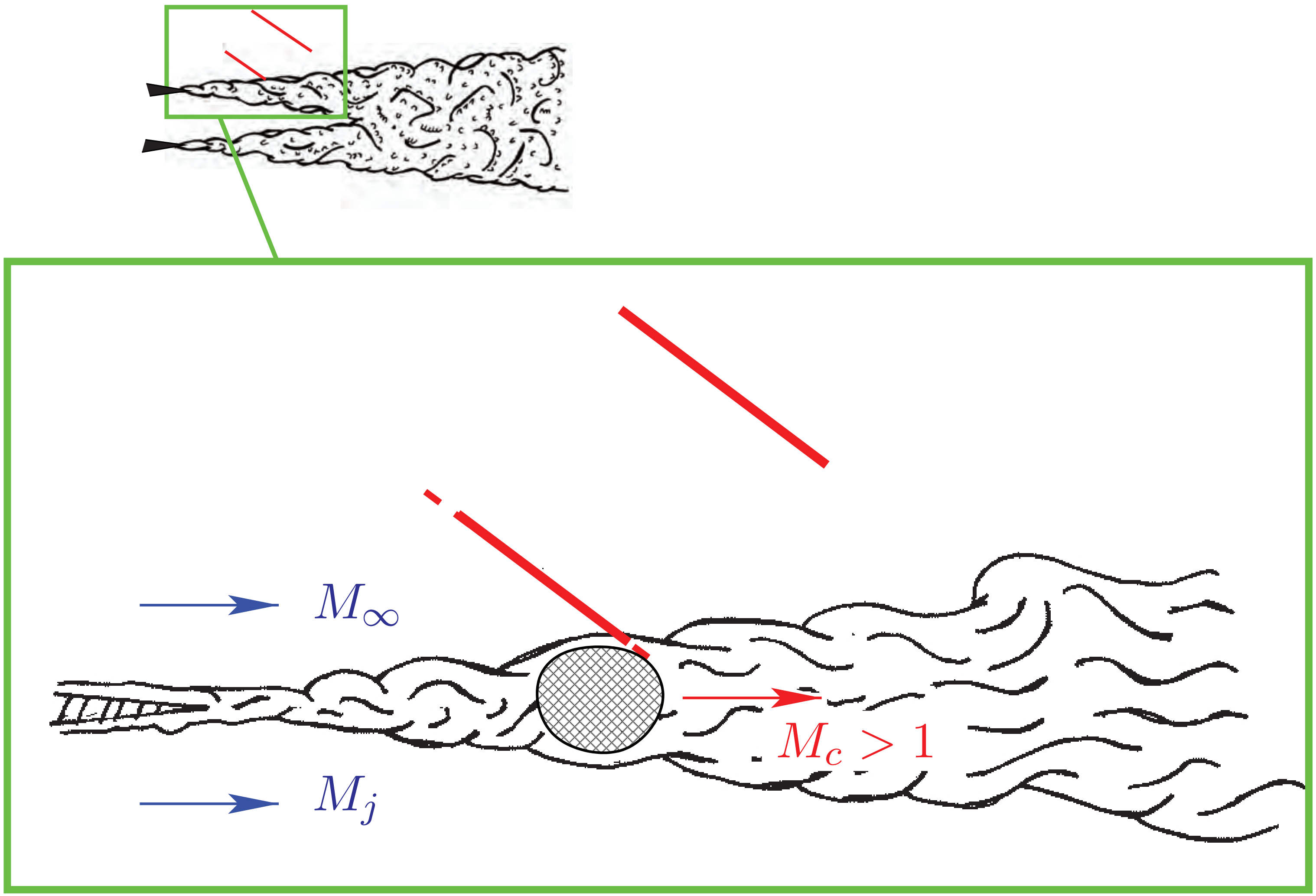
- Don't 'see' what makes the sound

$M = 2.5$: Supersonic Convection

- Eddies emit shock waves



$M_c > 1$ Basic Mechanism



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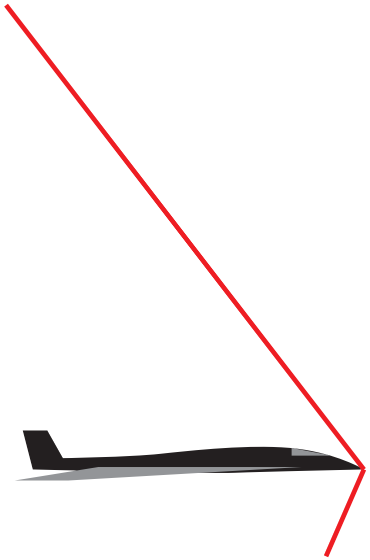
- Like sonic boom



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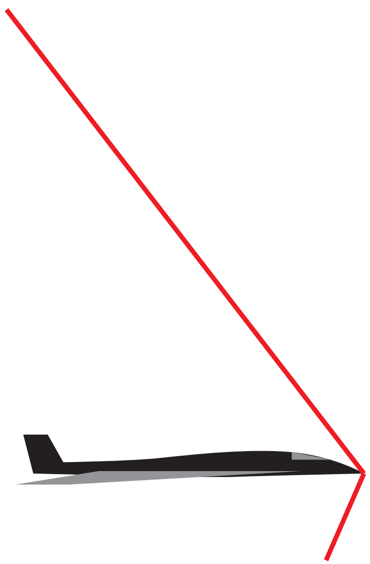

- ... but the aircraft–eddy appears and disappears

$M = 0.9$ Jet: Lighthill Source

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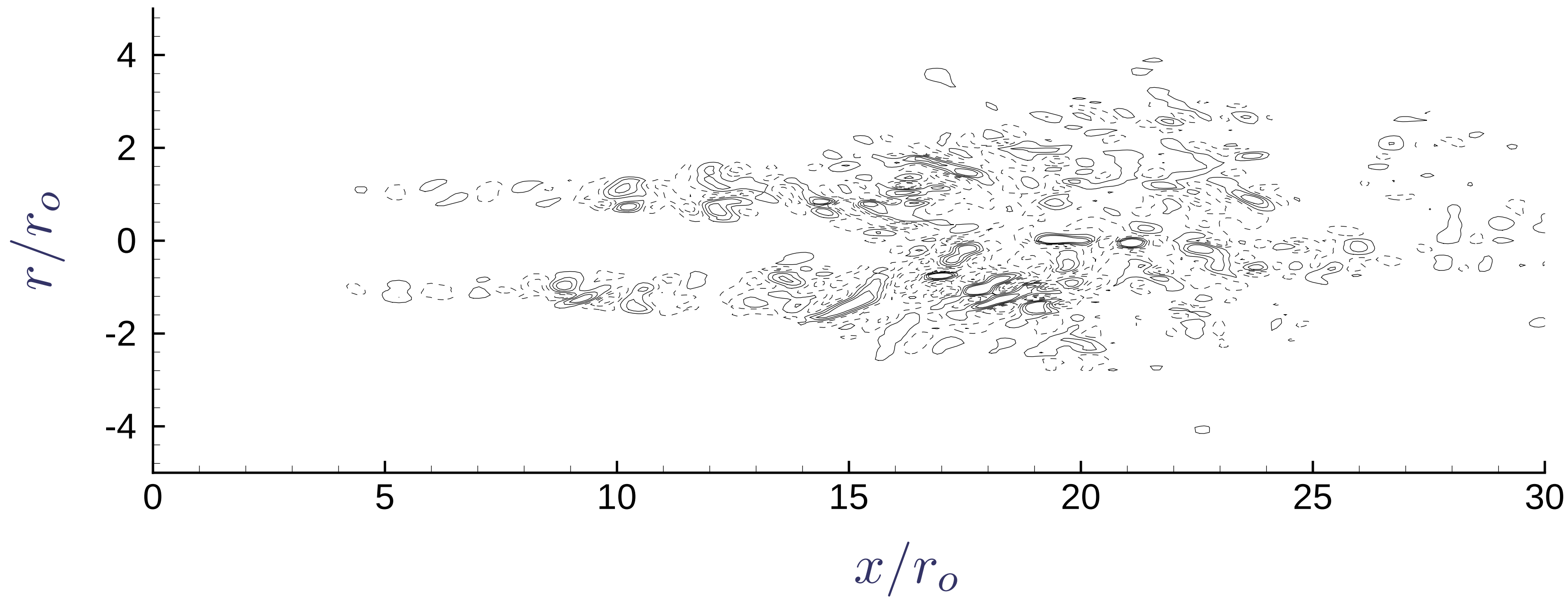

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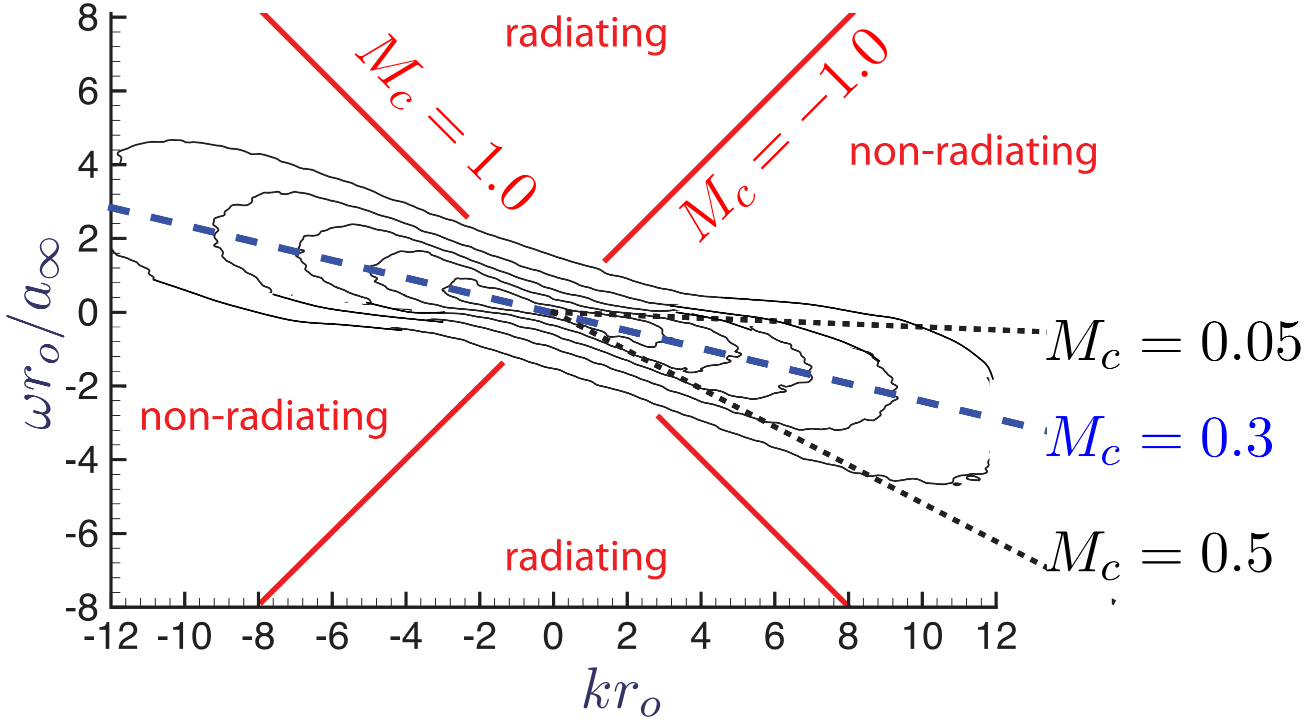
- Source in wavenumber-frequency space

$$S(x,t) \quad \Rightarrow \quad \hat{S}(k,\omega)$$



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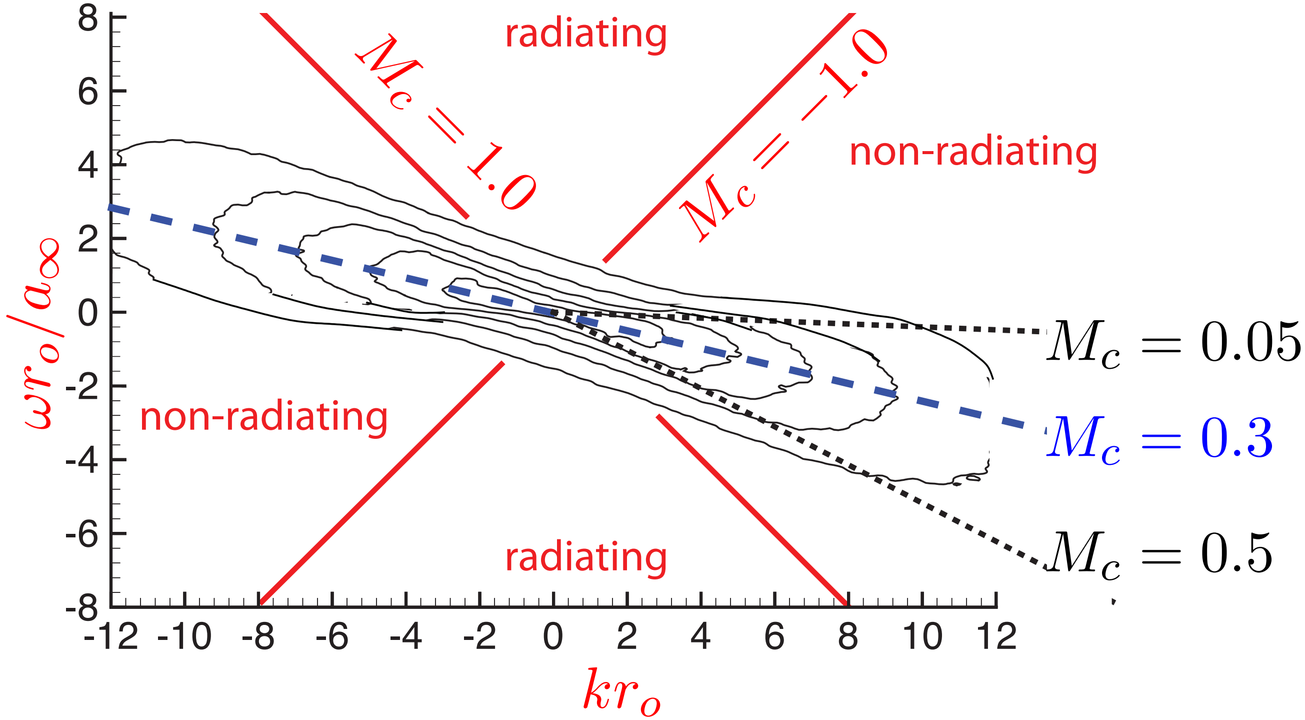
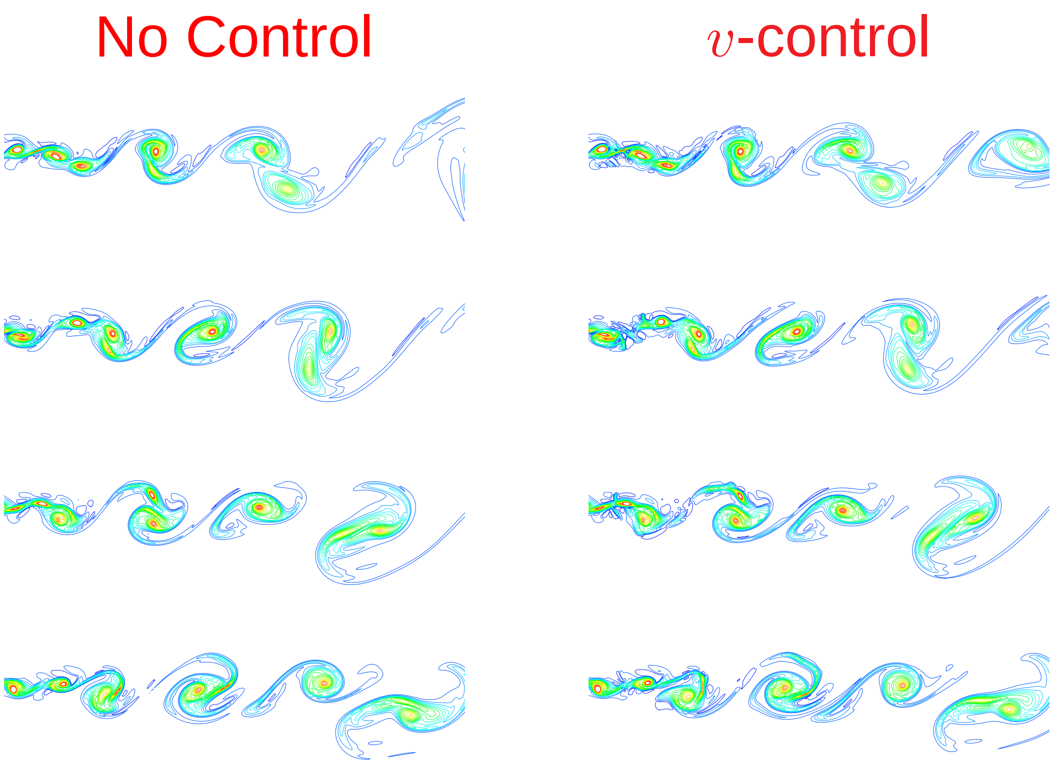


