Adjoint-Based Optimization For Understanding And Suppressing Jet Noise

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Acknowledgments

- Prof. Mingjun Wei
- Dr. Randall Kleinman
- Dr. Jeonglae Kim
- Prof. Daniel Bodony

Outline

- Background
- Equations
- Pictures

Simulation Prediction of Jet Noise

- Jet noise has defied simple mechanistic description
  - hampered design at fixed flow conditions (no U^8)
  - this is the turbulence problem: a problem of description

Simulation Prediction of Jet Noise

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  - this is the turbulence problem: a problem of description
- Large-eddy simulations making/nearly making quality predictions at engineering conditions
  - Bodony & Lele
  - Bogey, Bailly, Juvé
  - Shur, Spalart, *et al.*
  - Mendez, Lele, *et al.*
  - Uzun, Hussani
  - Karabasov, Dowling *et al.*
  - many others

Simulations

- Simulations have been only moderately helpful
  - have not substantively clarified turbulence noise source (*e.g.* Freund 2001)
  - do not point the ‘direction’ toward quiet
  - full space-time information nice but challenging to effectively harness

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- Adjoint-based optimization: circumvents turbulence complexity to provide direction of design improvement

Adjoint-Based Optimization

- Define 'good' quantitatively

$$\mathcal{J} = \mathcal{J}(\vec{q}, \vec{F})$$

  - $\vec{q}$ — flow solution, *e.g.* $\vec{q} = [\rho, \rho\mathbf{u}, e]^T$ at all $\mathbf{x}$ and t
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- Design/control problem: minimum $\mathcal{J}$ is quiet(er)

$$\delta\mathcal{J} = \left(\frac{\partial\mathcal{J}}{\partial\vec{q}}\right)_{\vec{F}} \delta\vec{q} + \left(\frac{\partial\mathcal{J}}{\partial\vec{F}}\right)_{\vec{q}} \delta\vec{F}$$

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    - ‘optimum’ constrained by existing hardware
  - simulations provide geometric/actuation flexibility
    - remain hopelessly expensive for extensive searches
- Key problem: each $\delta \vec{F}$ requires new $\vec{q}$ computation for $\delta \vec{q}$

Adjoint-Based Optimization

- $\vec{F}$ 'guess' and new $\vec{q}$ are constrained to solve flow equations:

$$\mathcal{N}(\vec{q}) = \vec{F}$$

- Define $\mathcal{M}$

$$\mathcal{M}(\vec{q}, \vec{F}) = \mathcal{N}(\vec{q}) - \vec{F} = 0$$

- Variation

$$\delta\mathcal{M} = \left(\frac{\partial\mathcal{M}}{\partial\vec{q}}\right)_{\vec{F}} \delta\vec{q} + \left(\frac{\partial\mathcal{M}}{\partial\vec{F}}\right)_{\vec{q}} \delta\vec{F} = 0$$

Adjoint-Based Optimization

- Apply $\vec{q}^{*} \cdot \delta\mathcal{M} = 0$ as constraint:

$$\delta\mathcal{J} = \delta\mathcal{J} - \vec{q}^{*} \cdot \delta\mathcal{M}$$
$$= \left[\left(\frac{\partial\mathcal{J}}{\partial\vec{q}}\right)_{\vec{F}} - \vec{q}^{*} \cdot \left(\frac{\partial\mathcal{M}}{\partial\vec{q}}\right)_{\vec{F}}\right]\delta\vec{q} + \left[\left(\frac{\partial\mathcal{J}}{\partial\vec{F}}\right)_{\vec{q}} - \vec{q}^{*} \cdot \left(\frac{\partial\mathcal{M}}{\partial\vec{F}}\right)_{\vec{q}}\right]\delta\vec{F}$$

- Remove $\delta\vec{q}$ dependence from $\delta\mathcal{J}$ by finding $\vec{q}^{*}$ that zeros first term

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$$\frac{\partial\mathcal{J}}{\partial\vec{q}} = \vec{q}^{*} \cdot \frac{\partial\mathcal{M}}{\partial\vec{q}}$$

- Removes need for repeated $\delta\vec{q}$ calculation to obtain

$$\frac{\delta\mathcal{J}}{\delta\vec{F}}$$ — 'direction' of better $\vec{F}$

Adjoint-Based Optimization

- An aeroacoustic cost function

$$\mathcal{J} = \int_{t_0}^{t_1} \int_{\mathbb{R}^3} W(\mathbf{x})[p(\mathbf{x},t) - p_o]^2 \, d\mathbf{x}dt$$

  - $W(\mathbf{x})$ weight localizes $\mathcal{J}$ in space – can be a point, surface, ...
  - Can add penalty, craft for shape optimization, target noise sources, ...

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- Functional differentiation yields

$$\frac{\partial \mathcal{J}}{\partial \vec{q}} \delta\vec{q} = \int_{t_0}^{t_1} \int_{\mathbb{R}^3} 2W(x)[p - p_o] \frac{\partial p}{\partial \vec{q}} \delta\vec{q} \, d\mathbf{x}dt$$

which is the l.h.s of the equation for $\vec{q}^*$: $\frac{\partial \mathcal{J}}{\partial \vec{q}} = \vec{q}^* \cdot \frac{\partial \mathcal{M}}{\partial \vec{q}}$

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- Take inner · product on r.h.s. of $\vec{q}^{*}$ equation to match $\mathcal{J}$ definition

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- Integrate by parts

$$\vec{q}^* \cdot \frac{\partial \mathcal{M}}{\partial \vec{q}} \delta\vec{q} = -\int_{t_0}^{t_1} \int_{\mathbb{R}^3} \delta\vec{q} \mathcal{M}^* \vec{q}^* \, d\mathbf{x} dt + b$$

  - $\mathcal{M}^*$ is the adjoint of the perturbed and linearized flow equations
  - $\vec{q}^*$ is now interpreted as the solution of the adjoint

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- Substitution with $b = 0$ yields adjoint differential equation for $\vec{q}^*$:

$$\mathcal{M}^*(\vec{q})\vec{q}^* = -2W(\mathbf{x})(p - p_o) \frac{\partial p}{\partial \vec{q}}$$

Boundary Terms

$$b = b_{|\mathbf{x}| \to \infty} + b_{t_0} + b_{t_1}$$

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- $b_{t_1} = 0$ by choice: start with $\vec{q}^* = 0$ at $t = t_1$, solve time reversed
  - need time-reversed information propagation to determine control needed

The Aeroacoustic Adjoint System

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- $\mathcal{M}^*(\vec{q})$ requires space/time dependent flow solution $\vec{q}(\mathbf{x}, t)$
  - needs resolved flow data at runtime
- Derivation tedious, but automatic
- Correctness testable:
  - can confirm by finite difference
  - anti-sound models

The Adjoint Equations

$$\left(\frac{\partial \rho^*}{\partial t} + u_i \frac{\partial {u^*}_i}{\partial t} + \frac{u_i u_i}{2}\frac{\partial p^*}{\partial t}\right) + u_j \frac{\partial \rho^*}{\partial x_j} + \left(u_i u_j + \frac{\tau_{ij}}{Re_\infty \rho}\right)\frac{\partial {u^*}_i}{\partial x_j} +$$

$$\left(\frac{u_i u_i}{2} u_j + n \frac{\tau_{jk} u_k - q_j / Pr}{Re_\infty \rho}\right)\frac{\partial p^*}{\partial x_j} - \frac{T}{Re_\infty Pr \rho}\frac{\partial}{\partial x_j}\left(\mu \frac{\partial p^*}{\partial x_j}\right) = {f^*}_{\rho^*}$$

$$\left(\rho \frac{\partial {u^*}_i}{\partial t} + \rho u_i \frac{\partial p^*}{\partial t}\right) + \rho \frac{\partial \rho^*}{\partial x_i} + \rho u_j \left(\frac{\partial {u^*}_i}{\partial x_j} + \frac{\partial {u^*}_j}{\partial x_i}\right) + \left[\rho u_i u_j \left(\rho E + p\right)\delta_{ij} - \frac{\tau_{ij}}{Re_\infty}\right]\frac{\partial p^*}{\partial x_j} +$$

$$\frac{1}{Re_\infty}\frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial {u^*}_i}{\partial x_j} + \frac{\partial {u^*}_j}{\partial x_i}\right) + \lambda \delta_{ij}\frac{\partial {u^*}_k}{\partial x_k} + \mu\left(u_j \frac{\partial p^*}{\partial x_i} + u_i \frac{\partial p^*}{\partial x_j}\right)\lambda \delta_{ij}\left(u_k \frac{\partial p^*}{\partial x_k}\right)\right] = {f^*}_{{u^*}_i}$$

$$\frac{1}{\gamma - 1}\frac{\partial p^*}{\partial t} + \frac{p\delta_{ij} - n\tau_{ij}/Re_\infty}{p}\frac{\partial {u^*}_i}{\partial x_j} + \left(\frac{\gamma}{\gamma - 1} u_j - n \frac{\tau_{jk} u_k - q_j / Pr}{Re_\infty p}\right)\frac{\partial p^*}{\partial x_j}$$

$$+ \frac{T}{Re_\infty Pr p}\frac{\partial}{\partial x_j}\left(\mu \frac{\partial p^*}{\partial x_j}\right) = {f^*}_{p^*}$$

Optimization

- Substituting $\mathcal{J}$, $\mathcal{M}$, $\vec{q}^{*}$ into $\delta\mathcal{J}$ equation:

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- Thus we have a direction in which to improve control/design:

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$$\vec{F}^{\text{new}} = \vec{F}^{\text{old}} - r\,\frac{\delta\mathcal{J}}{\delta\vec{F}}$$