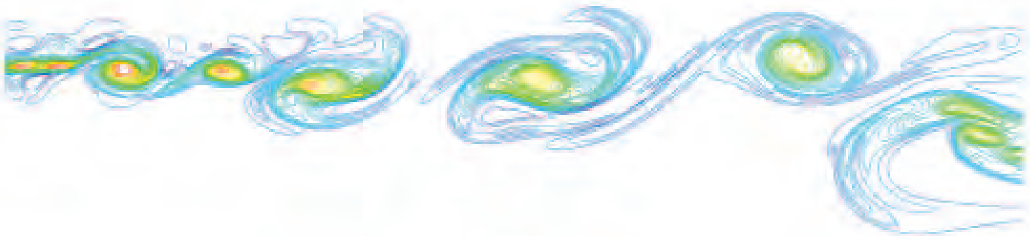
- Filtering down to radiating-only modes breaks locality
- Flows can look the same and yet have very different sound

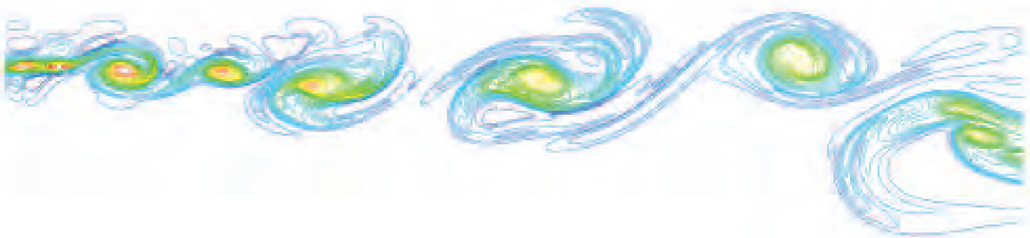
Large Turbulent Structures

- Very similar looking ‘turbulence’ can have entirely different sound
- Two-dimensional mixing layer (Wei & Freund, *JFM*, 2006)
- Controlled flow is $\gtrsim 6$ dB quieter



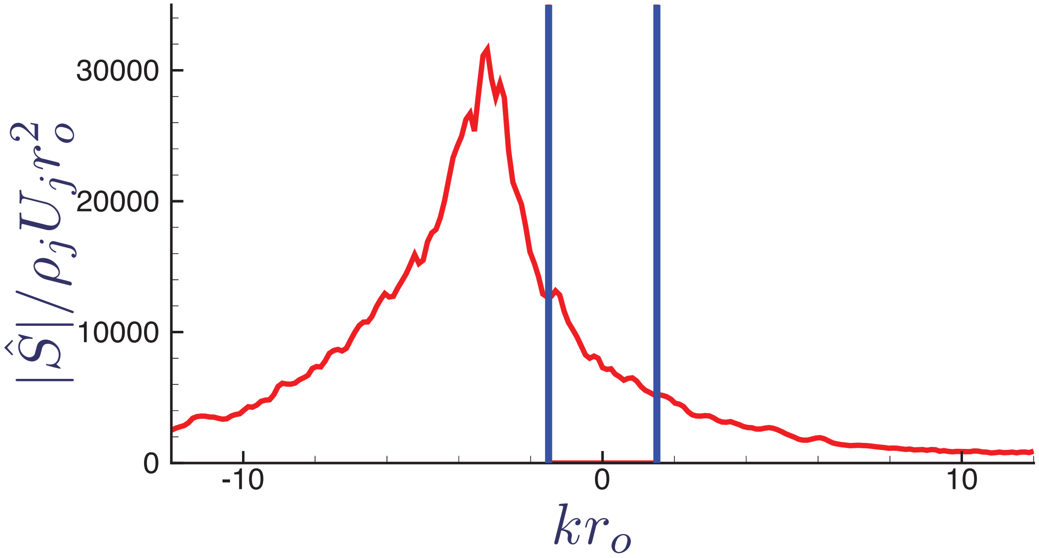
Large-scale Structures





Makes Prediction Challenging

- Need to predict subtle aspects of turbulence....
- $\omega = 1.5 a_\infty / r_o$:



- Need to faithfully represent components with small energy

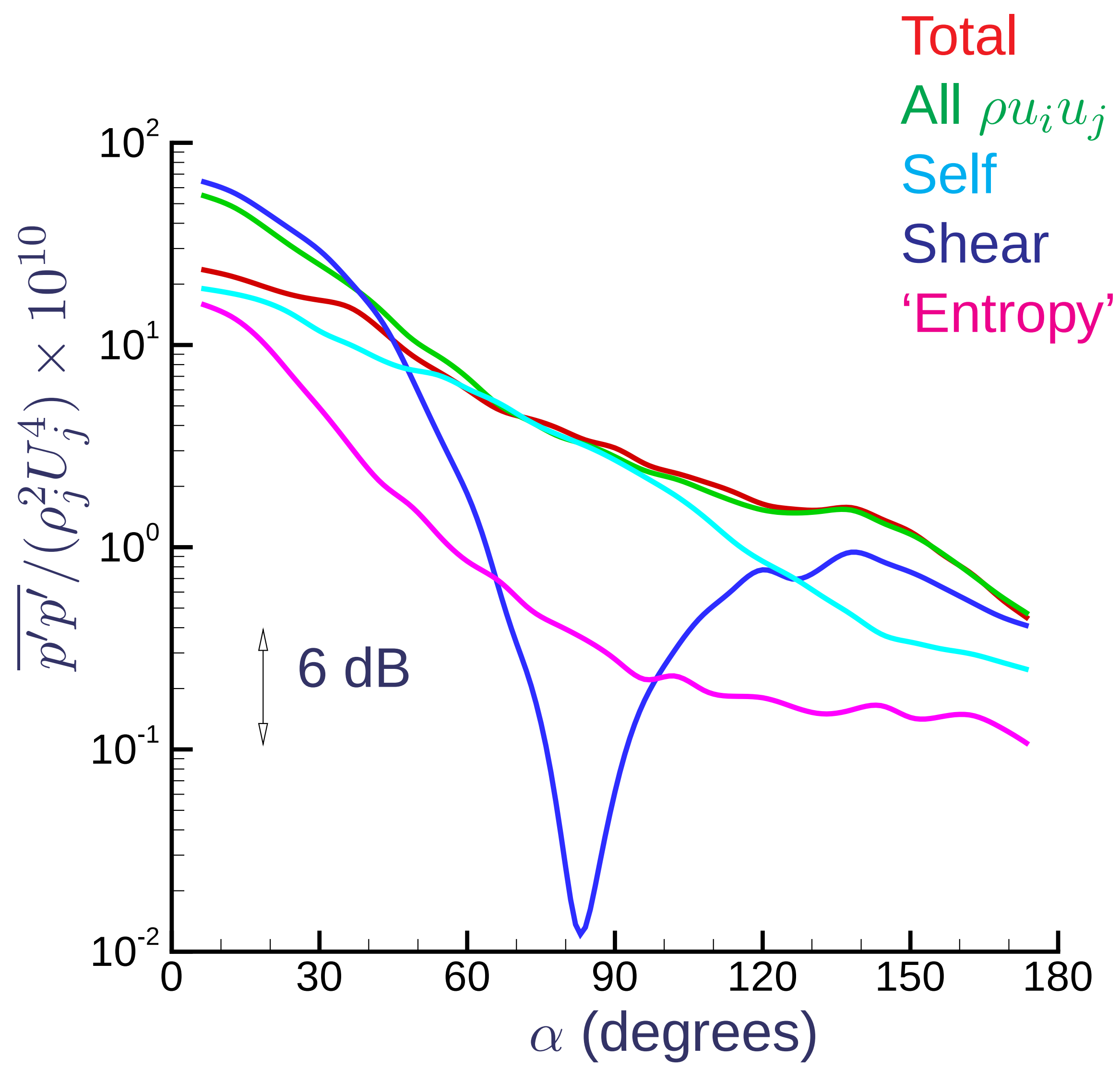
Simplify Source

- Common mean + perturbation turbulence decomposition

$$T_{ij} = \bar{T}_{ij} + \underbrace{\rho(\bar{u}_i u'_j + u'_i \bar{u}_j)}_{\text{shear}} + \underbrace{\rho u'_i u'_j}_{\text{self}} + \underbrace{(p' - a_\infty^2 \rho')\delta_{ij}}_{\text{`entropy'}} - \underbrace{\tau'_{ij}}_{\text{viscous}}$$

- Neglect viscous source (universally accepted)
  - Implicit result of Colonius & Freund (2000) even for $Re = 2000$

Directivity ($M = 0.9$ Jet)



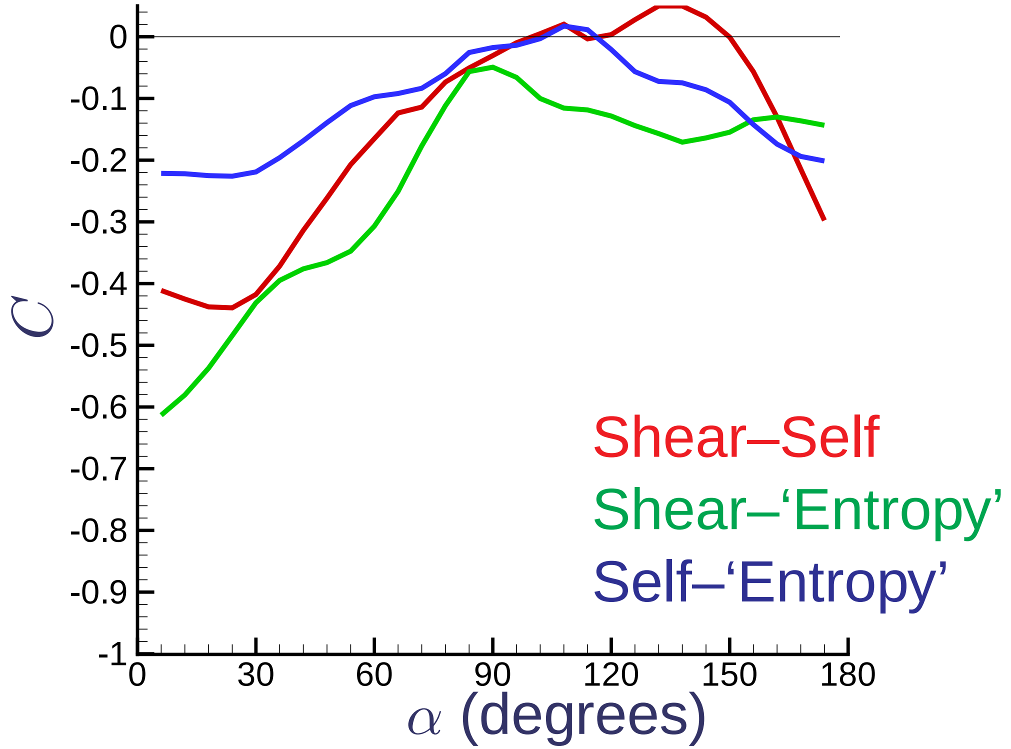
Net Power

	Component	Power$/\rho_j {U_j}^3 A_j$	Power/Power T_{ij}
Total:	T_{ij}	8.3 $\times 10^{-5}$	1.00
Shear:	T_{ij}^l	8.7 $\times 10^{-5}$	1.05
Self:	T_{ij}^n	6.9 $\times 10^{-5}$	0.83
Entropy:	T_{ij}^s	2.0 $\times 10^{-5}$	0.25

- Net powers of different components to not “add up”

Correlation Coefficients

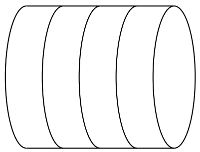
$$C_{\beta-\gamma} = \frac{\overline{\rho^\beta \rho^\gamma}}{\rho^\beta_{\mathrm{rms}} \rho^\gamma_{\mathrm{rms}}}$$

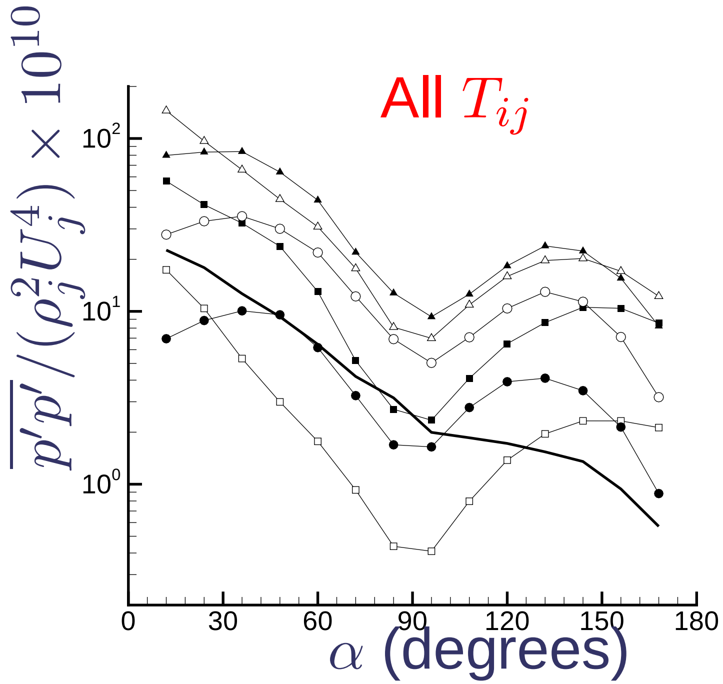


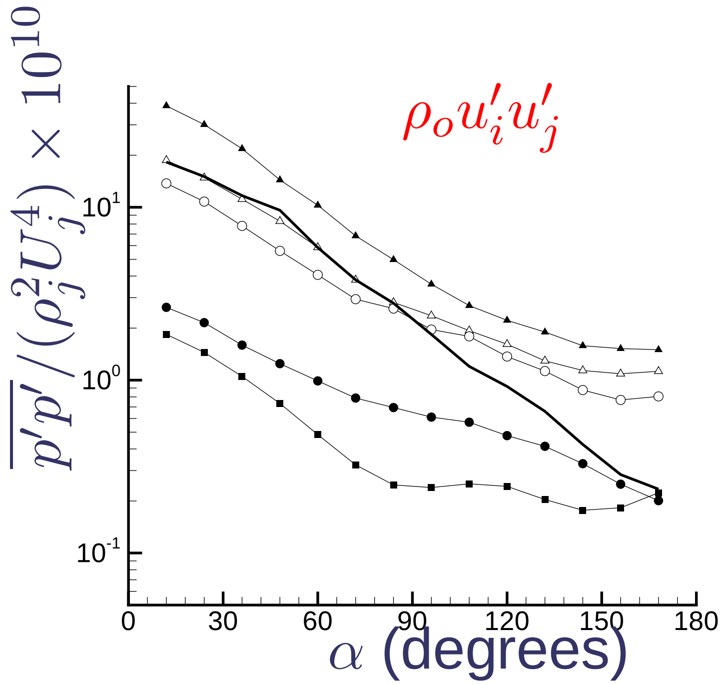
- Need to model terms and correlations

Non-local

Should be local if quadrupole: $p = T_{ij} * G_{,ij}$







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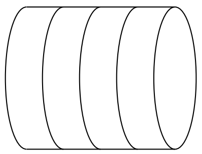

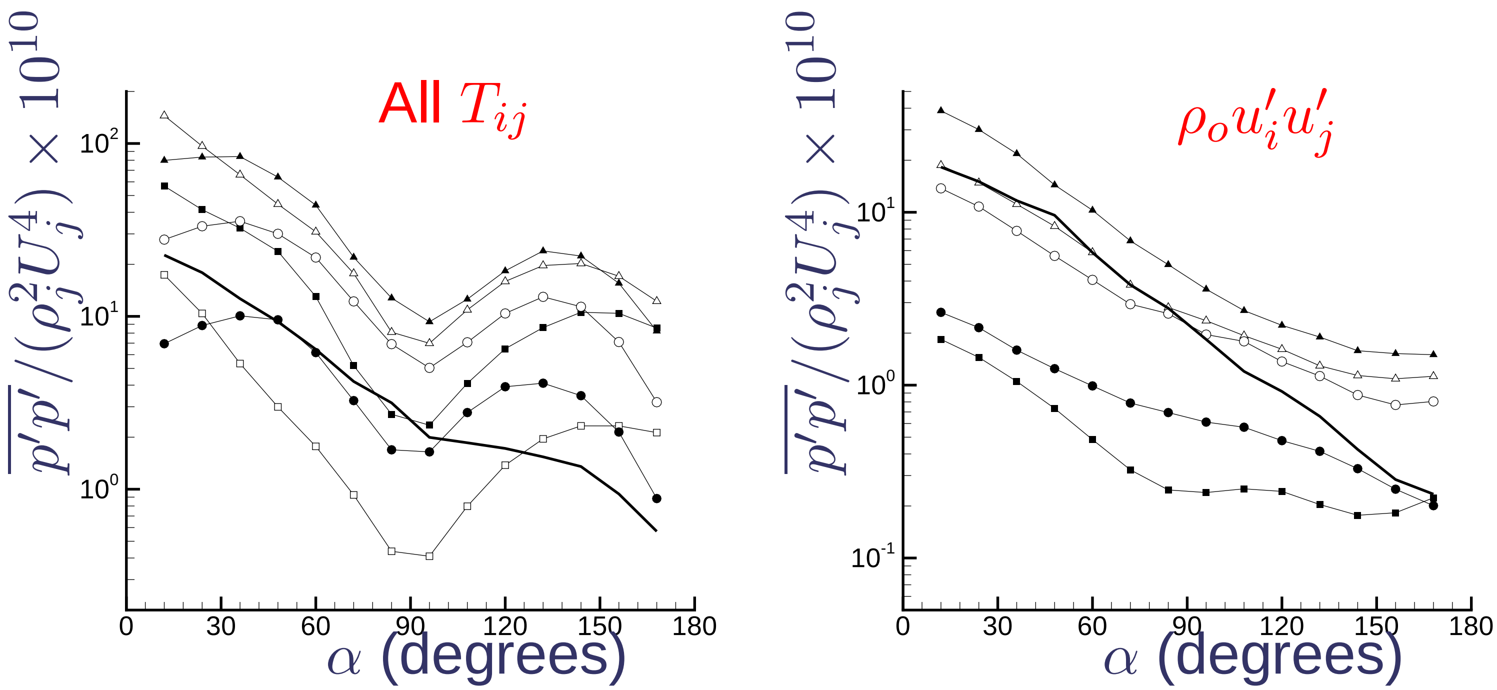

- Compact sufficient for U^8 but not for all details

Other options for $\mathcal{L}\vec{q} = S(\vec{q})$?

$$\mathcal{N}(\vec{q}) = 0 \qquad \Rightarrow \qquad \mathcal{L}\vec{q} = S(\vec{q})$$

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  - Lighthill: $\mathcal{L}$ – homogeneous-medium wave operator
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  - Why when exact relations can be a starting point? – unjustified

Which $\mathcal{L}\vec{q} = S(\vec{q})$ best?

- All $\mathcal{L}\vec{q} = S(\vec{q})$ are exact
  - Given $S(\vec{q})$, $\mathcal{L}^{-1}S(\vec{q})$ gives sound
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- Simplest? $\longrightarrow$ Lighthill (or related)
  - $\mathcal{L}$ easily inverted
  - $S(\vec{q})$ seems no more complex than others
  - solutions of $\mathcal{L}\vec{q} = 0$ well behaved
  - disturbing that so much non-source stuff is in S

Anything Simpler?

- Better differentiation of source and propagation?
  - complicates $\mathcal{L}^{-1}S(\vec{q})$
  - may simplify $S(\vec{q})$ – more like true source (unexplored)
- Inconvenient truth: the turbulent flow that constitutes $S(\vec{q})$ remains mysterious

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- Most robust?

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- Use formulation most robust to unavoidable errors in S
  - Samanta, Freund, Wei, Lele, *AIAA J.* (2006)

Robustness

- Many potential ways to evaluate robustness...
- For now: empirical robustness evaluation using DNS data
- Work with time dependent formulations
  - SGS noise models
  - large-scale dynamics models (POD Galerkin projection, PSE)

Formulation

- Goldstein (2003) general acoustic analogy $\mathcal{L}\vec{q} = S(\vec{q})$:

$$\bar{\rho}\frac{\bar{D}}{Dt}\frac{\rho'}{\bar{\rho}} + \frac{\partial}{\partial x_j}\bar{\rho}u'_j = 0$$

$$\bar{\rho}\left(\frac{\bar{D}}{Dt}u'_i + u'_j\frac{\partial \tilde{v}_i}{\partial x_j}\right) + \frac{\partial p'_e}{\partial x_i} - \frac{\rho'}{\bar{\rho}}\frac{\partial \tilde{\tau}_{ij}}{\partial x_j} = \frac{\partial}{\partial x_j}(e'_{ij} - \tilde{e}_{ij})$$

$$\frac{1}{\gamma - 1}\left(\frac{\bar{D}p'_e}{Dt} + \gamma p'_e\frac{\partial \tilde{v}_j}{\partial x_j} + \gamma\frac{\partial}{\partial x_j}\bar{p}u'_j\right) - u'_i\frac{\partial \tilde{\tau}_{ij}}{\partial x_j}$$
$$= \frac{\partial}{\partial x_j}(\eta'_j - \tilde{\eta}_j) + (e'_{ij} - \tilde{e}_{ij})\frac{\partial \tilde{v}_i}{\partial x_j}$$

- Exact consequence of flow equations

Formulation

- Base flow ($\bar{\rho}$, $\bar{p}$, $\tilde{v}_i$)
  - "user" specified
  - for explicit mean-flow refraction (*e.g.*)
  - satisfies exact equations with sources $\widetilde{T}_{ij}$, $\widetilde{H}_{ij}$ and $\widetilde{H}_0$

- Introduced new dependent variables

$$p'_e \equiv p' + \frac{\gamma - 1}{2}\rho v_i v_i + (\gamma - 1)\widetilde{H}_0 \qquad \text{and} \qquad u'_i \equiv \rho \frac{v'_i}{\bar{\rho}},$$

Formulation

- Noise source $S(\vec{q})$:

$$
\begin{aligned}
e'_{ij} &\equiv -\rho v'_i v'_j + \frac{\gamma - 1}{2}\delta_{ij}\rho v'_k v'_k + \sigma'_{ij} \\
\tilde{e}_{ij} &\equiv \widetilde{T}_{ij} - \delta_{ij}(\gamma - 1)\widetilde{H}_0 \\
\eta'_i &\equiv -\rho v'_i h'_0 - q'_i + \sigma_{ij} v'_j \\
\tilde{\eta}_i &\equiv \widetilde{H}_i - \widetilde{T}_{ij}\tilde{v}_j
\end{aligned}
$$

  - zero mean for time averaged base flow

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- Step II: base flow defines source $S(\vec{q})$

$$e'_{ij} - \tilde{e}_{ij} \qquad \text{and} \qquad \eta'_i - \tilde{\eta}_i$$

- Step III: solve $\mathcal{L}\vec{q} = S(\vec{q})$
  - same high-order schemes at DNS
  - same mesh
  - same wave-equation extrapolation to far field
  - no special treatment of $\mathcal{L}\vec{q} = 0$ solutions (!?!?)
  - neglect diffusive transport

Locally Parallel Base Flow

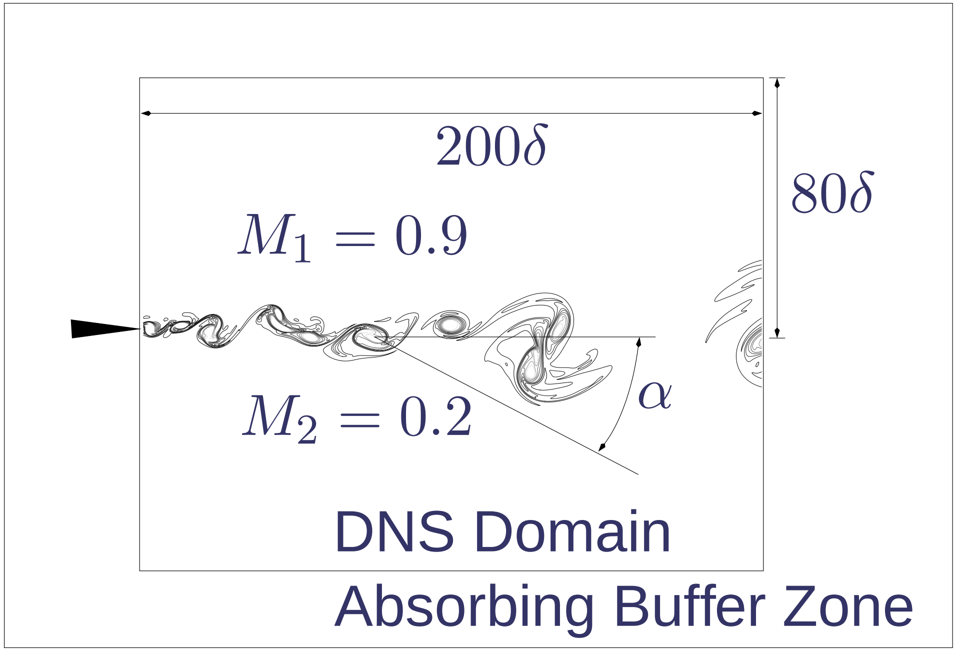
- Mean-flow base flow but neglect streamwise derivatives

$$\frac{\partial \bar{q}}{\partial x_1} = 0$$

- Rational approximation of mean-flow analogy
- Used by Tam & Auriault with *ad hoc* $S(\vec{q})$
- Analyze in same way as true acoustic analogies using actual S subject to same approximation

DNS

- Two-dimensional mixing layer



- Randomly excited
- Wei (2004) PhD dissertation; Wei & Freund, *JFM* (2006)
- 3907 fields stored every $4\Delta t$

Source Errors

- Decompose DNS flow into empirical eigenfunctions (POD modes)