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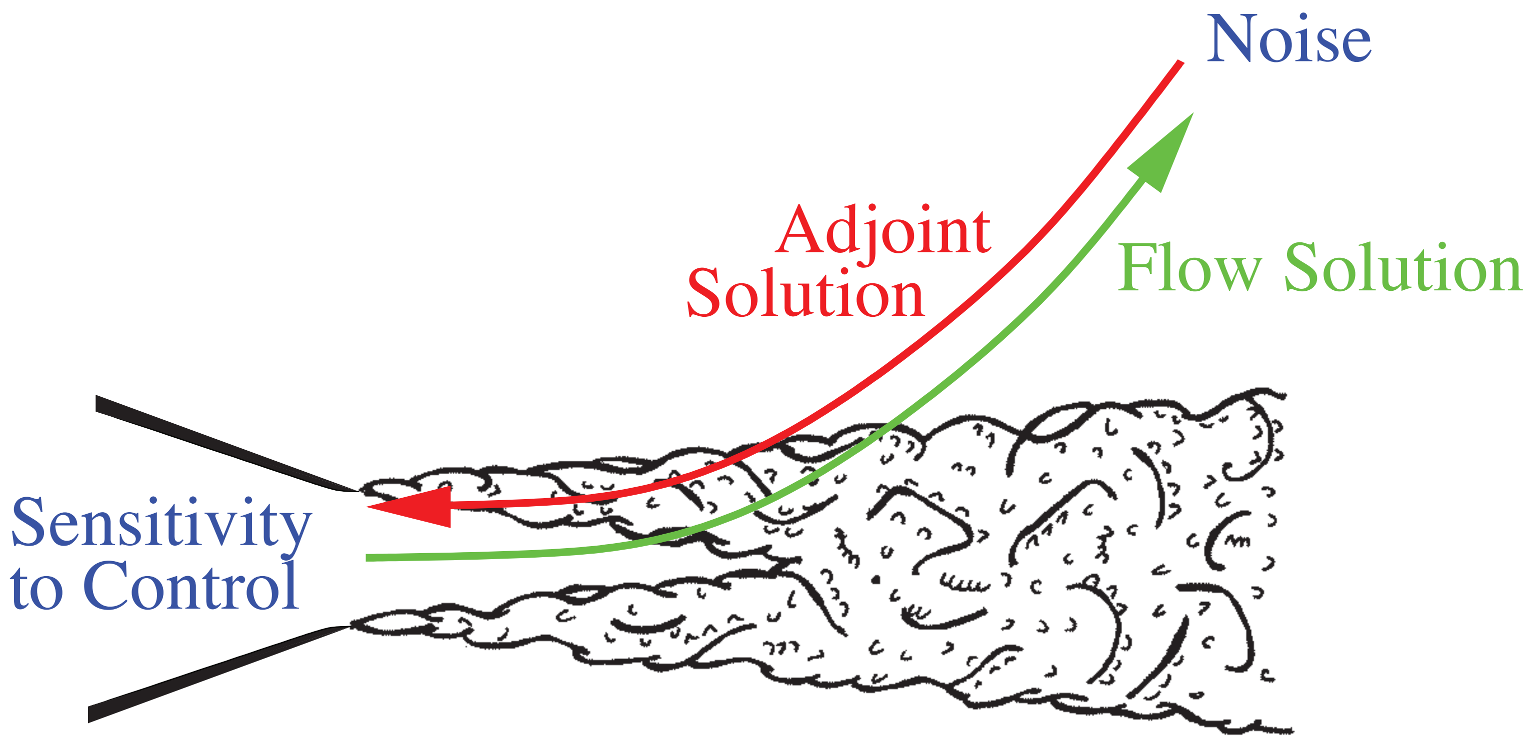
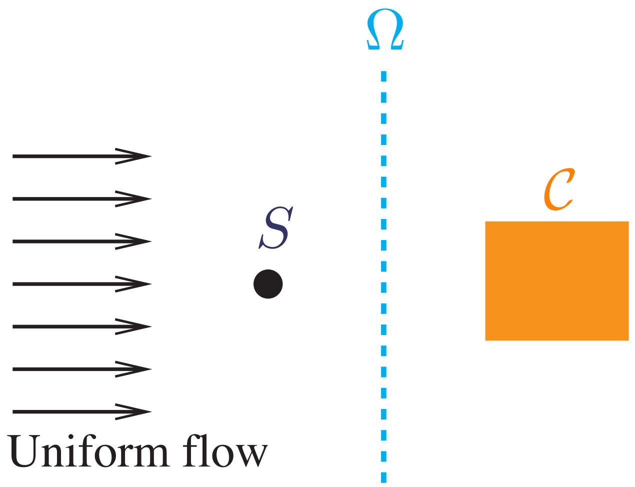
- Standard conjugate gradient has been effective

Jet Noise


Noise
Adjoint
Solution
Flow Solution
Sensitivity
to Control

Demonstration: Anti-Sound



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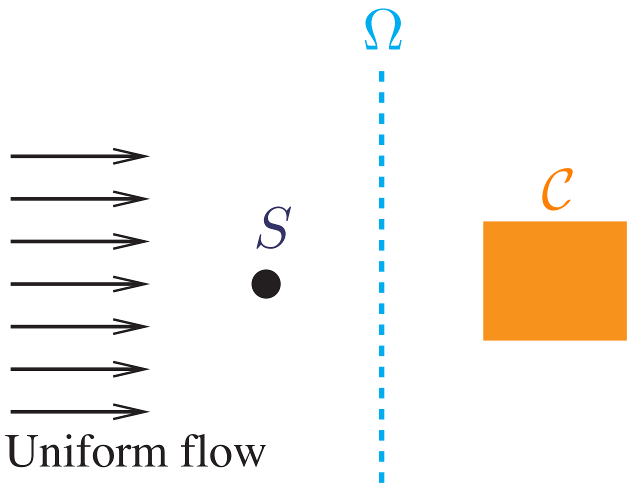
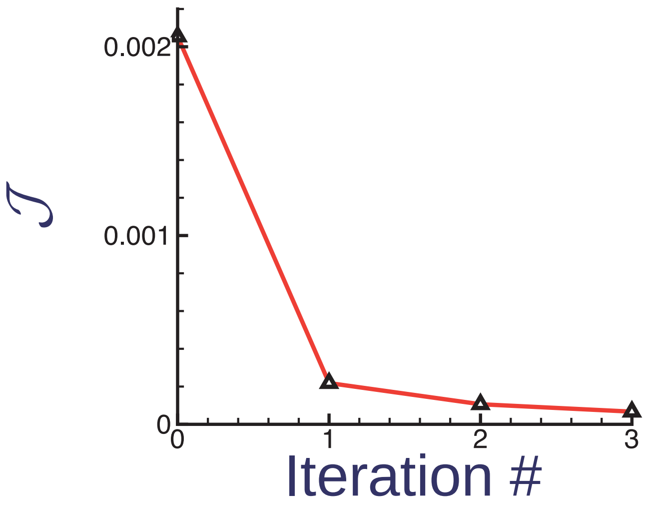
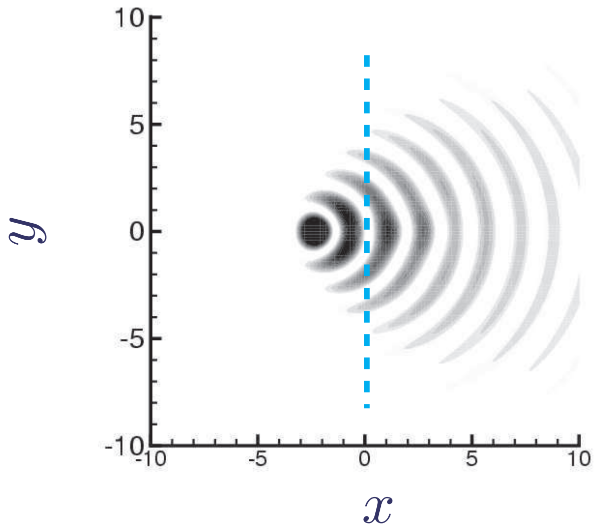
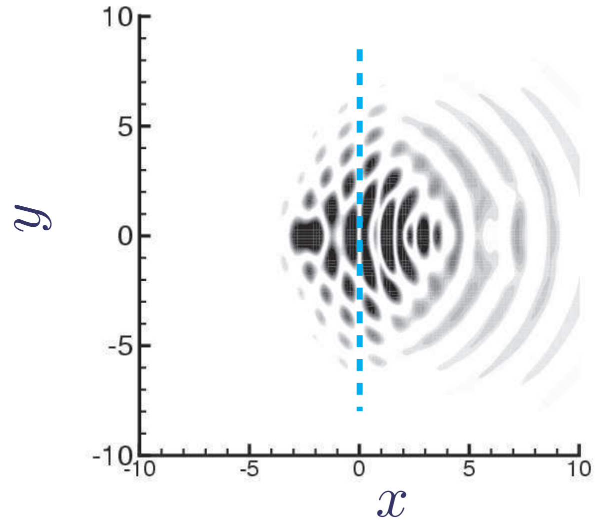
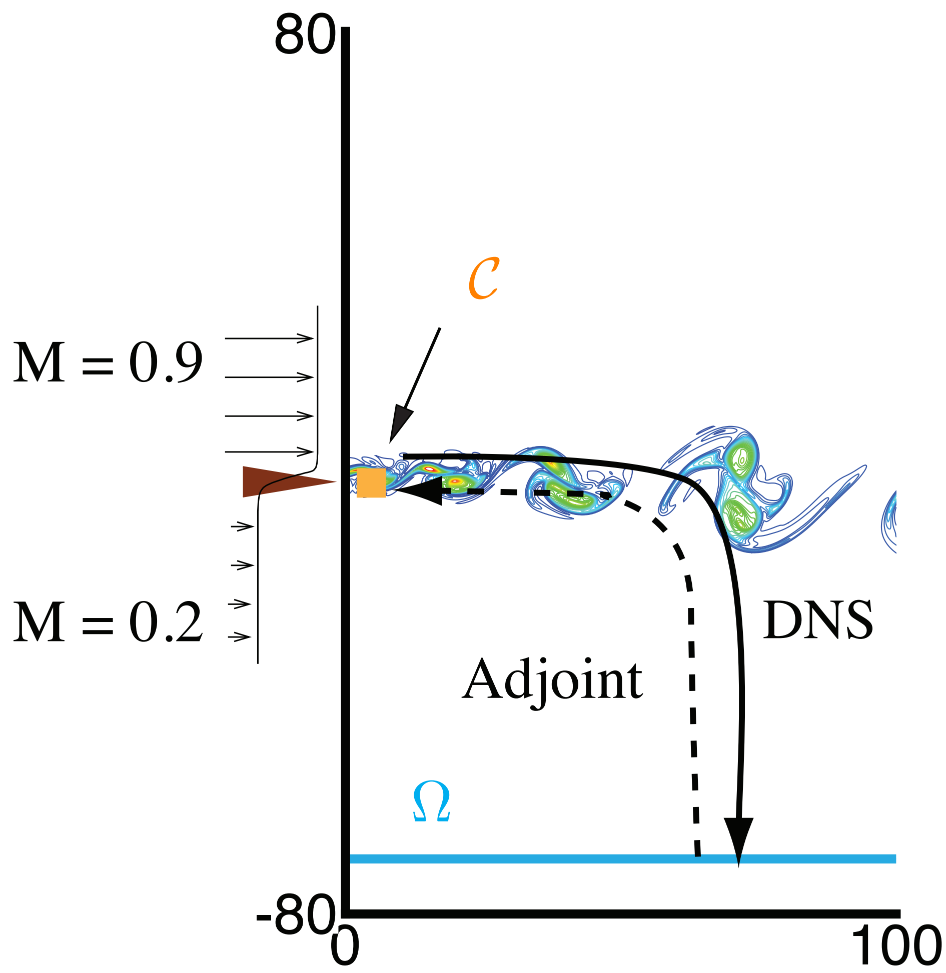








2-D Mixing Layer



- Models near-nozzle flow
- Reduce:

$$\mathcal{J}(\vec{q},\vec{F}) = \int_{t_0}^{t_1}\int_{\Omega}(p-p_\infty)^2\, d\Omega dt$$

with control $\vec{F}$ in $\mathcal{C}$:

$$\frac{\partial \vec{q}}{\partial t} = \tilde{\mathcal{N}}(\vec{q}) + \vec{F}(\mathbf{x},t)$$

- Each space/time point of $\vec{F}$ is a control parameter (10^7)