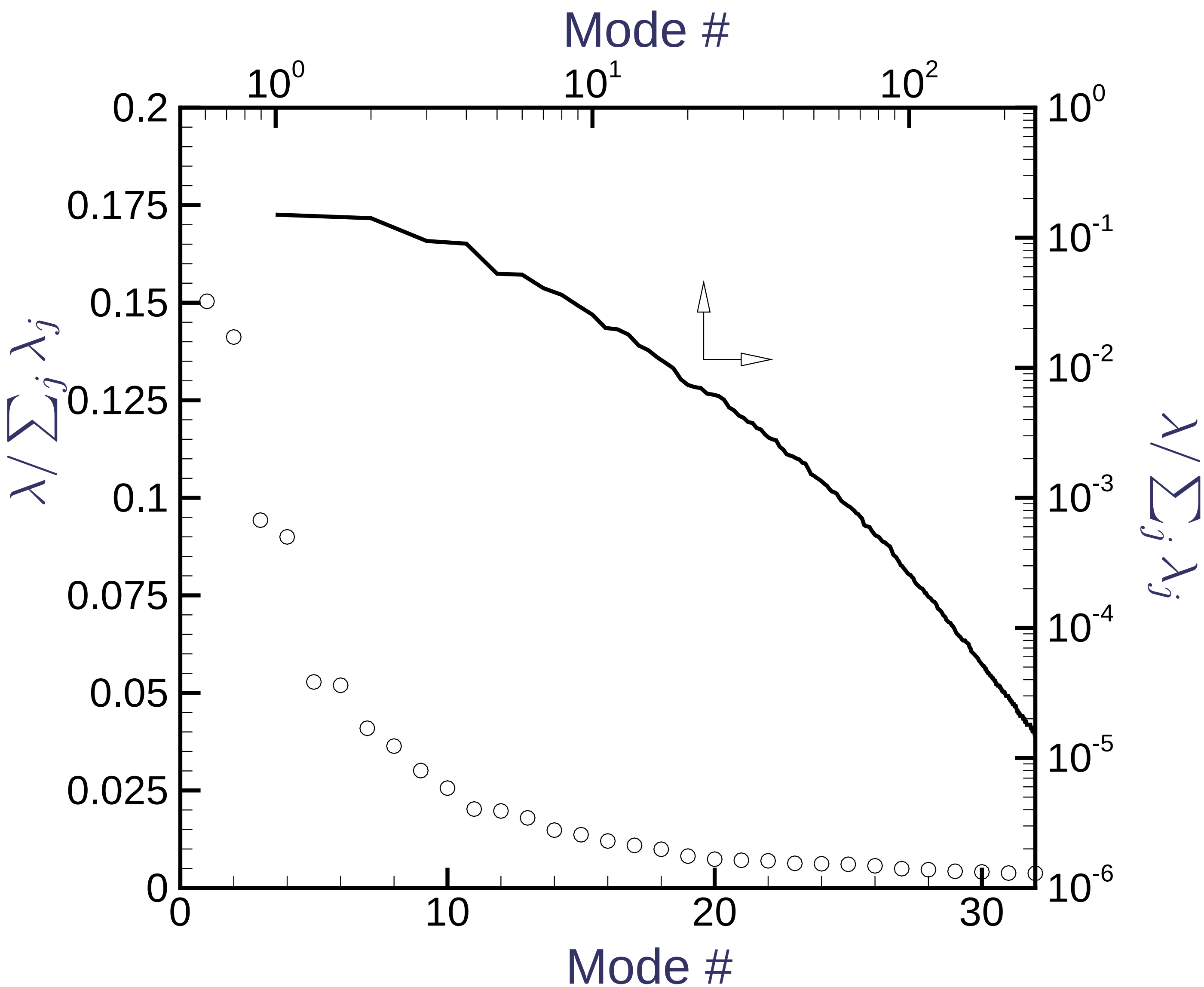
$$\vec{q}(\mathbf{x},t) = \sum_{i=1}^{N} a_i(t)\vec{\psi}_i(\mathbf{x}) \qquad N = 587$$

where $\vec{\psi}$ modes are constructed using snapshots and KE norm

$$E = \int_{\mathcal{V}} \rho u_i u_i \, d\mathbf{x}$$

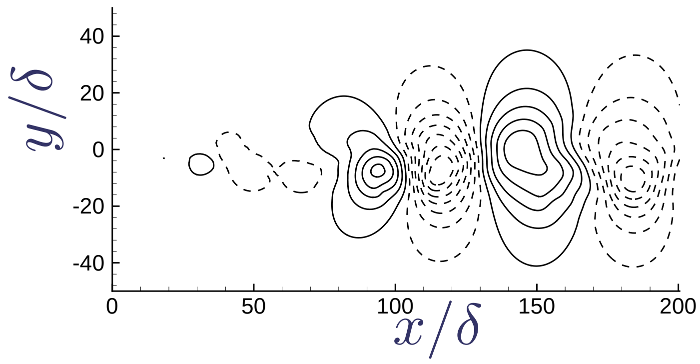
- Expect:
  - low modes: large scale, low frequency, high energy
  - high modes: small scale, high frequency, low energy

Mode Spectrum

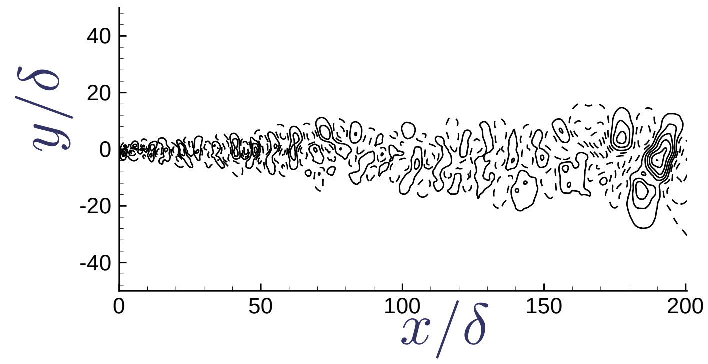


Mode Shapes and $a(t)$

MODE 1

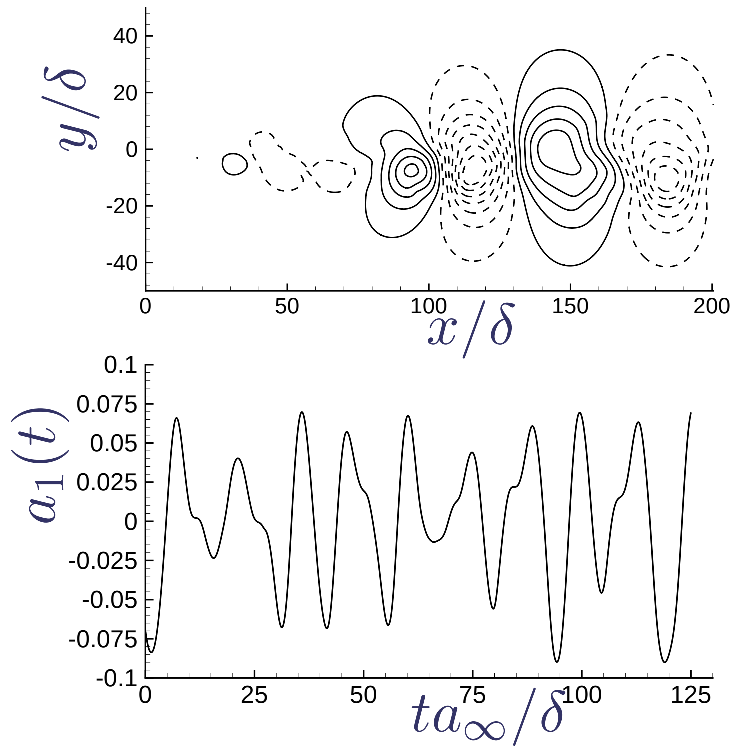


MODE 128

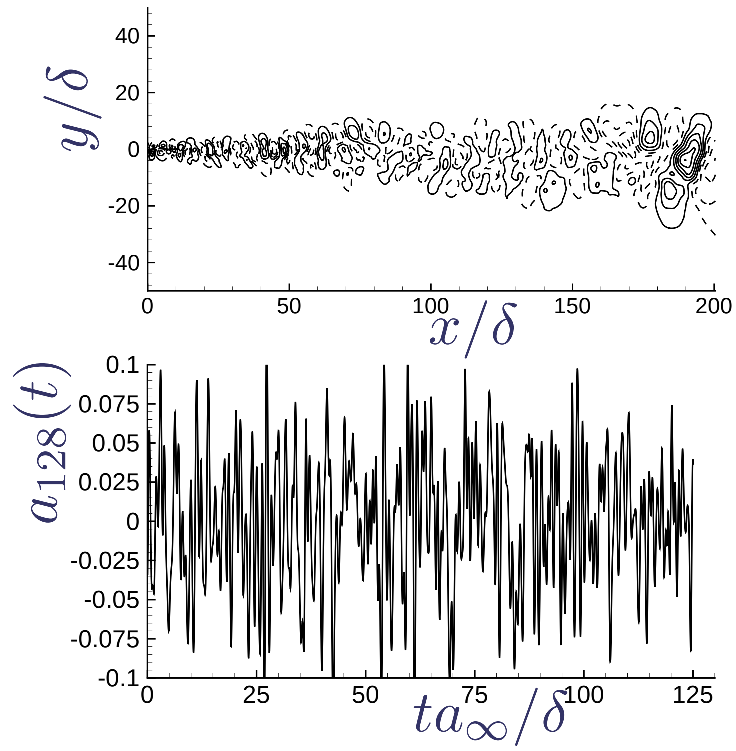


Mode Shapes and $a(t)$

MODE 1

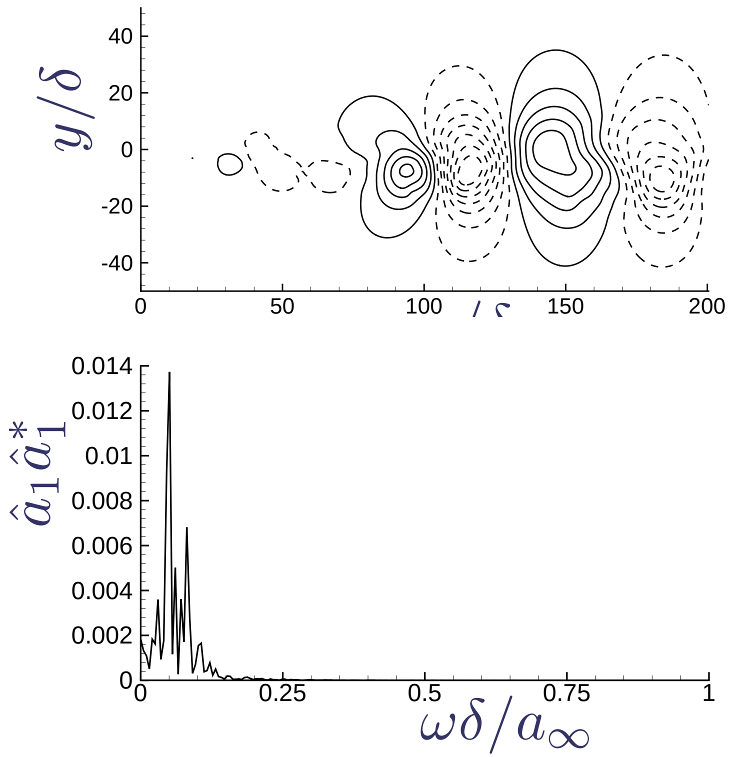


MODE 128

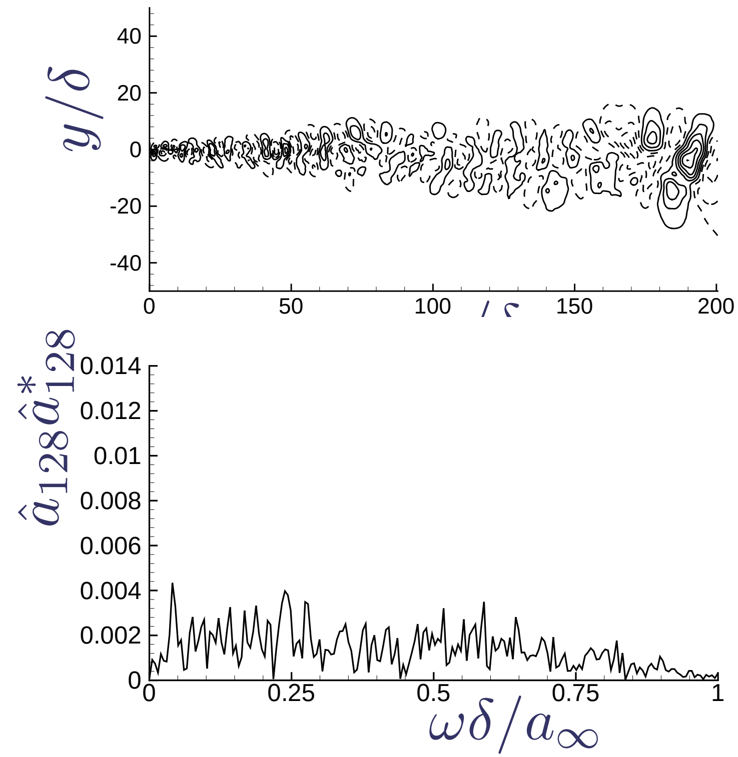


Mode Shapes and $a(t)$

MODE 1



MODE 128



- Lower modes: larger scale, lower frequency, higher energy
- Higher modes: smaller scale, higher frequency, lower energy

Errors to Assess Robustness

- High-frequency / small-scale errors: truncate series

$$\vec{q}_e(\mathbf{x},t) = \sum_{i=1}^{N_t} a_i(t)\vec{\psi}_i(\mathbf{x})$$

  - *e.g.* missing scales in LES

- Low-frequency / large-scale errors: mess with mode 1 and/or 2

$$\vec{q}_e(\mathbf{x},t) = \vec{q}(\mathbf{x},t) - \frac{a_1(t)}{2}\vec{\psi}_1(\mathbf{x})$$

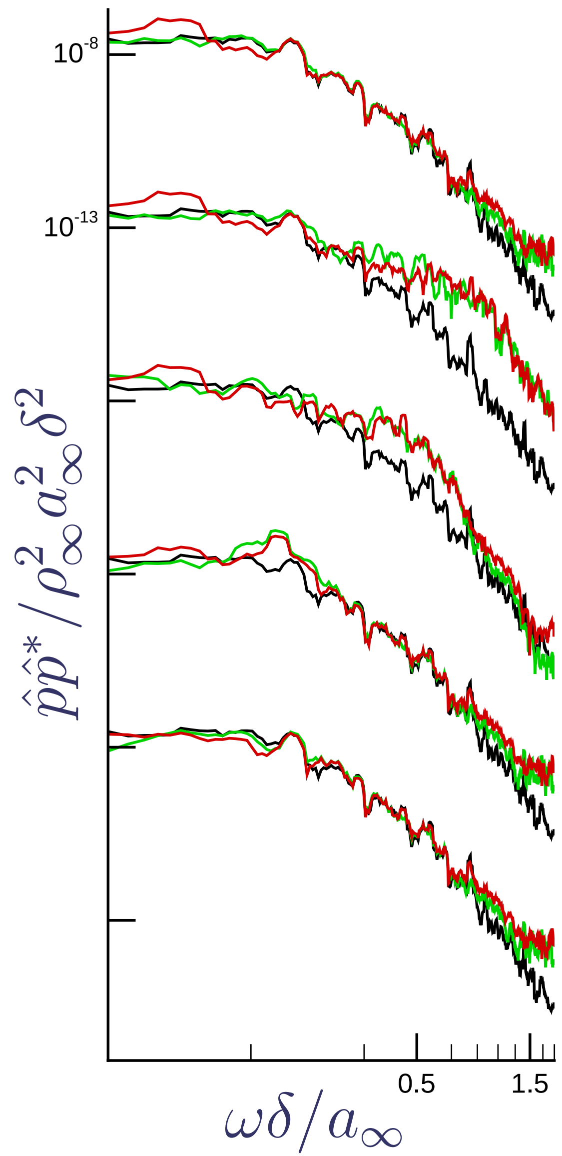
$$\vec{q}_e(\mathbf{x},t) = \vec{q}(\mathbf{x},t) - \frac{a_1(t)}{2}\vec{\psi}_1(\mathbf{x}) - \frac{a_2(t)}{2}\vec{\psi}_2(\mathbf{x})$$