Numerical Methods

- Both the flow and adjoint use the same discrete operators

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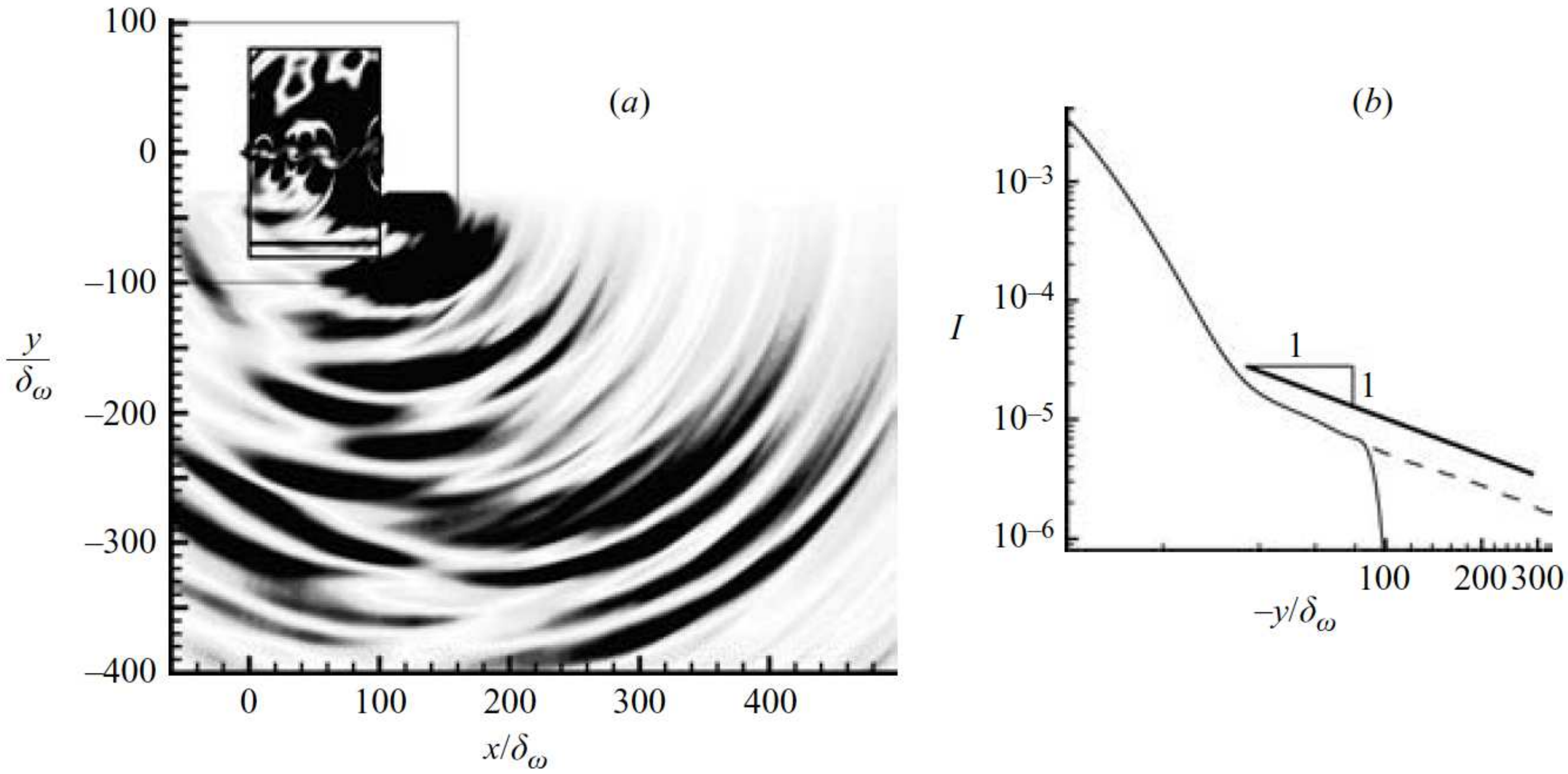
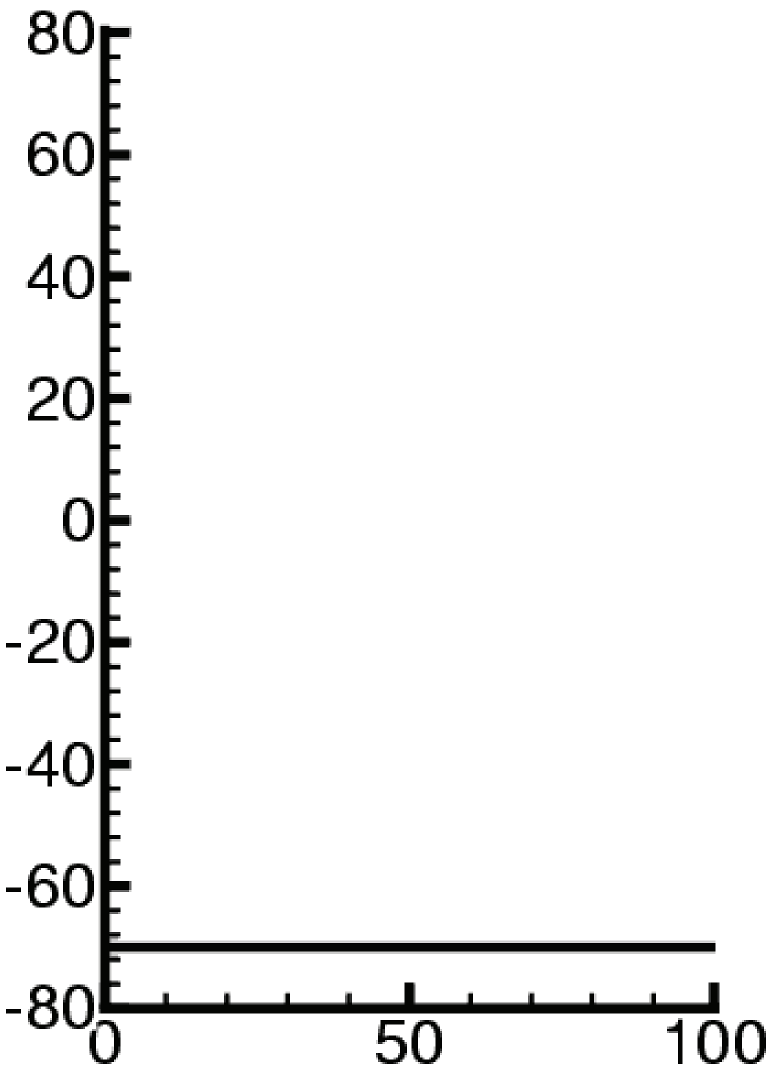
- Sixth-order, coefficient optimized, finite-difference schemes
- Fourth-order Runge–Kutta time advancement
- Absorbing buffer zone boundary conditions

Sound Field

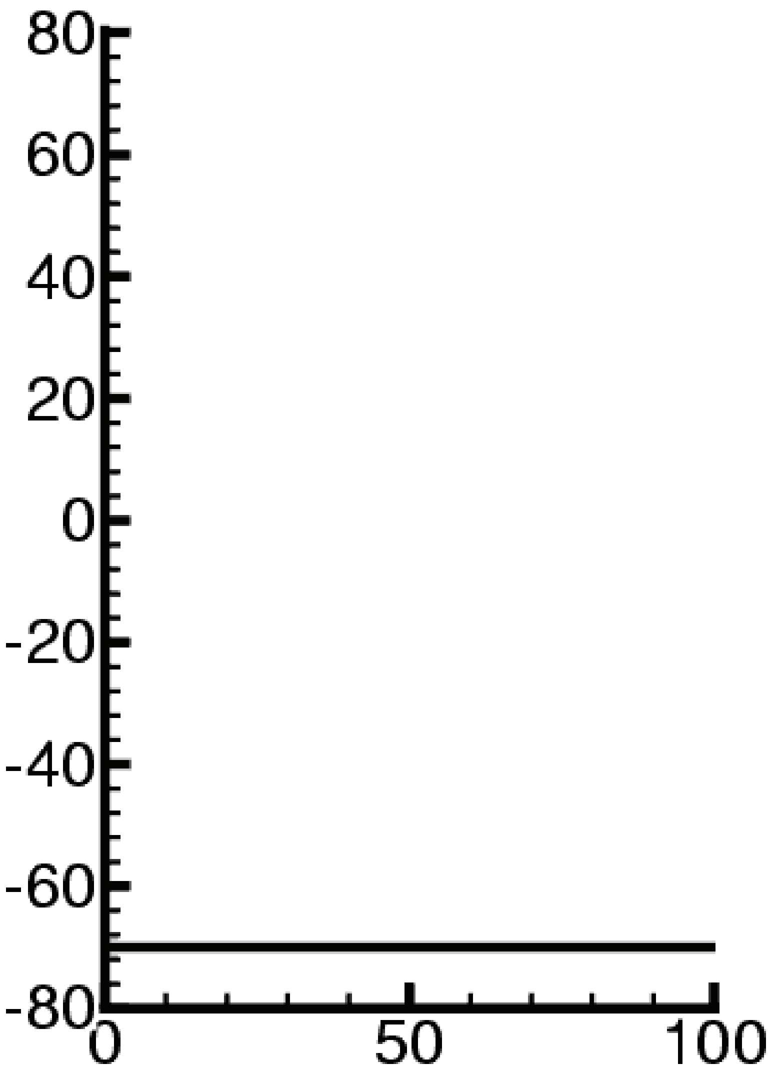


(a)
100
0
–100
–200
–300
–400
y/δω
0
100
200
300
400
x/δω
(b)
10–3
10–4
10–5
10–6
I
1
1
100
200 300
–y/δω

Adjoint Pressure

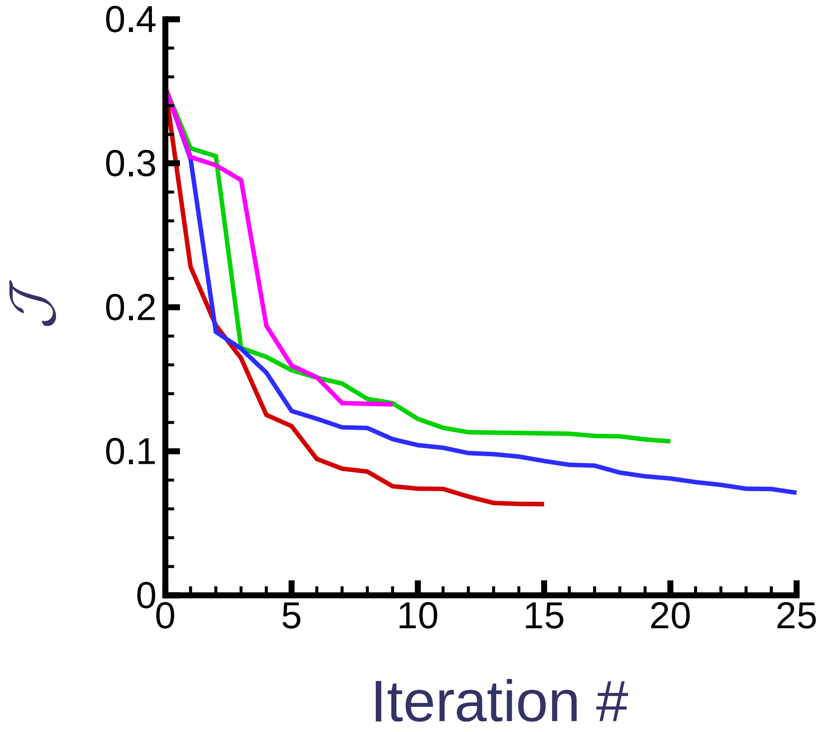


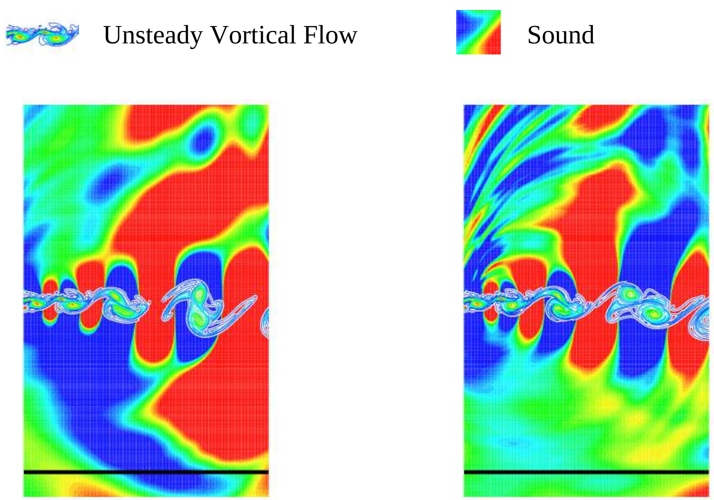
Adjoint Pressure



- $\mathcal{J}$ on Ω reduced by ~ 10dB
- Reduced by $\gtrsim 5$dB in *all* directions (not anti-sound)

- 7–11 dB noise reduction



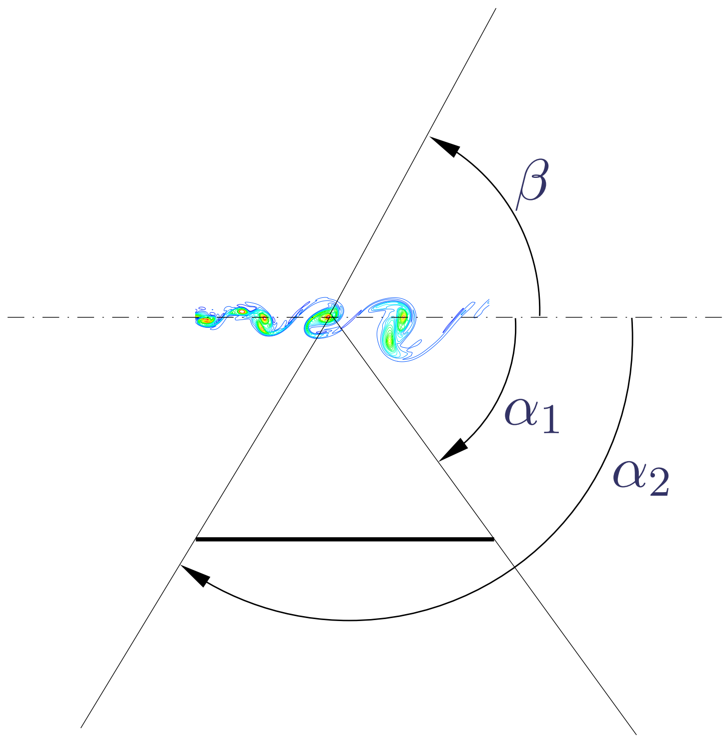


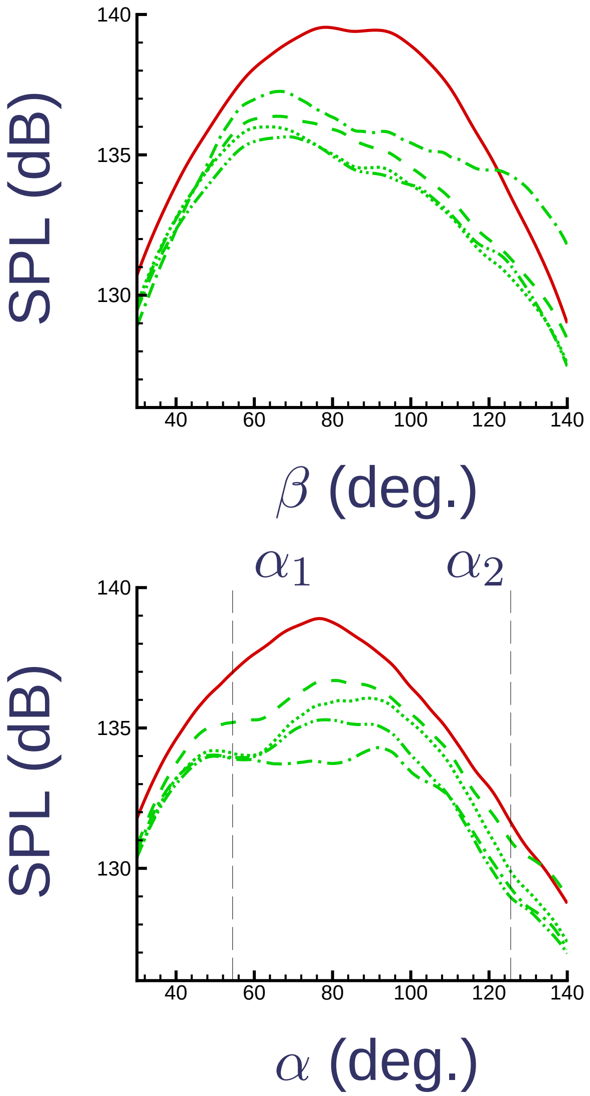
Before:
Noisy on the line

After:
7.4 dB quieter

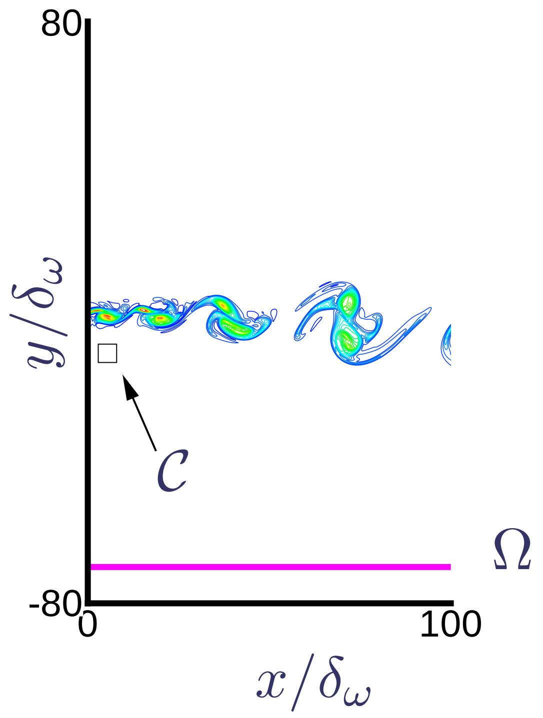
Internal Energy Control
Y-momentum Control
X-momentum Control
Mass Control

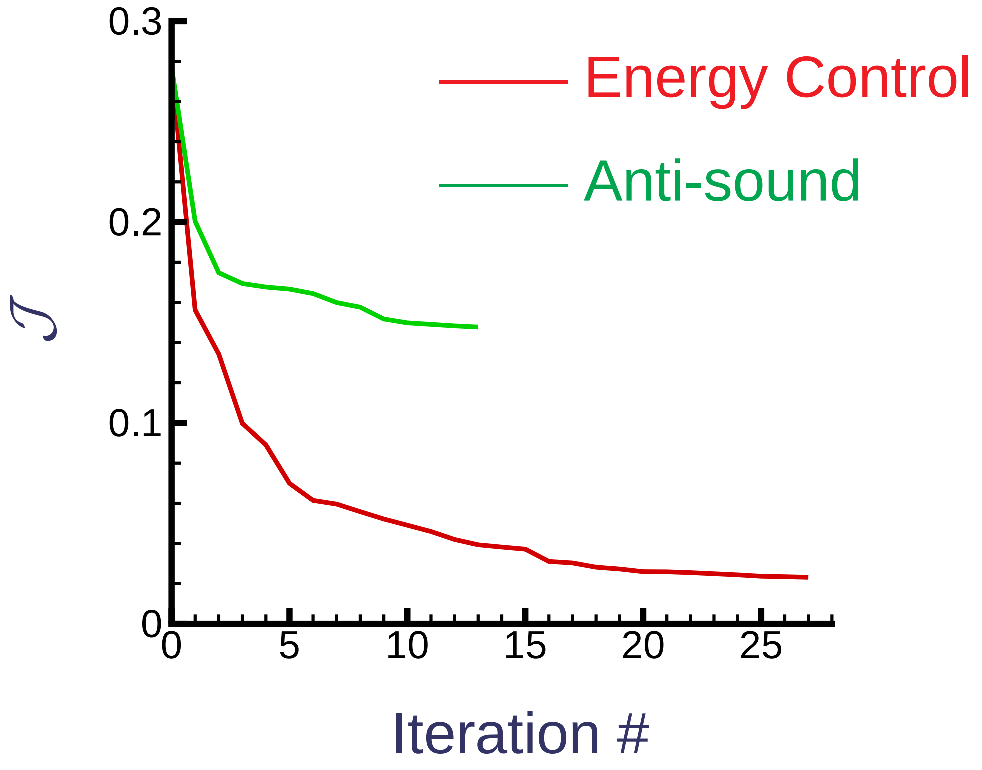
Sound Directivity



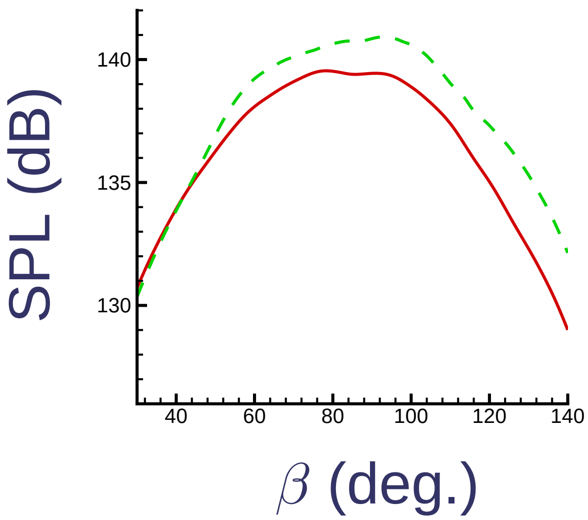


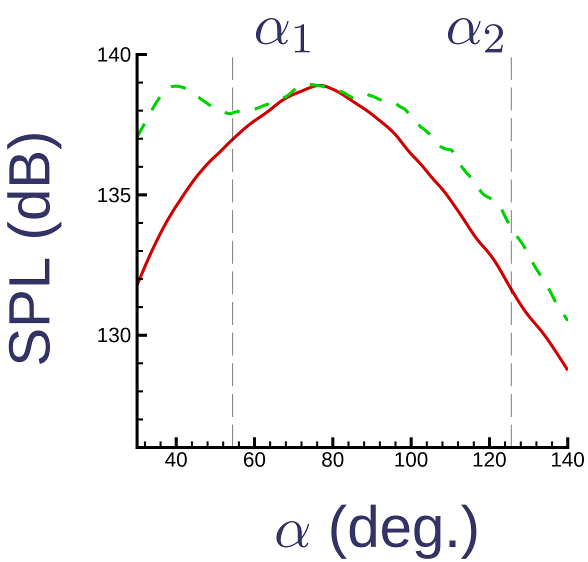
Anti-Sound?