  - *e.g.* POD dynamical model, PSE

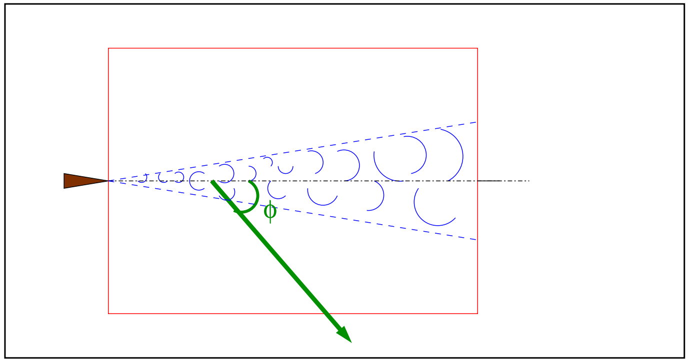
Errors

Case	Energy Retained	Description
A	100.0%	Full source
B	99.3%	128 modes
C	91.3%	32 modes
D	92.5%	$a_1' = a_1/2$
E	85.5%	$a_{1,2}' = a_{1,2}/2$

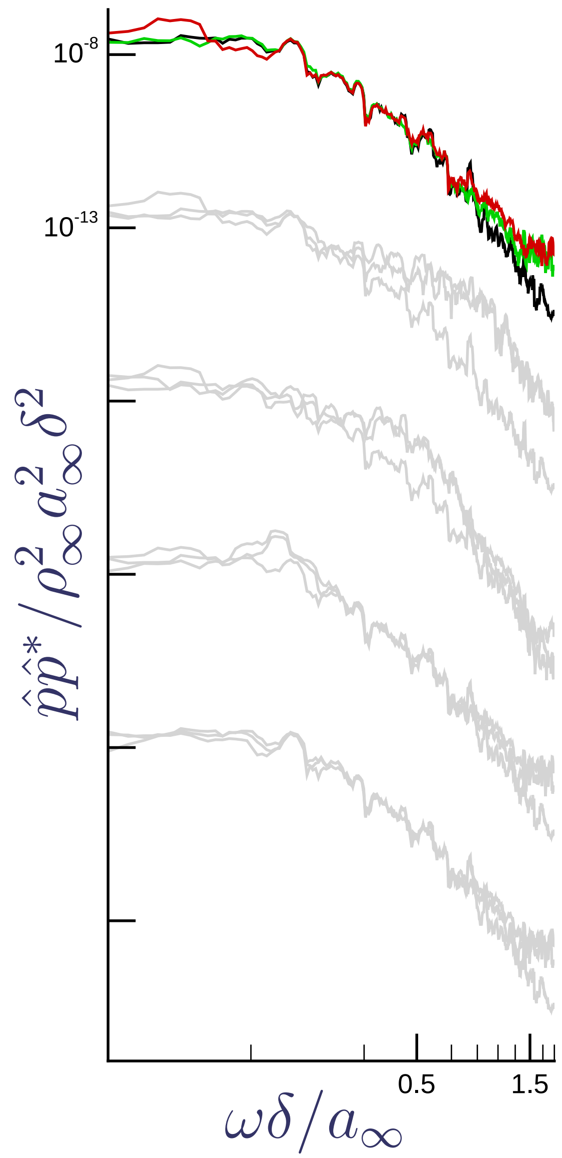
Sound Pressure Spectra: $\phi = 50^\circ$



DNS
DNS-mean base flow
Uniform base flow



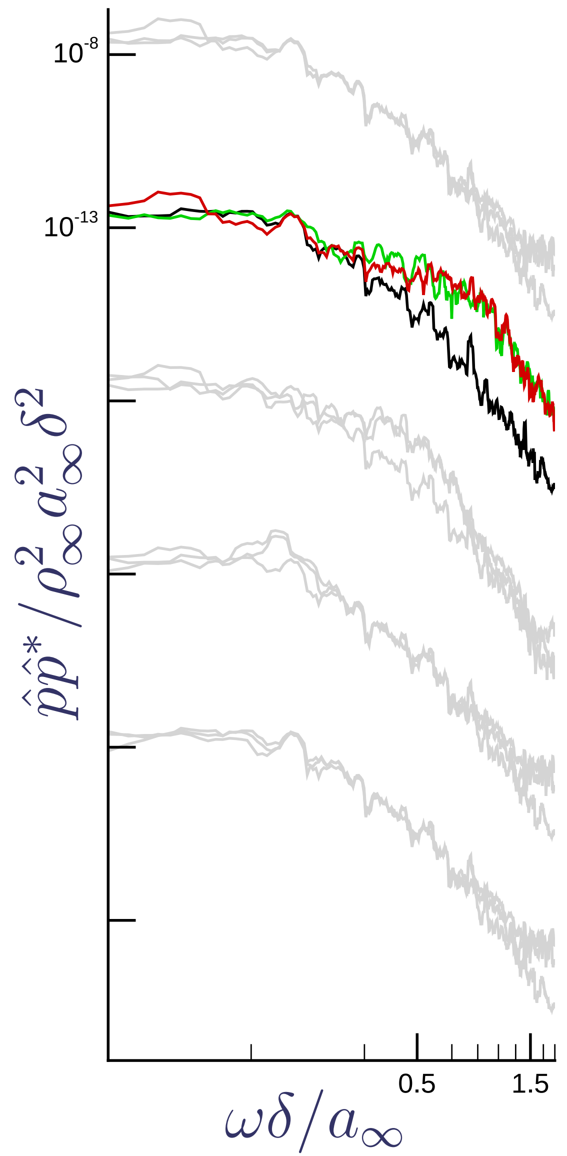
Sound Pressure Spectra: $\phi = 50°$



DNS
DNS-mean base flow
Uniform base flow

Case A: Full source

Sound Pressure Spectra: $\phi = 50^\circ$



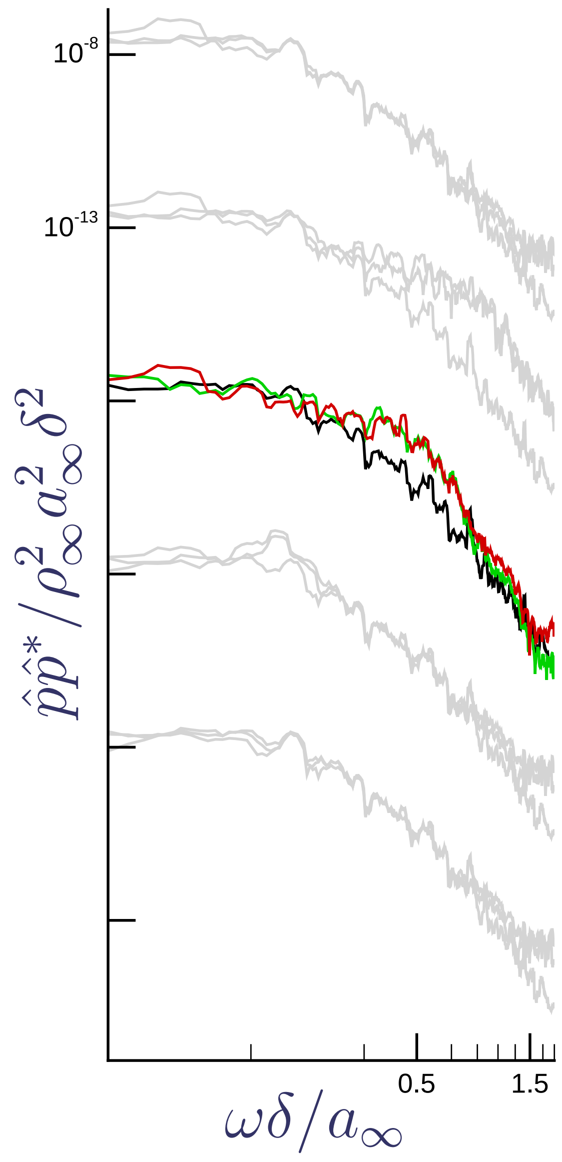
DNS

DNS-mean base flow

Uniform base flow

Case B: 128 modes

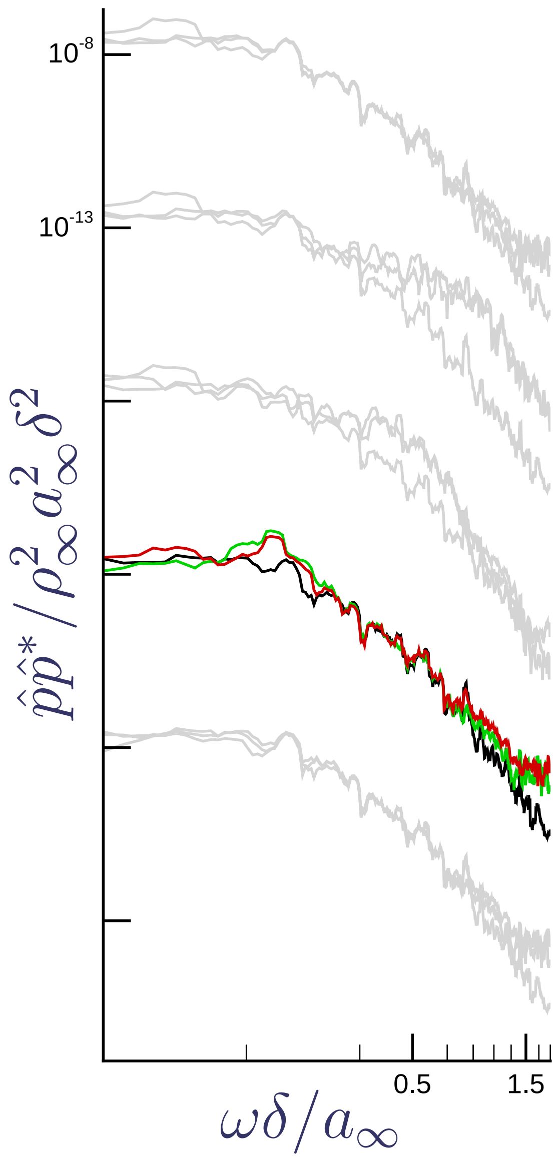
Sound Pressure Spectra: $\phi = 50°$



DNS
DNS-mean base flow
Uniform base flow

Case C: 32 modes

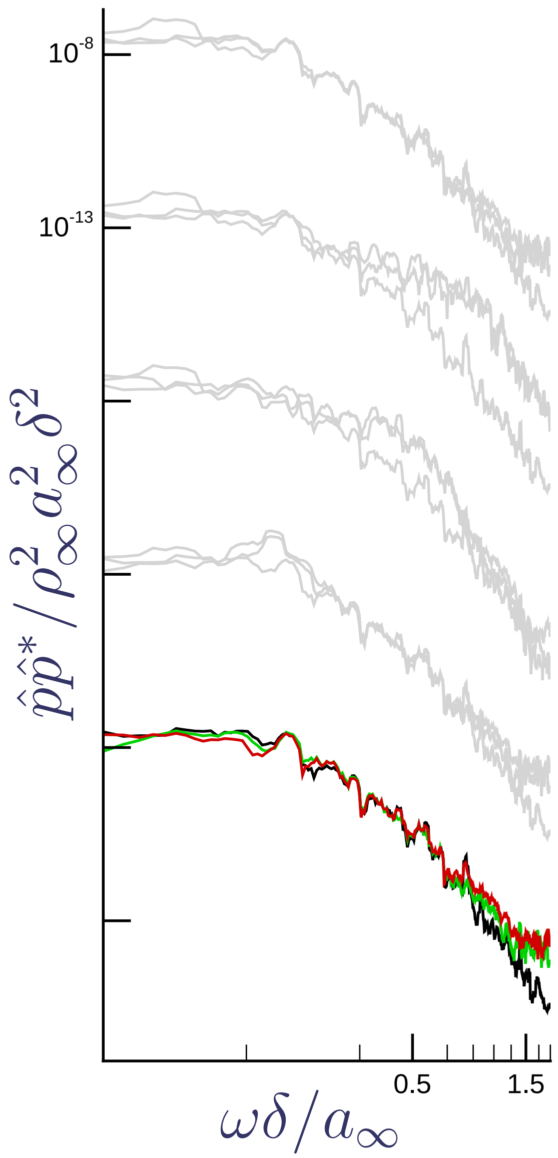
Sound Pressure Spectra: $\phi = 50°$



DNS
DNS-mean base flow
Uniform base flow

Case D: $a_1' = a_1/2$

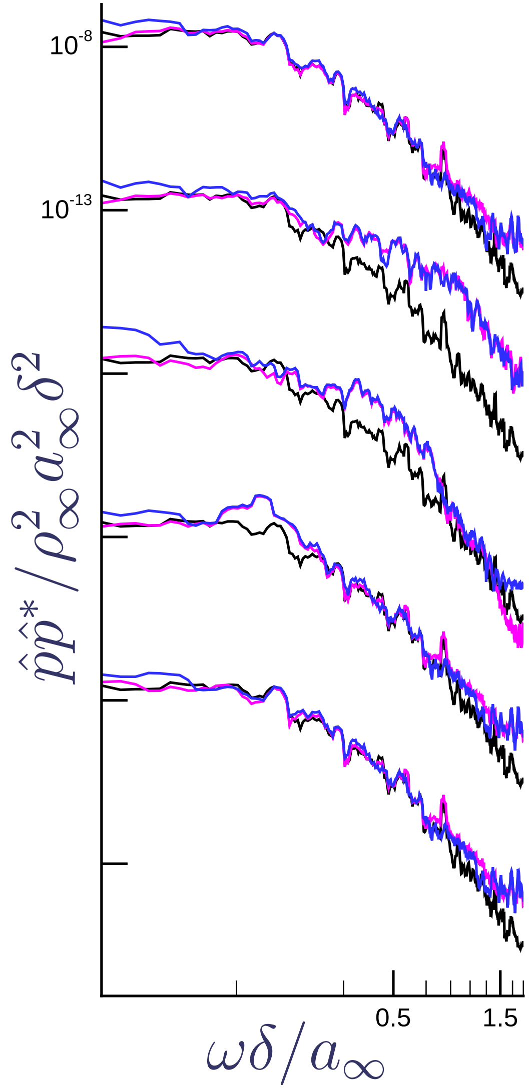
Sound Pressure Spectra: $\phi = 50°$



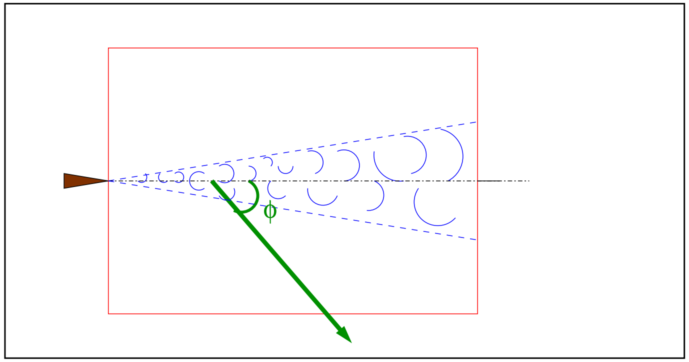
DNS
DNS-mean base flow
Uniform base flow