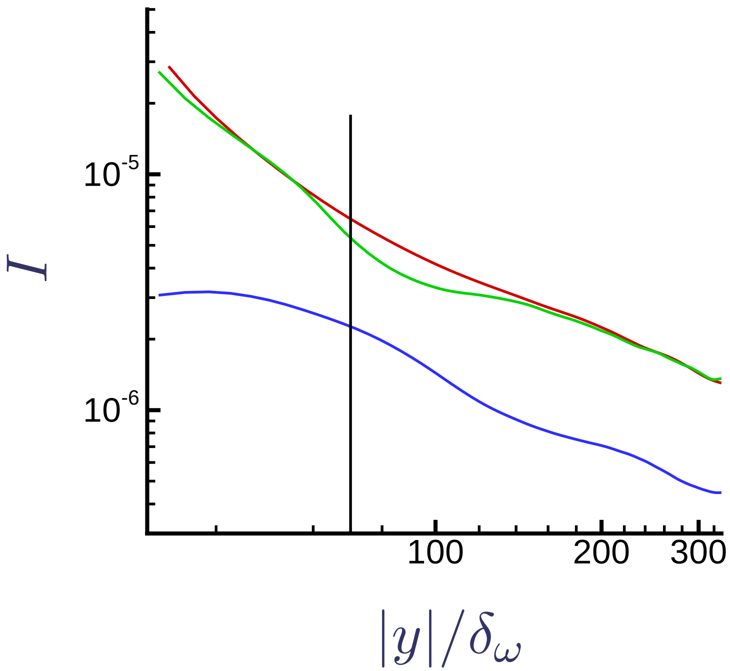

80
-80
0
100
y/δ_ω
x/δ_ω
$\mathcal{C}$
Ω


0.3
0.2
0.1
0
0
5
10
15
20
25
$\mathcal{J}$
Iteration #
Energy Control
Anti-sound