Case E: $a'_{1,2} = a_{1,2}/2$

Sound Pressure Spectra: $\phi = 50^{\circ}$



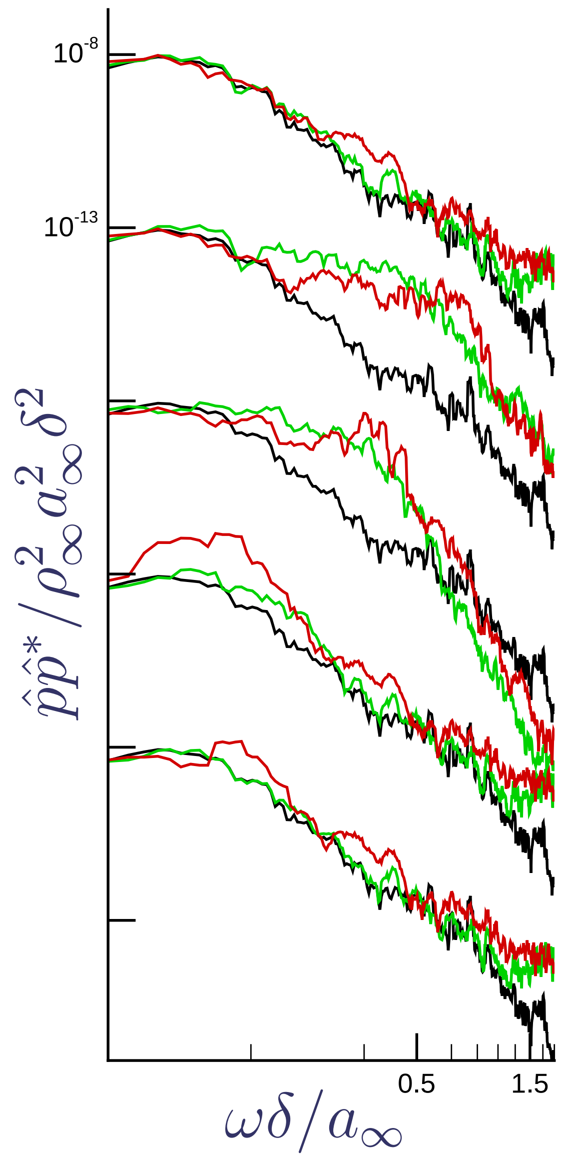
DNS
Parallel base flow
Locally parallel flow



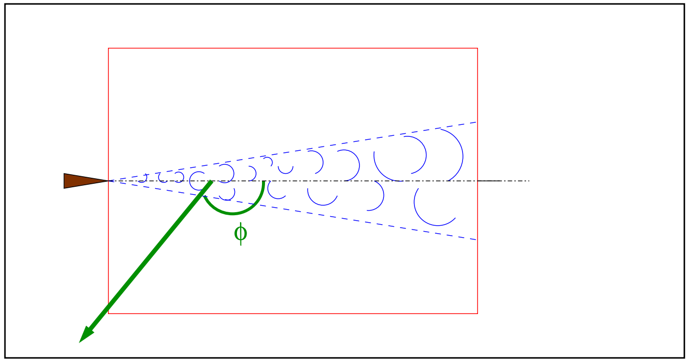
Sound Spectra: $\phi = 50^o$

None clearly more robust at 50°

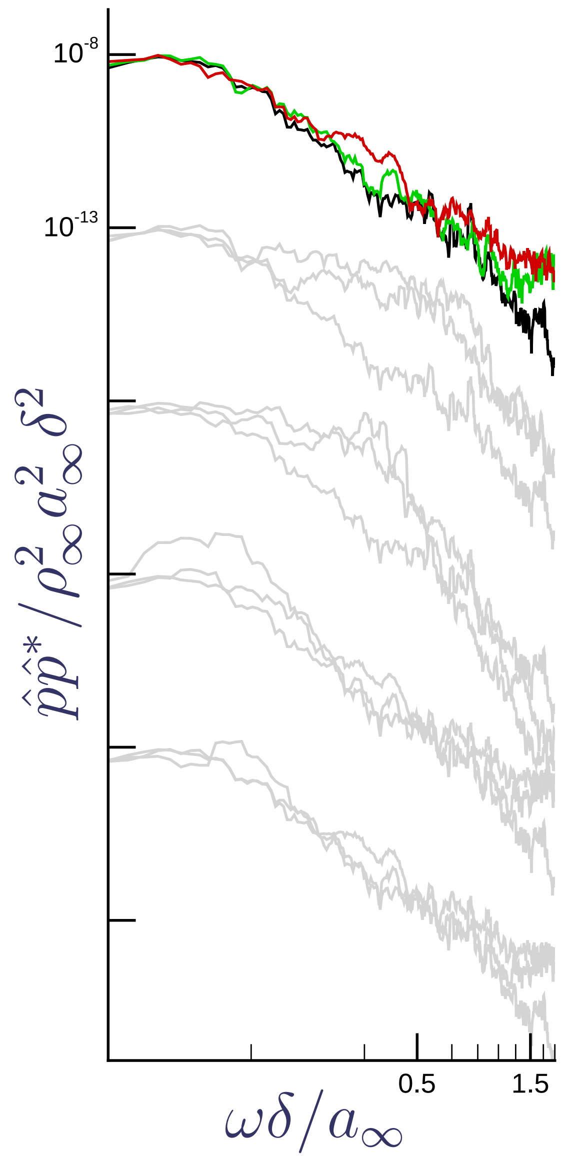
Sound Pressure Spectra: $\phi = 130^{\circ}$



DNS
DNS-mean base flow
Uniform base flow



Sound Pressure Spectra: $\phi = 130°$



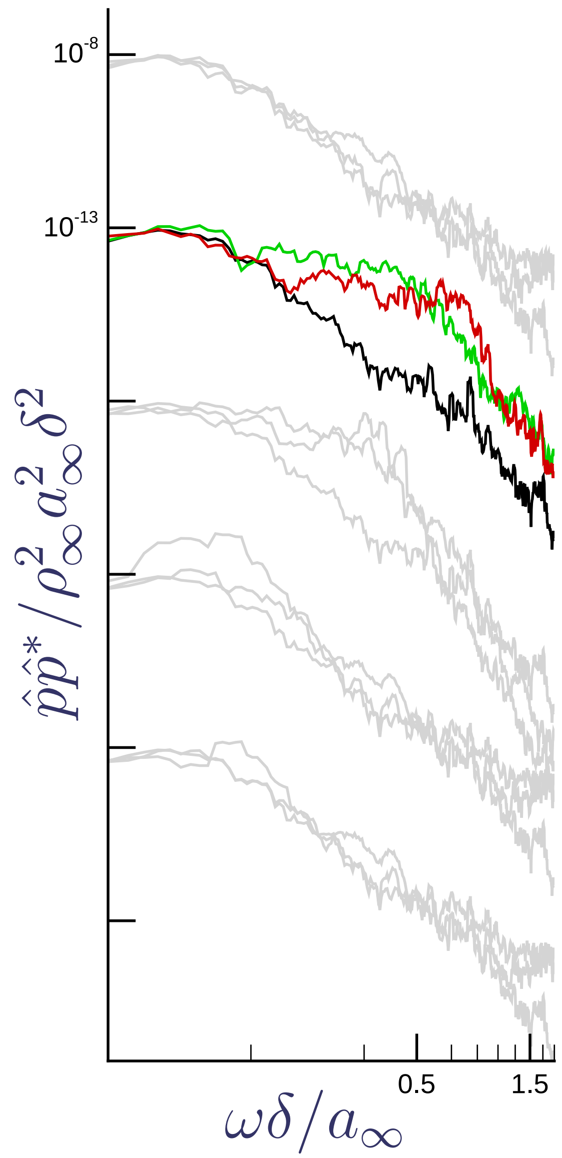
DNS

DNS-mean base flow

Uniform base flow

Case A: Full source

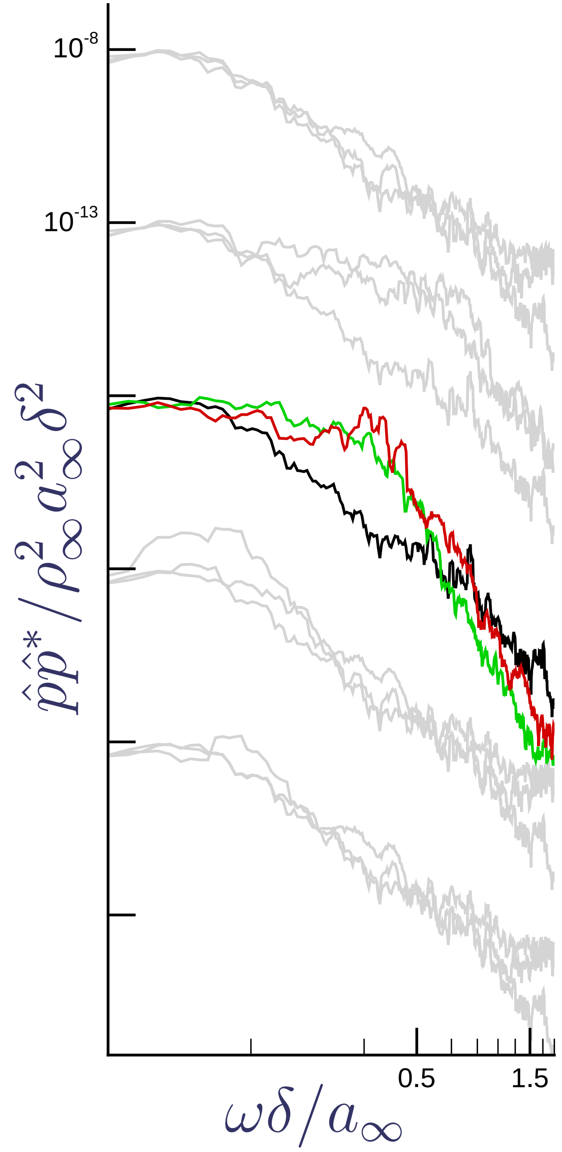
Sound Pressure Spectra: $\phi = 130^\circ$



DNS
DNS-mean base flow
Uniform base flow

Case B: 128 modes

Sound Pressure Spectra: $\phi = 130°$



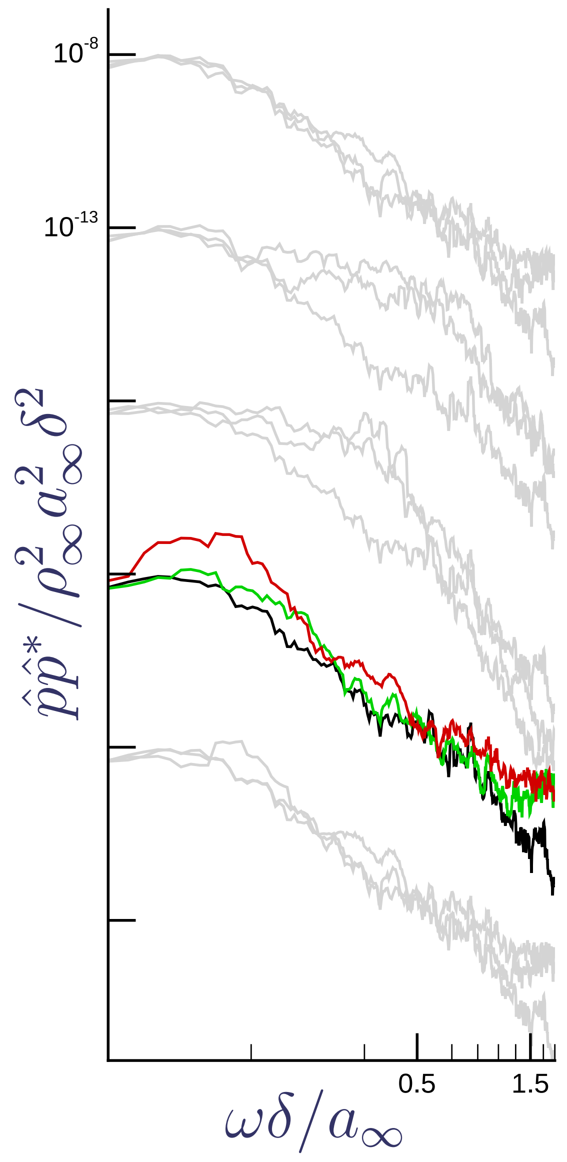
DNS

DNS-mean base flow

Uniform base flow

Case C: 32 modes

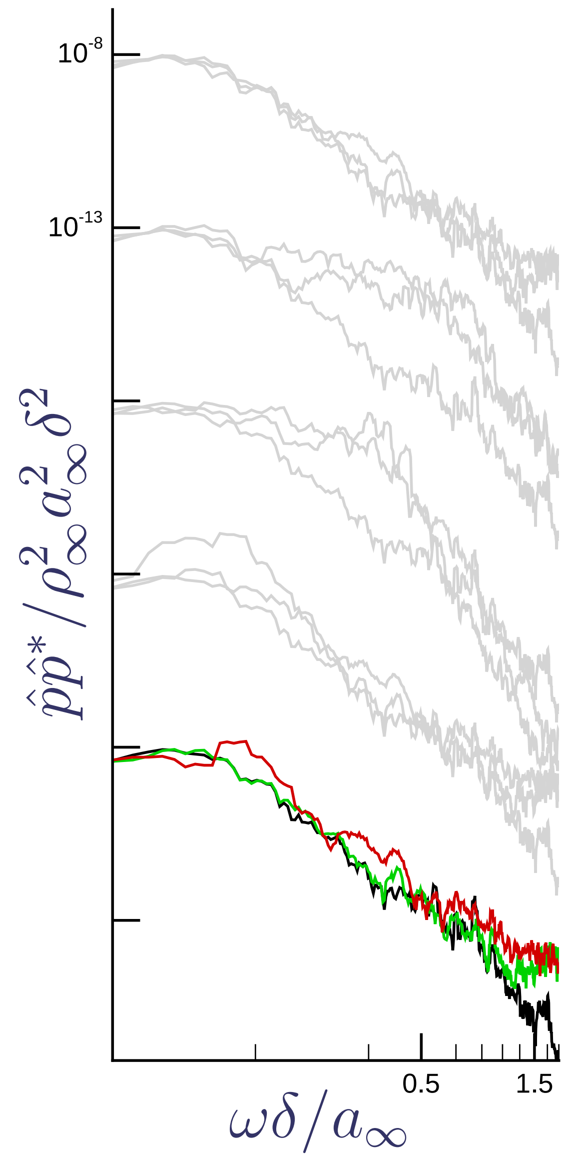
Sound Pressure Spectra: $\phi = 130^\circ$



DNS
DNS-mean base flow
Uniform base flow

Case D: $a_1' = a_1/2$

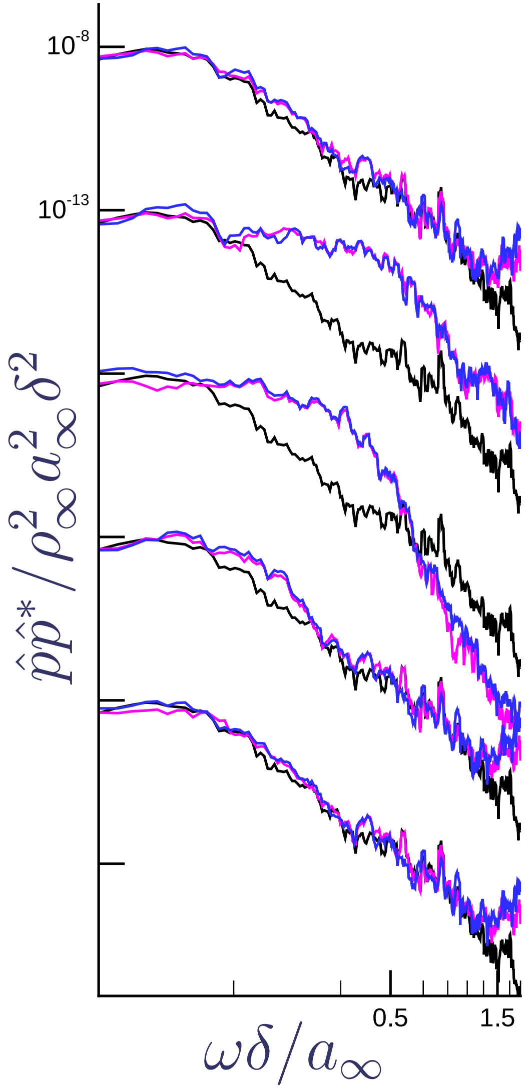
Sound Pressure Spectra: $\phi = 130^\circ$



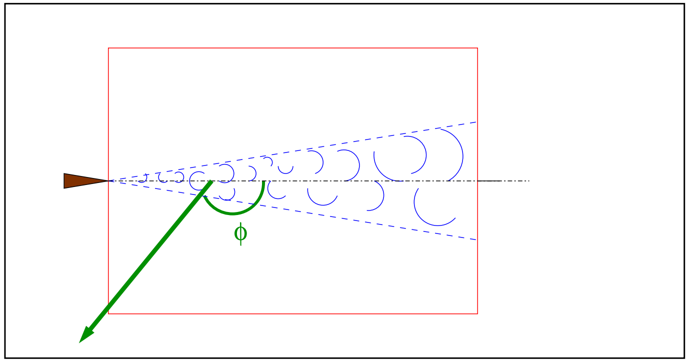
DNS
DNS-mean base flow
Uniform base flow

Case E: $a'_{1,2} = a_{1,2}/2$

Sound Spectra: $\phi = 130^o$



DNS
Parallel base flow
Locally parallel flow

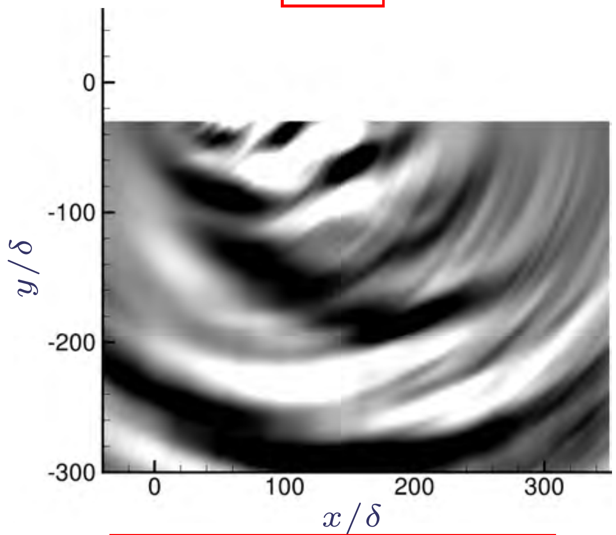


Sound Spectra: $\phi = 130^{\rm o}$

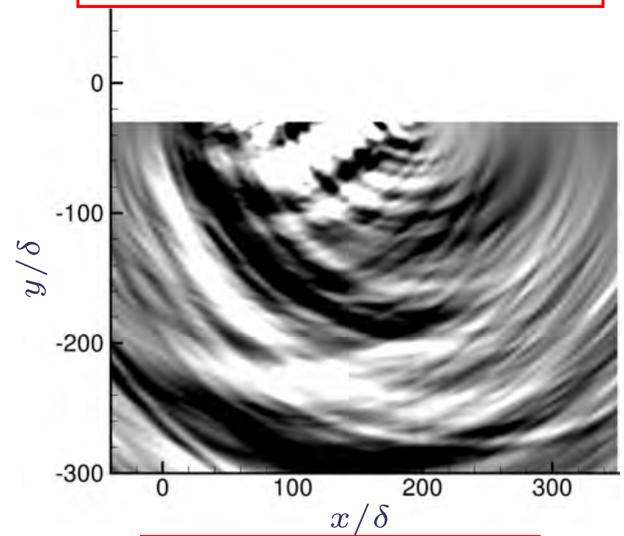
Lighthill-like analogy pathologically sensitive to S errors

Sound Field Visualization

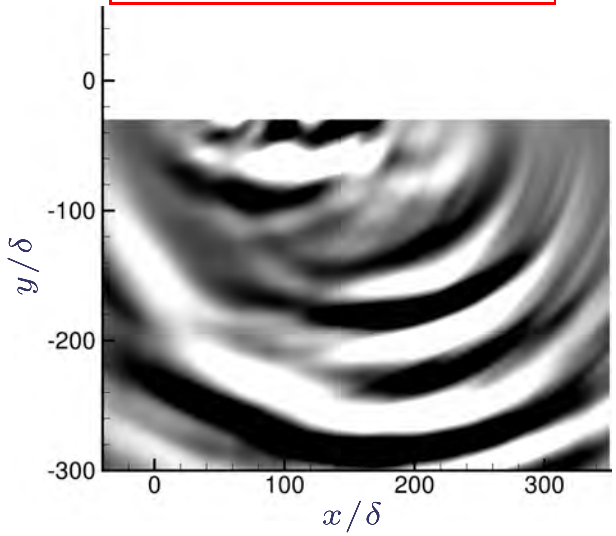
- DNS



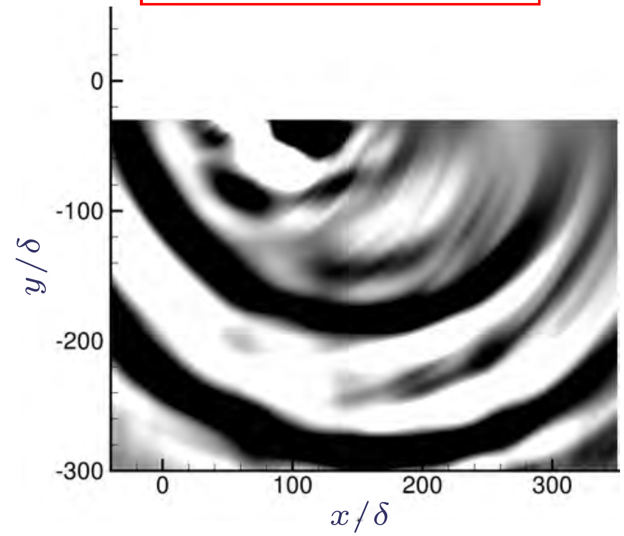
- DNS-mean base flow, 128 modes



- DNS-mean base flow, $a_1/2$



- Uniform base flow, $a_1/2$



Error: Filtering

- Filter flow variables:
  $\beta \hat{f}_{i-2} + \alpha \hat{f}_{i-1} + \hat{f}_i + \alpha \hat{f}_{i+1} + \beta \hat{f}_{i+2} = \sum_{j=0}^{N} \frac{a_j (f_{i-j} + f_{i+j})}{2}$

- Transfer function: $T(k\Delta x) = \frac{\sum_{n=0}^{N} a_n \cos(nk\Delta x)}{1 + 2\alpha\cos(k\Delta x) + 2\beta\cos(2k\Delta x)}$

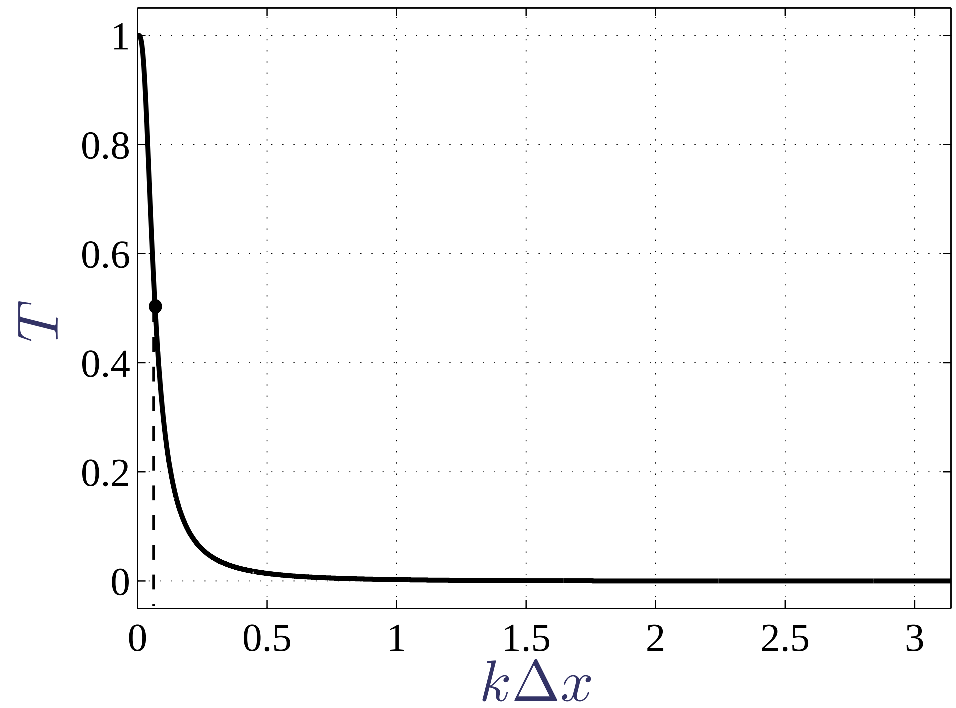
$$T(k_1 \Delta x) = s_1$$
$$T(k_2 \Delta x) = s_2$$

- $\beta = 0.16645, \quad \alpha = -0.66645$

- $a_0 = \frac{1}{4}(2 + 3\alpha), \quad a_1 = \frac{1}{16}(9 + 16\alpha + 10\beta),$

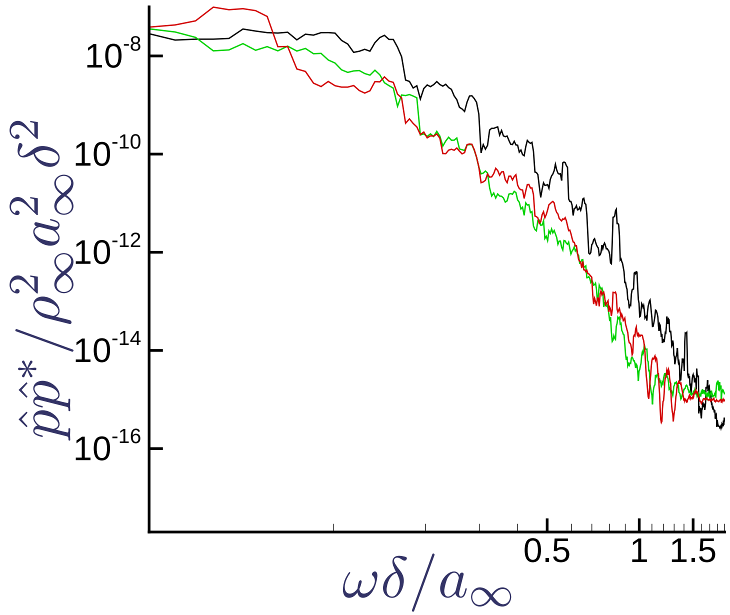
- $a_2 = \frac{1}{4}(\alpha + 4\beta), \quad a_3 = \frac{1}{16}(6\beta - 1).$

Error: Filtering

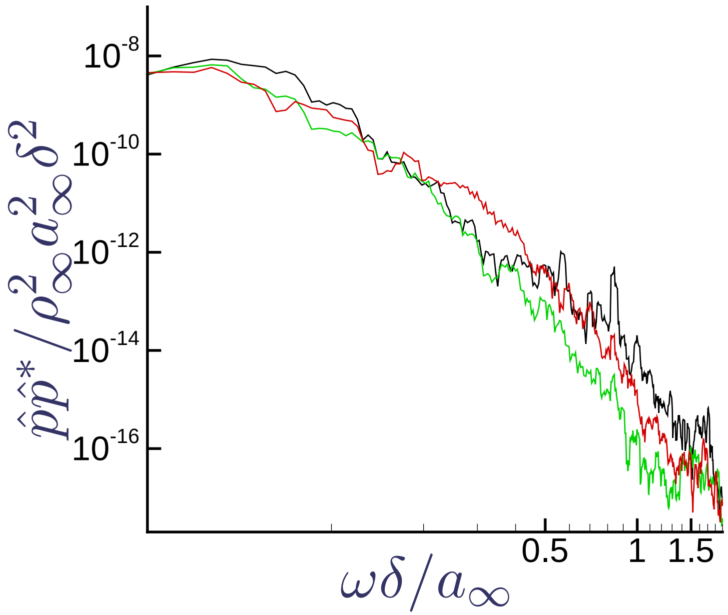


- $T(\lambda) = 0.5$ for $\lambda = 15.7\delta_\omega$
- Filter applied directly to DNS data
- Filtered data used to compute means, correlations and sources