Anti-Sound: Far Field



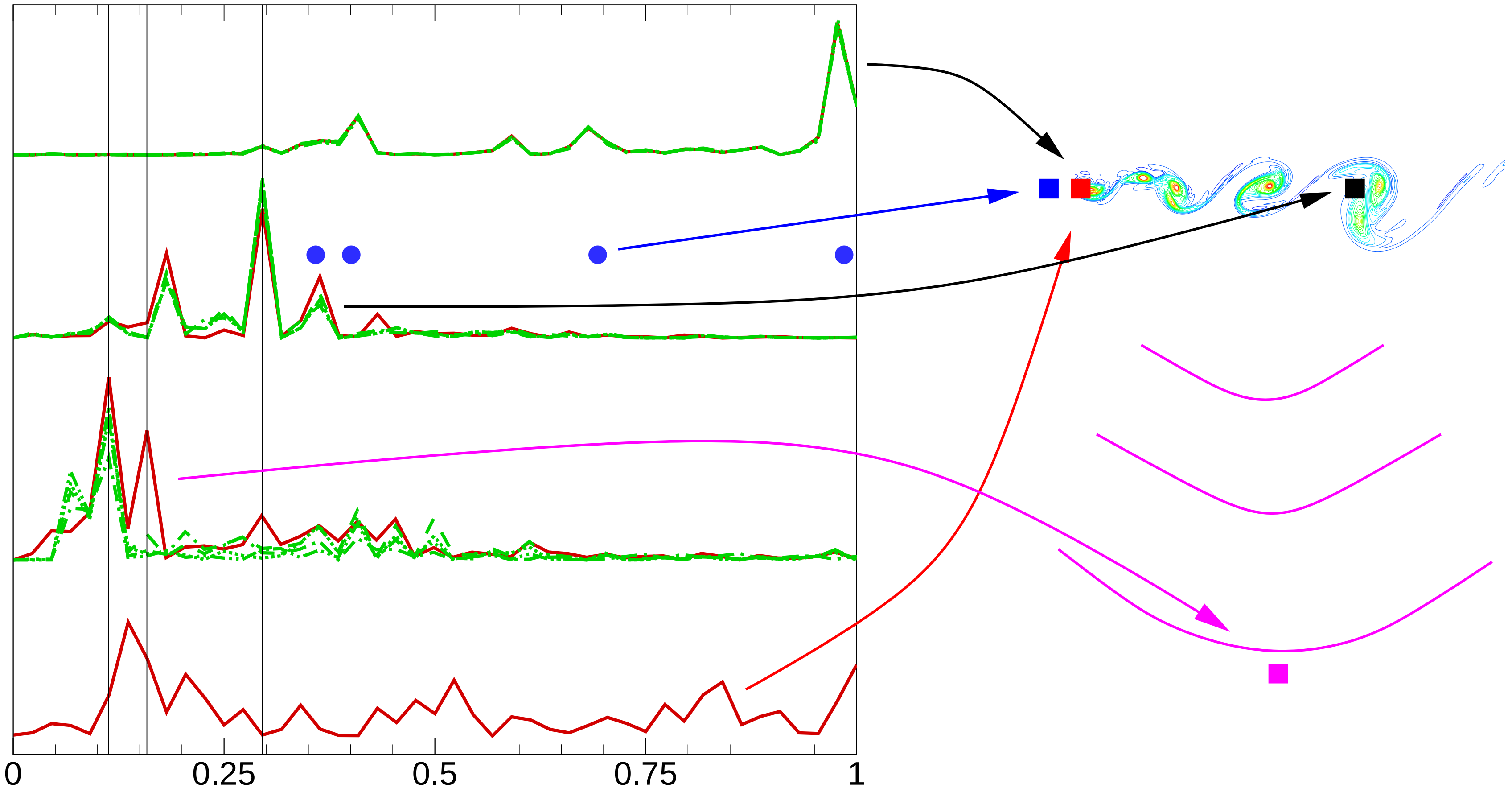




- Genuine change of flow as source of sound

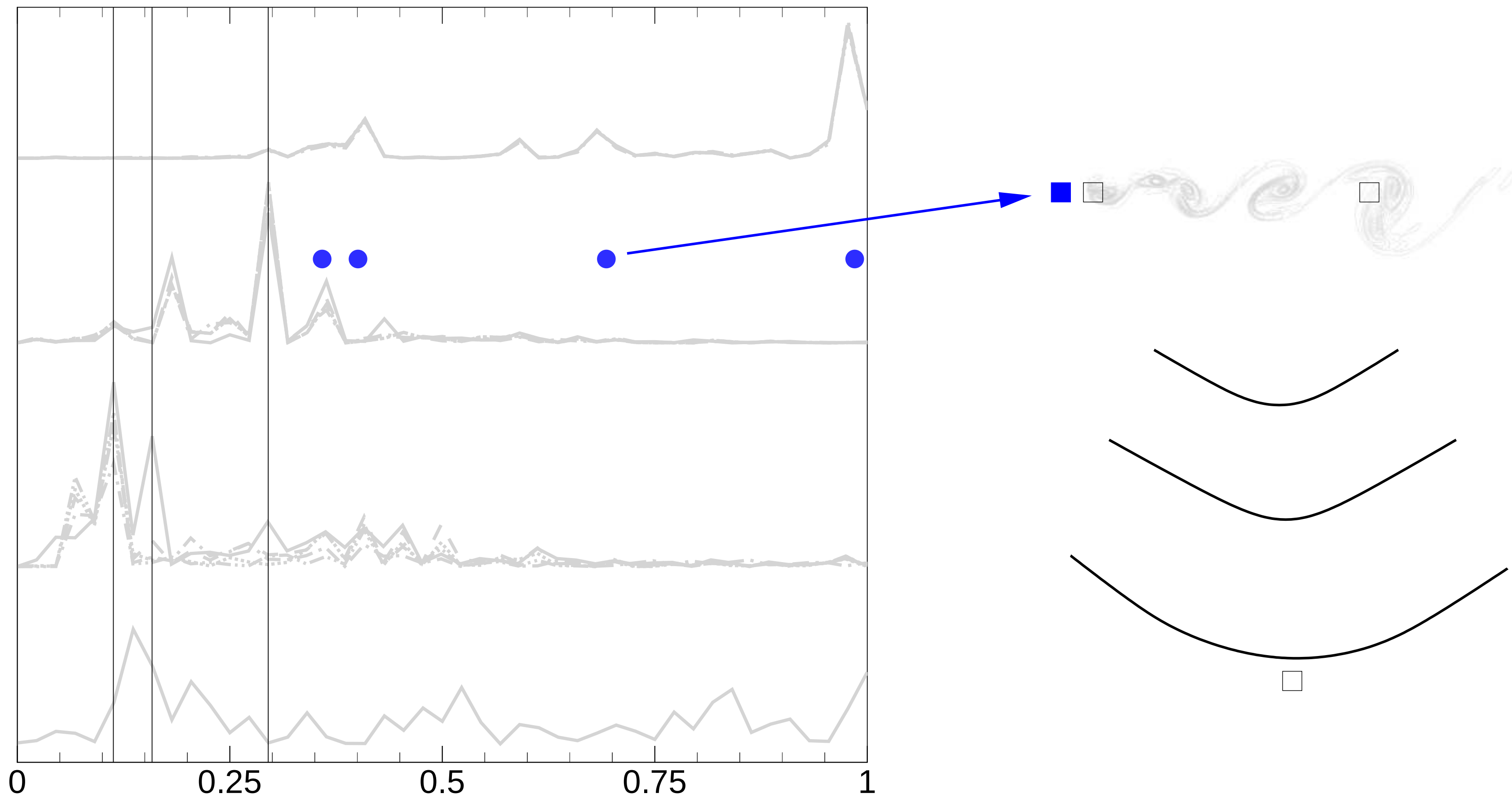
Spectra

ALL



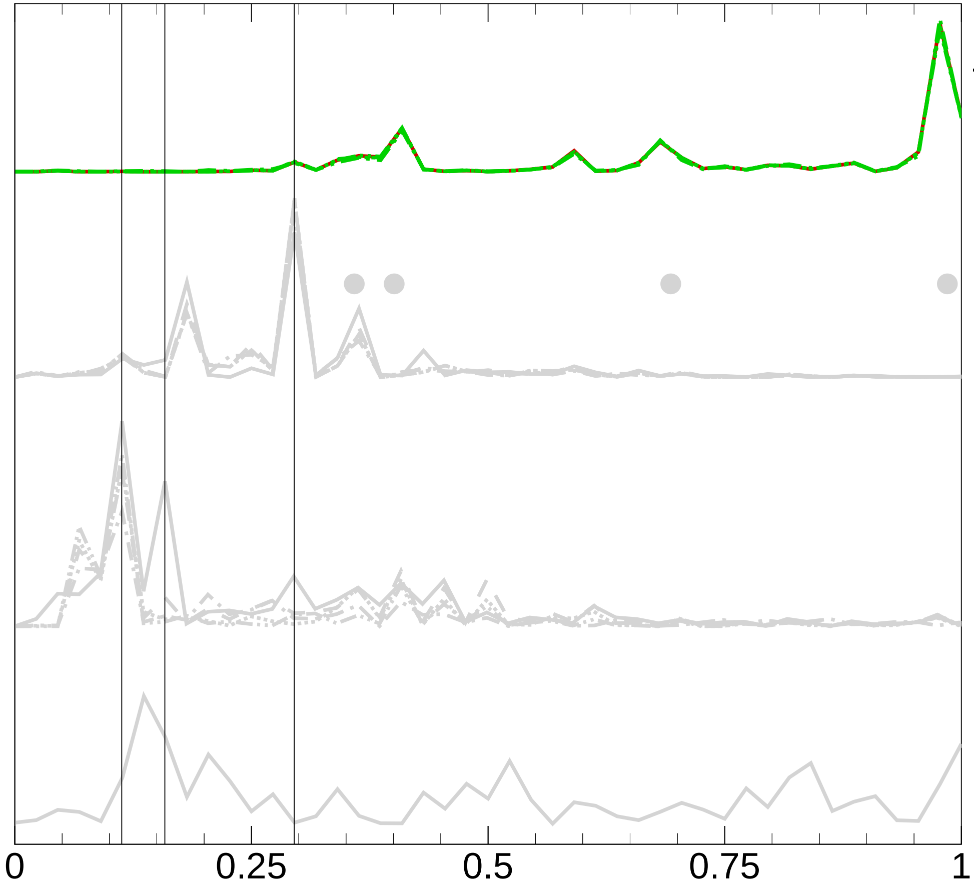
Spectra

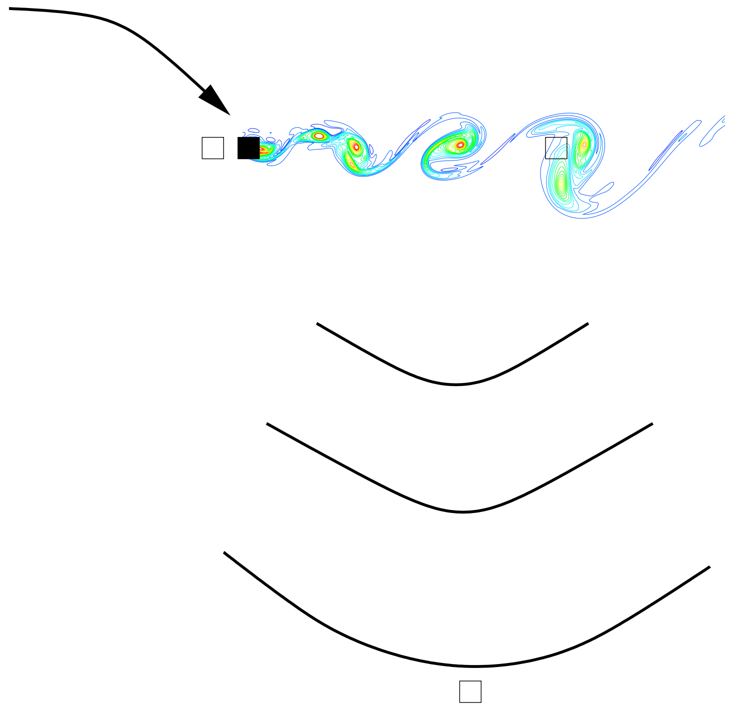
INFLOW EXCITATION



Spectra

RESPONSE IN $\mathcal{C}$: UNCHANGED BY CONTROL

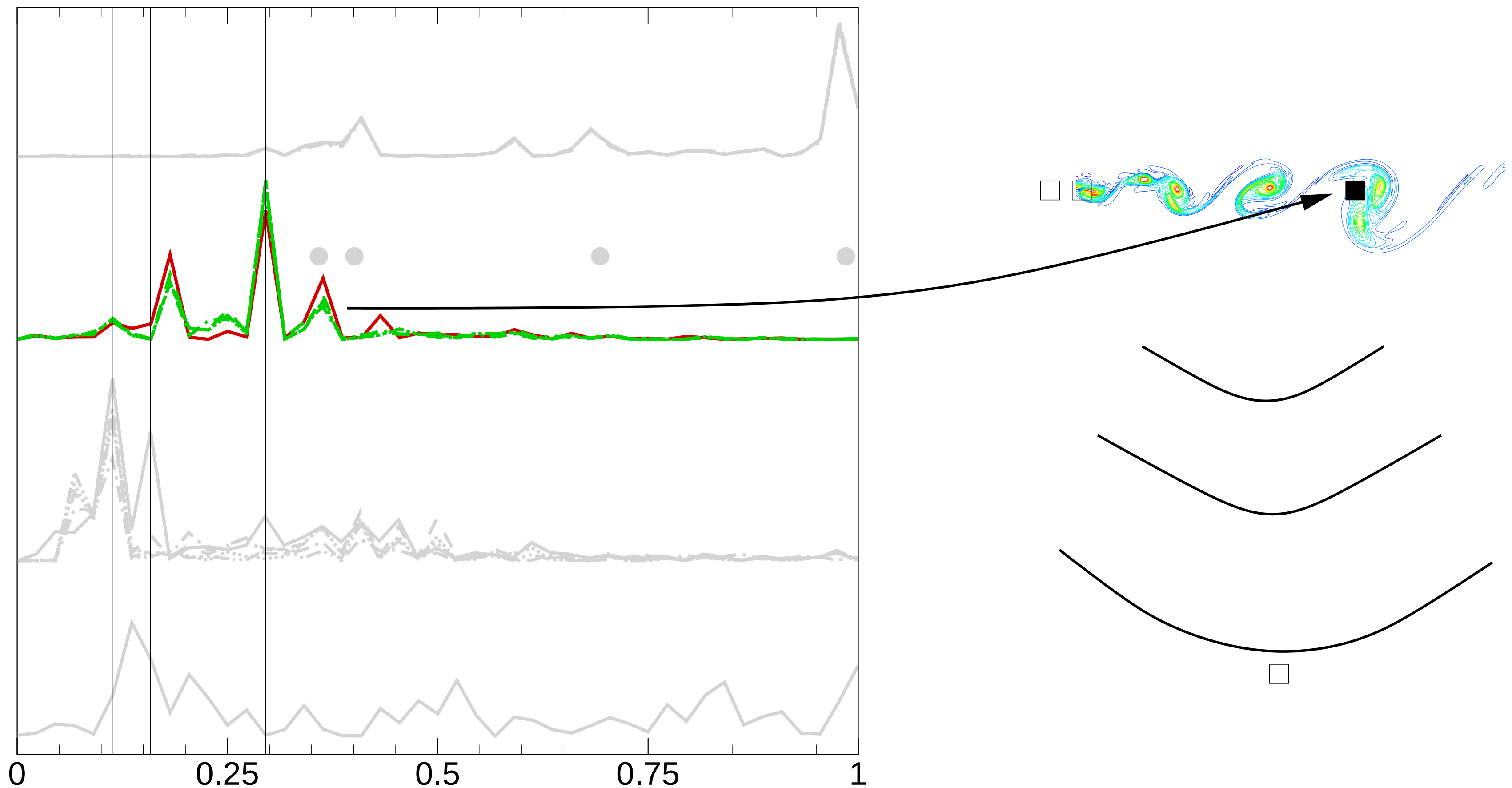




No Control; Control

Spectra

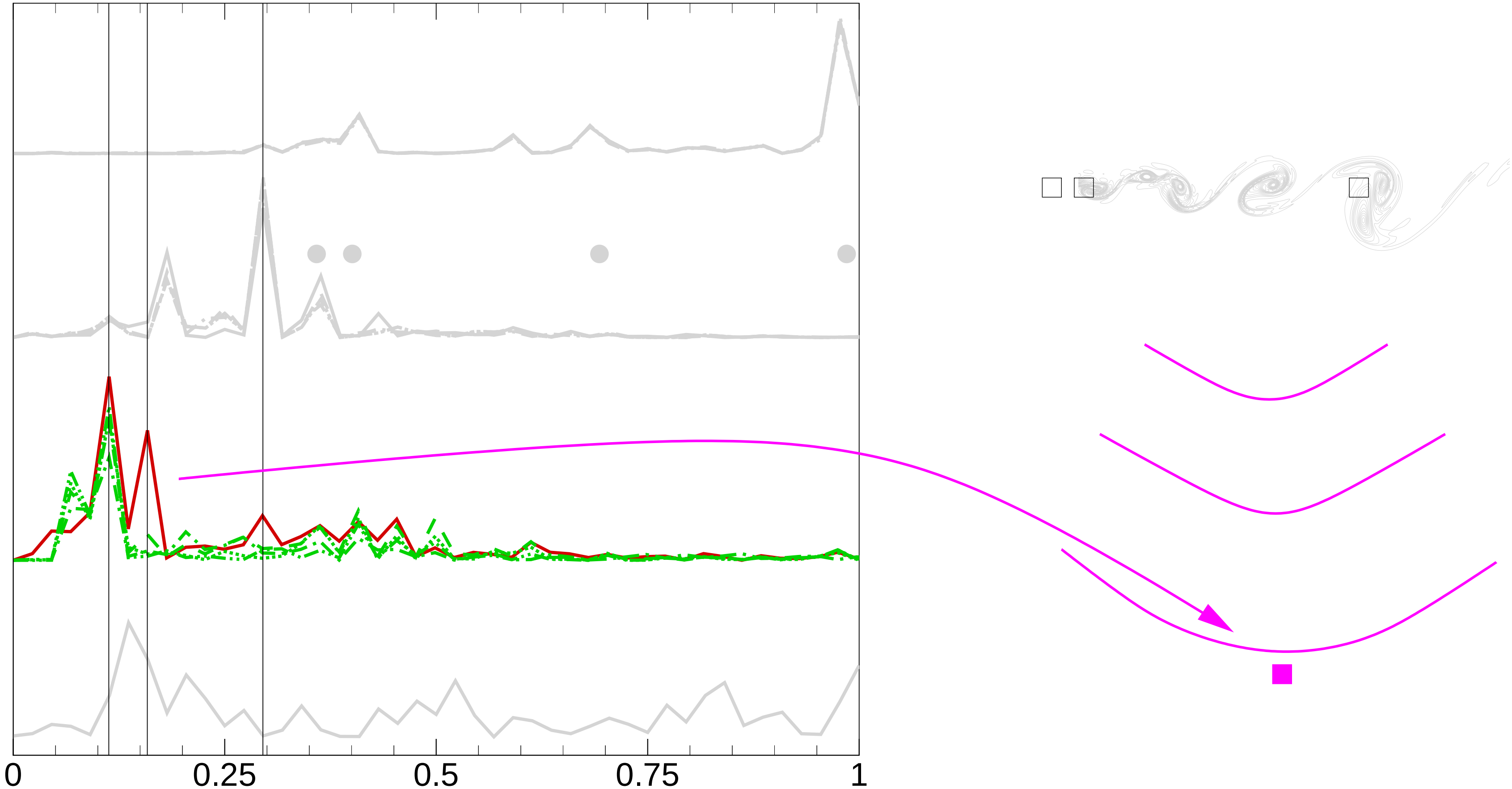
RESPONSE DOWNSTREAM: NONLINEARITY



No Control; Control

Spectra