Sound Pressure Spectra



$\boxed{\alpha = 50^{\circ}}$

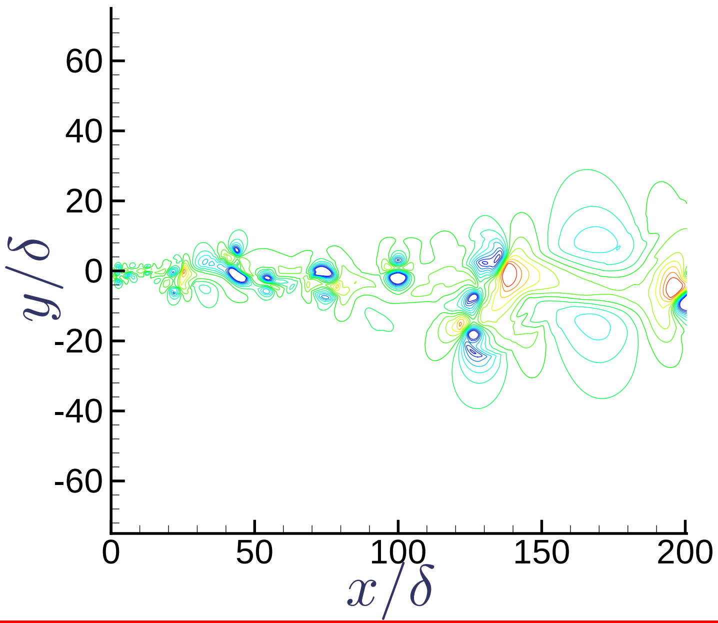


$\boxed{\alpha = 130^{\circ}}$

DNS-mean base flow   Uniform base flow

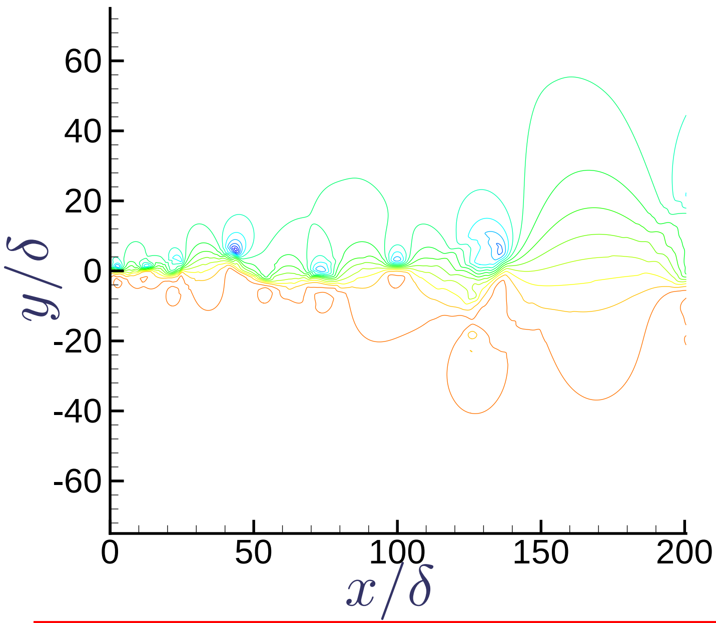
Sensitivity?

- Why is the Lighthill-like (uniform flow) so sensitive?



DNS-mean base flow analogy

Bubbly Structure



Uniform base flow analogy

Wavy Structure

A Crude Model

- $a_1(t) = -\sin\omega t$ and $a_2(t) = \cos\omega t \Rightarrow$ form suggested by actual POD analysis

- Convected harmonic wave modulated by a Gaussian envelope: $y_p = e^{-\eta x^2}[a_1(t)\cos kx + a_2(t)\sin kx]$

- Velocity field: $u_1 = \frac{1}{2}(M_1 - M_2)[\tanh(\sigma(y - y_p)) + 1] + M_2$

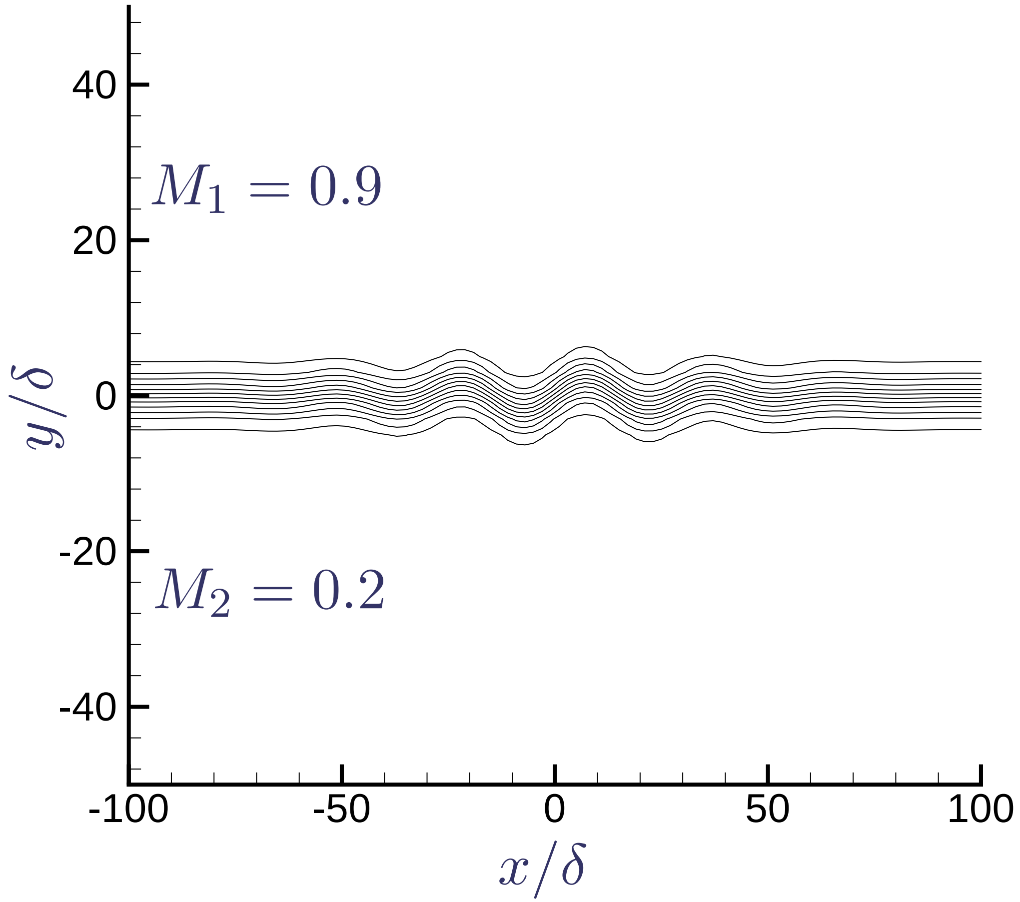
- Construct T_{11}

- Solve Lighthill's equation: $\left(\nabla^2 - \frac{1}{a_\infty^2}\frac{\partial^2}{\partial t^2}\right)\rho(\mathbf{x}, t) = -\frac{1}{a_\infty^2}\frac{\partial^2 \rho u_1 u_1}{\partial x_1 \partial x_1}$

- Solution: $\rho(\mathbf{x}, \omega) = -\frac{1}{4i}\int \mathcal{S}(\mathbf{y}, \omega) H_0^{(1)}(k_\omega \mid \mathbf{x} - \mathbf{y} \mid) d\mathbf{y}$

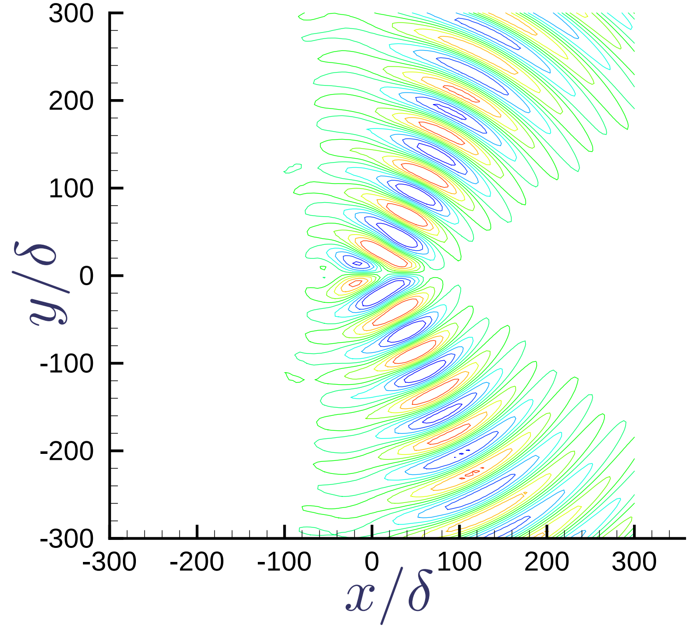
- Two cases: (1) a_1, a_2; (2) $a_1/2$, a_2

Model Source

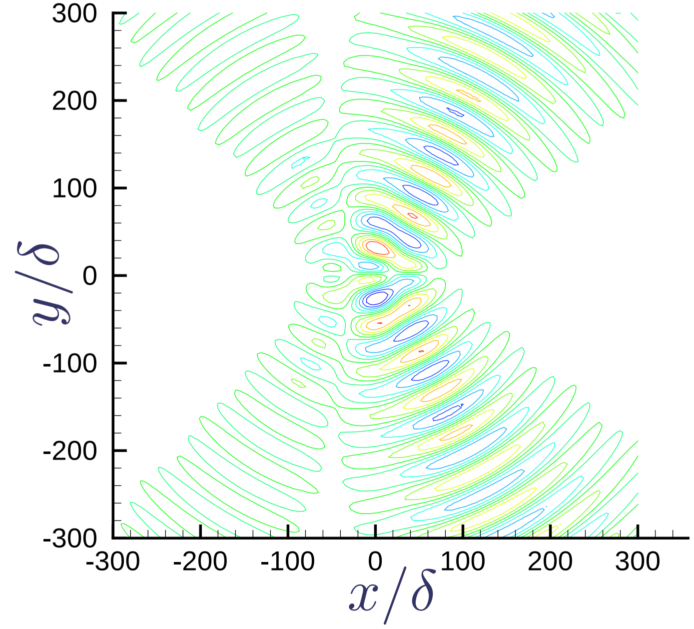


Model Source

Full source

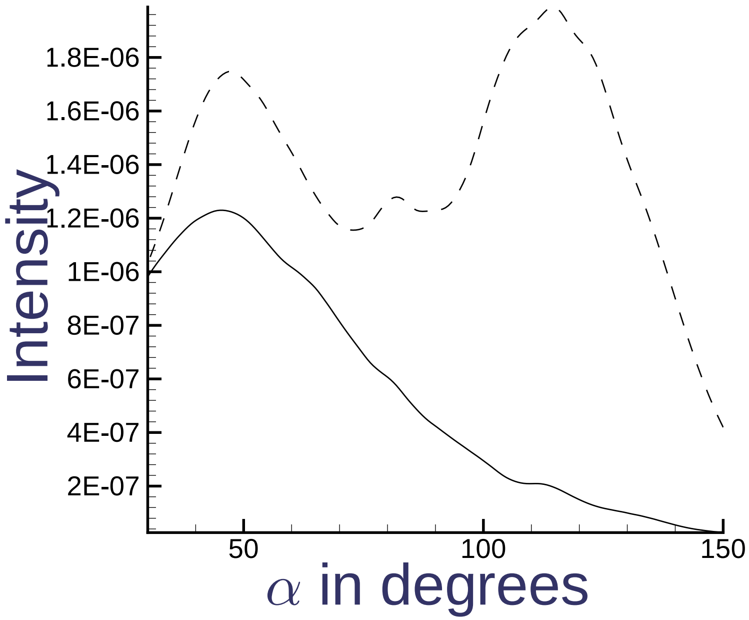


Source with error

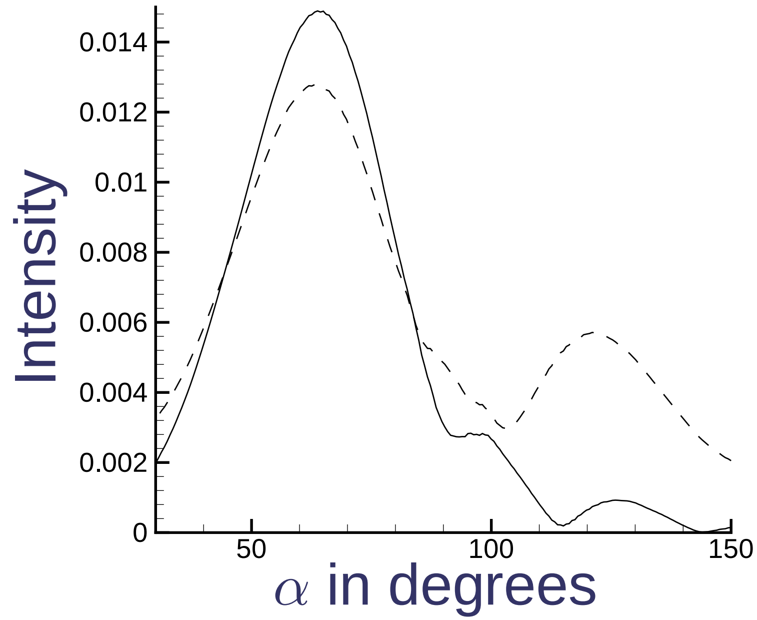


Model Source

Numerical simulation



Model



Summary

- Sound constitutes a tiny amount of a flow's energy
- Defining sound involves splitting the flow solution into source and propagation
  - resulting formulas for prediction
    - predict U^8 scaling observed
    - challenging: turbulence complexity, phase velocity,....
  - source simplifications have not significantly improved predictions
  - $\mathcal{N}(\vec{q}) = 0 \quad \Rightarrow \quad \mathcal{L}\vec{q} = S(\vec{q})$ not unique

Summary

- Assessed choice of $\mathcal{L}\vec{q} = S(\vec{q})$ based upon robustness criterion
- Small-scale errors
  - all analogies behaved similarly
  - potential implications for SGS-noise modeling
- Large-scale errors
  - large errors for uniform-flow base flow (Lighthill)
  - analogies with principal shear in $\mathcal{L}$ were similarly robust
  - potential implication for POD-dynamic, PSE models
- The high sensitivity of uniform base flow due to non-compact wavy character
- Homogeneous solutions ($\mathcal{L}\vec{q} = 0$) did not hinder predictions

Wither Prediction?

- Large-eddy simulation..... we are at the dawn of affordability

Wither Prediction?

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- Without engineering insights, there still wont be guidance regarding what to do with predictions.......

Wither Prediction?

- Large-eddy simulation..... we are at the dawn of affordability
- Without engineering insights, there still wont be guidance regarding what to do with predictions....... next talk