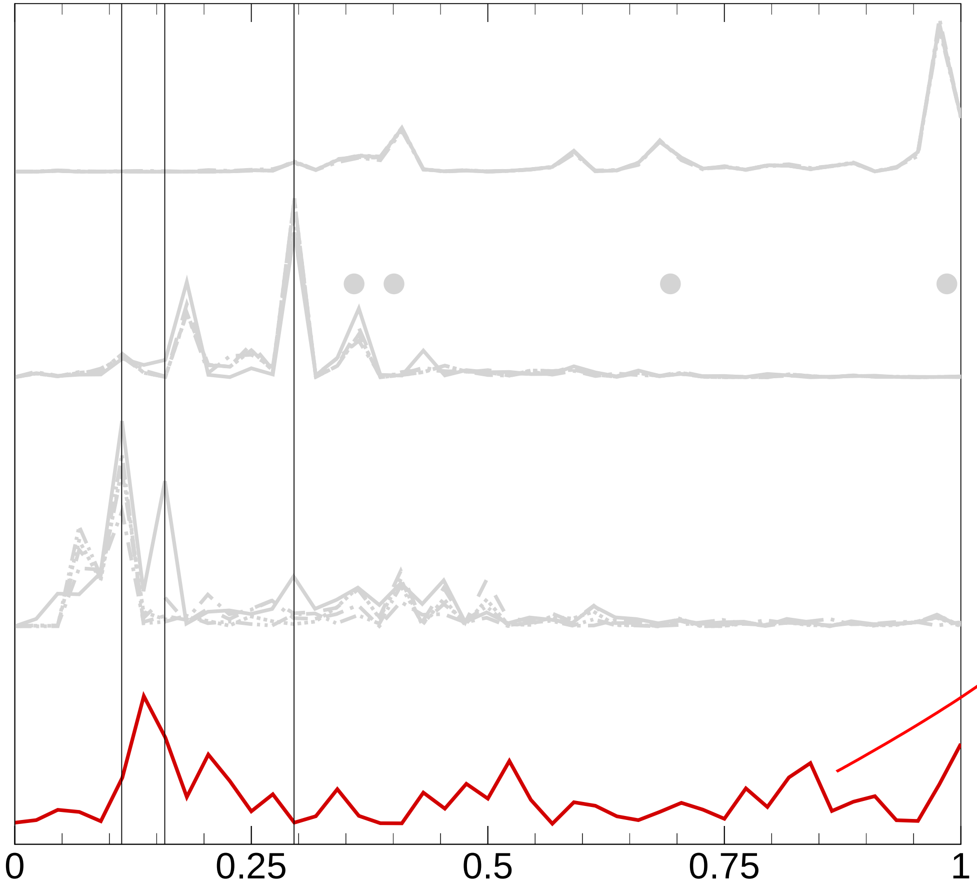
SOUND FIELD

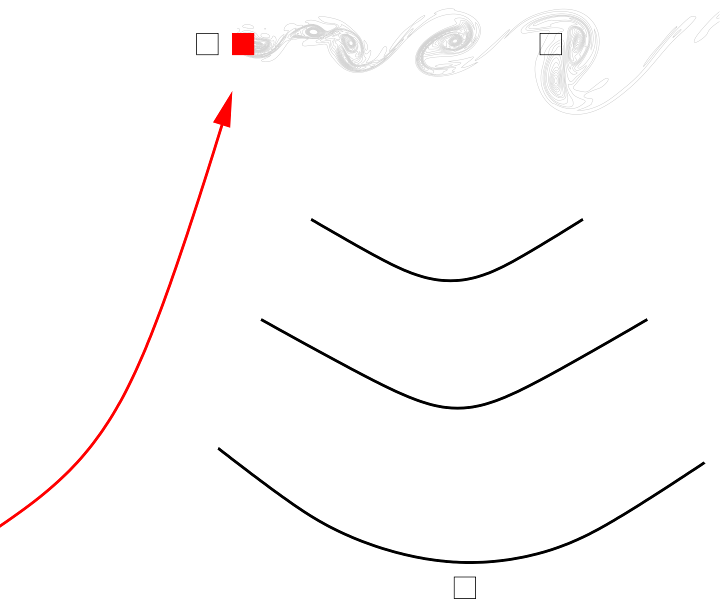


No Control; Control

Spectra

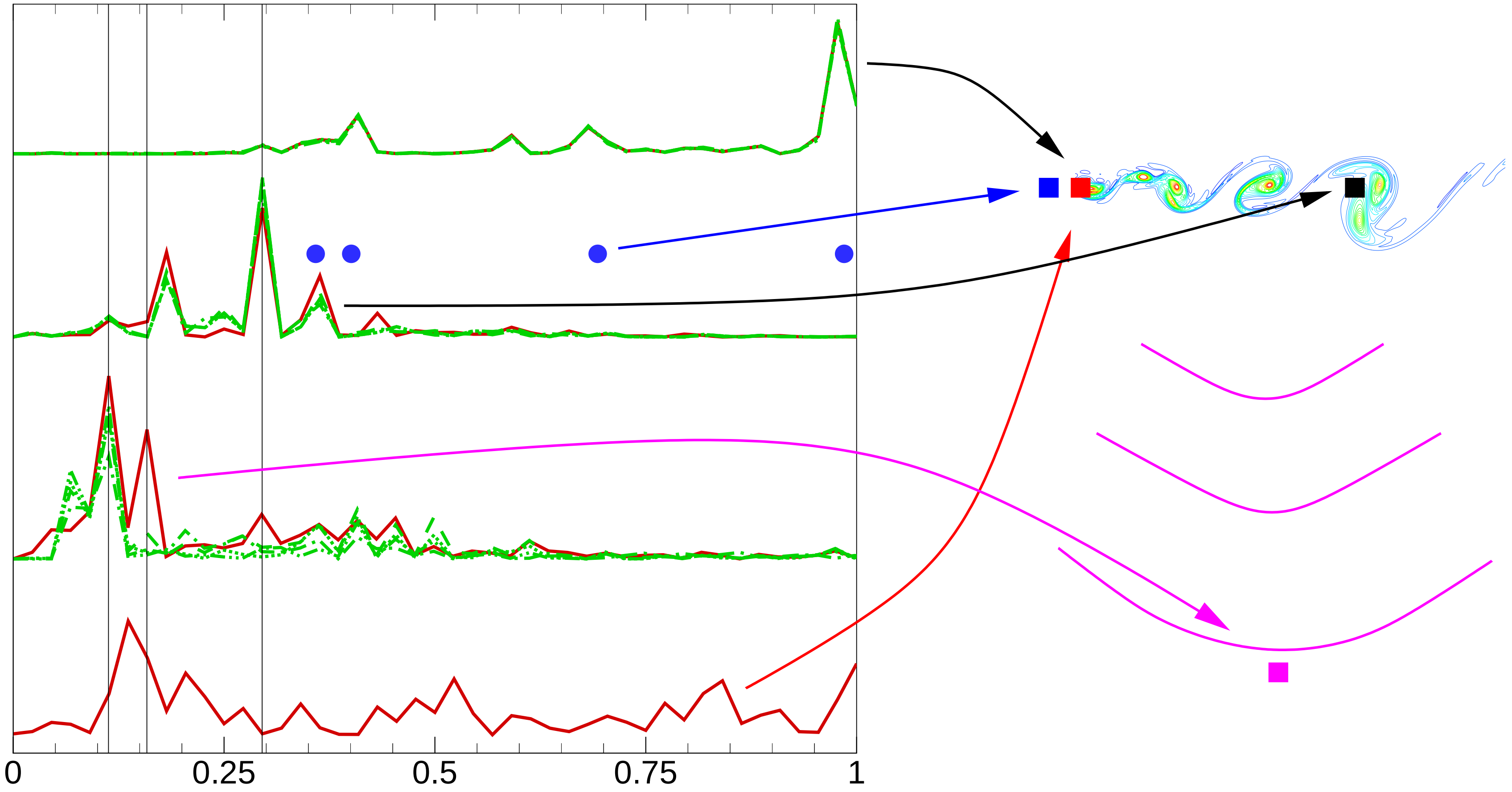
CONTROL





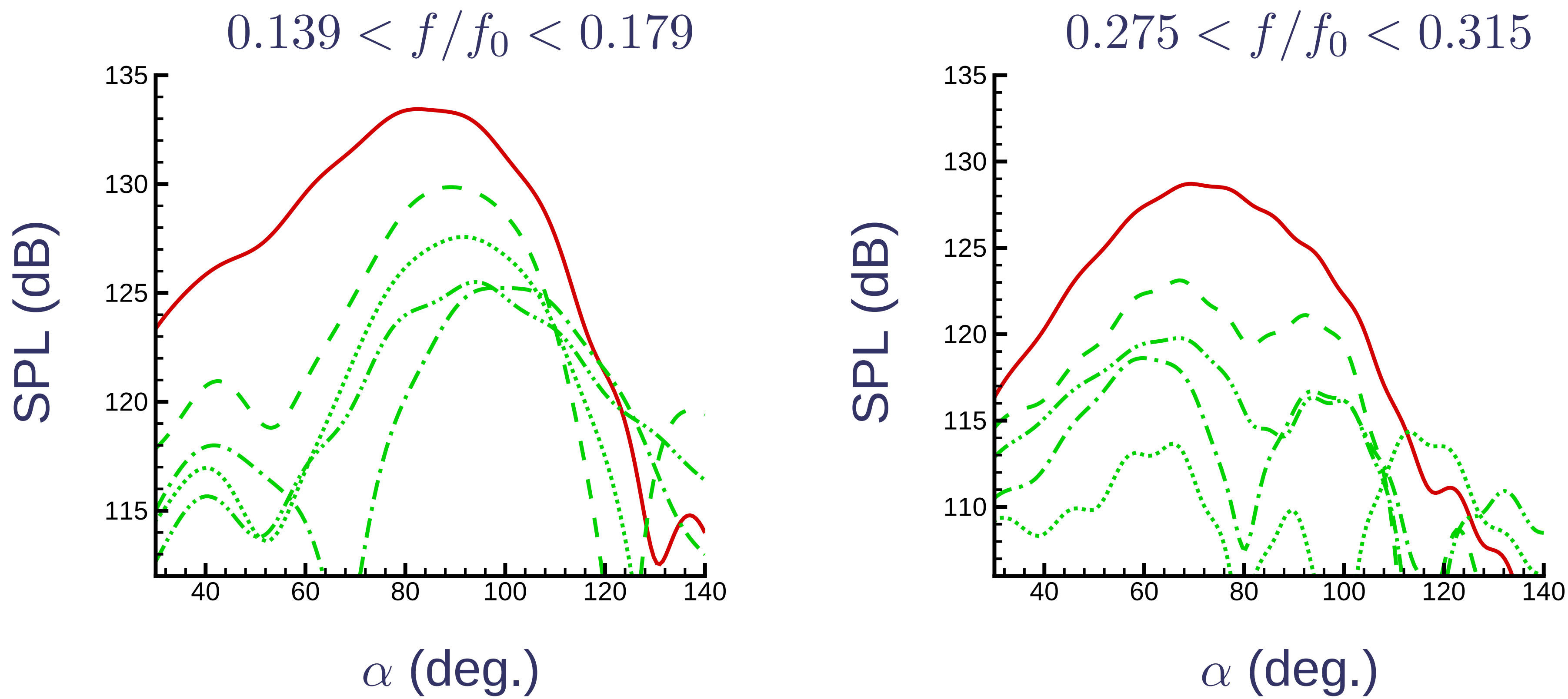
Spectra

...frequency mismatch implicates nonlinearity



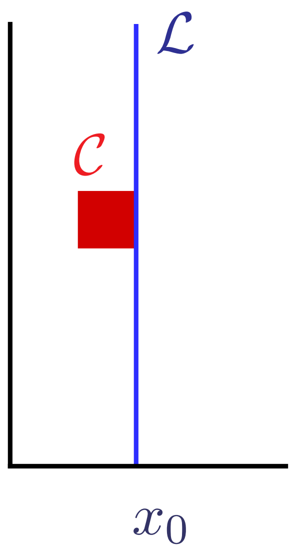
Directivity at Frequencies

No Control; Control



Control Power

$$\tilde{F} = \int_{\mathcal{L}} E_k(x_0, y, t) u(x_0, y, t)\, dy \quad , \qquad E_k = \frac{1}{2}\rho[(u - \bar{u})^2 + (v - \bar{v})^2]$$



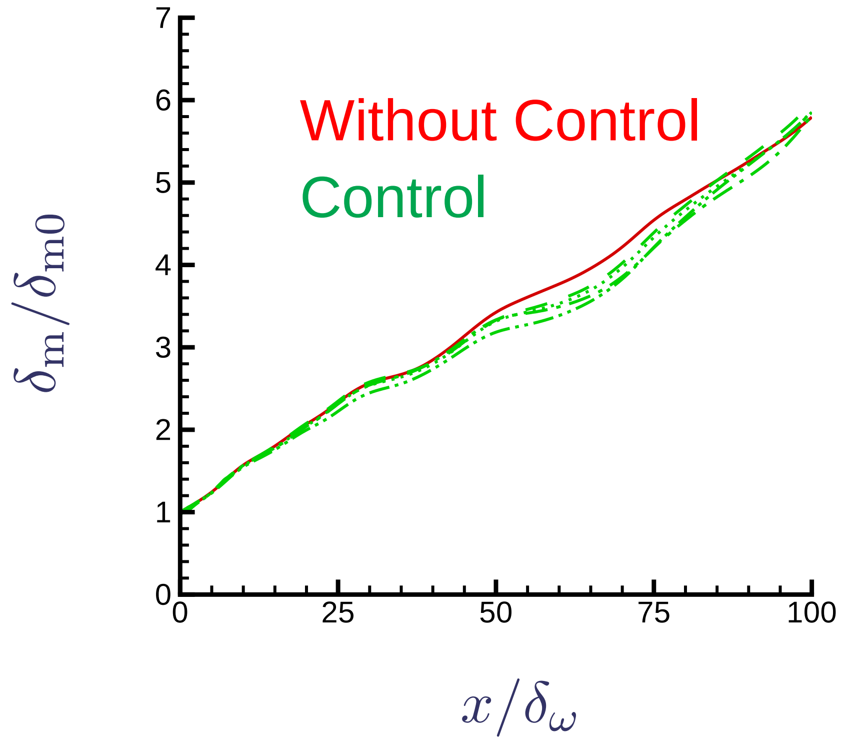
$$\begin{aligned}
\eta_\rho(t) &= \frac{1}{\tilde{F}} \int_{\mathcal{C}} \phi_\rho(x, y, t) T_0/\gamma\, dxdy \\
\eta_u(t) &= \frac{1}{\tilde{F}} \int_{\mathcal{C}} \phi_u(x, y, t) u(x, y, t)\, dxdy \\
\eta_v(t) &= \frac{1}{\tilde{F}} \int_{\mathcal{C}} \phi_v(x, y, t) v(x, y, t)\, dxdy \\
\eta_e(t) &= \frac{1}{\tilde{F}} \int_{\mathcal{C}} \phi_e(x, y, t)\, dxdy
\end{aligned}$$

	$\lvert\eta_\rho\rvert$	$\lvert\eta_u\rvert$	$\lvert\eta_v\rvert$	$\lvert\eta_e\rvert$
maximum	2.25×10^{-2}	5.13×10^{-3}	1.90×10^{-4}	2.27×10^{-1}
average	1.94×10^{-3}	4.39×10^{-4}	1.87×10^{-5}	2.75×10^{-2}

So what changed?

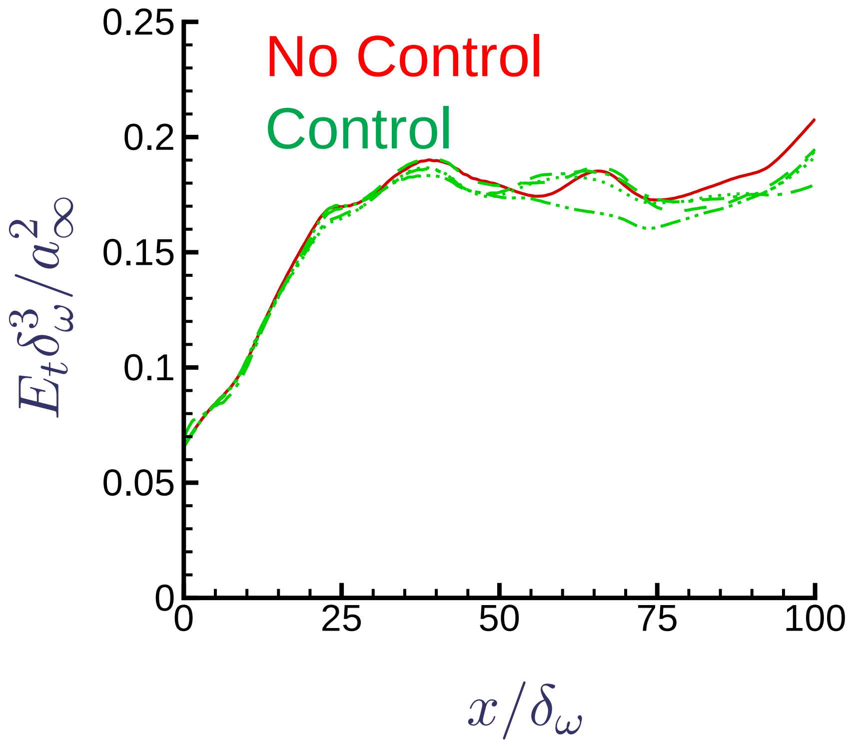
Mean Flow Spreading

$$\delta_m = \int_{y_a}^{y_b} \frac{\rho(u - U_a)(U_b - u)}{\rho_\infty \Delta U^2}\, dy$$

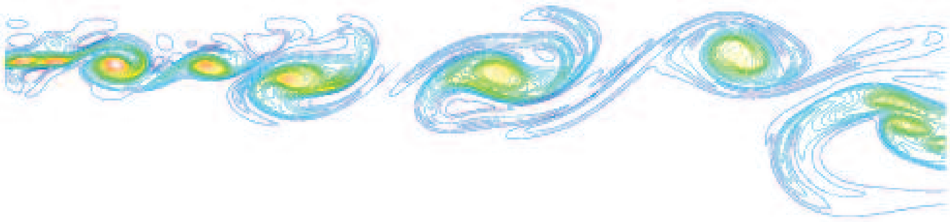


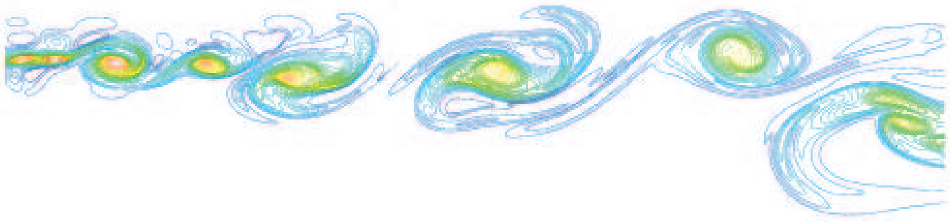
TKE Developing in Space

$$E_t(x) = \frac{\int_{-80\delta_\omega}^{80\delta_\omega} \overline{E_k}\, dy}{\delta_m(x)}$$



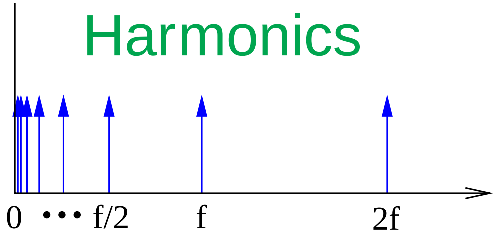
Large-scale Structures: Before/After

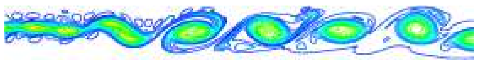


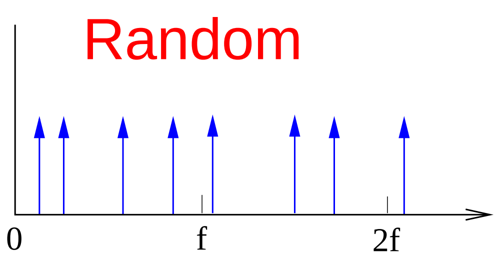


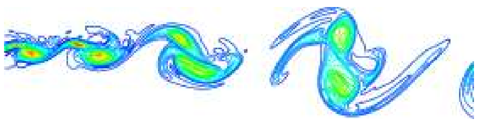
Harmonic Excitation

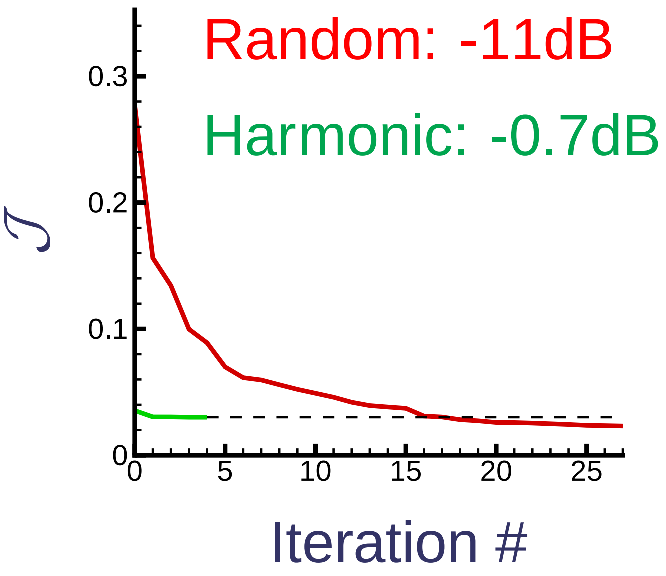
- Excite base flow with harmonics to induce order (*e.g.* Colonius *et al.* 1997)











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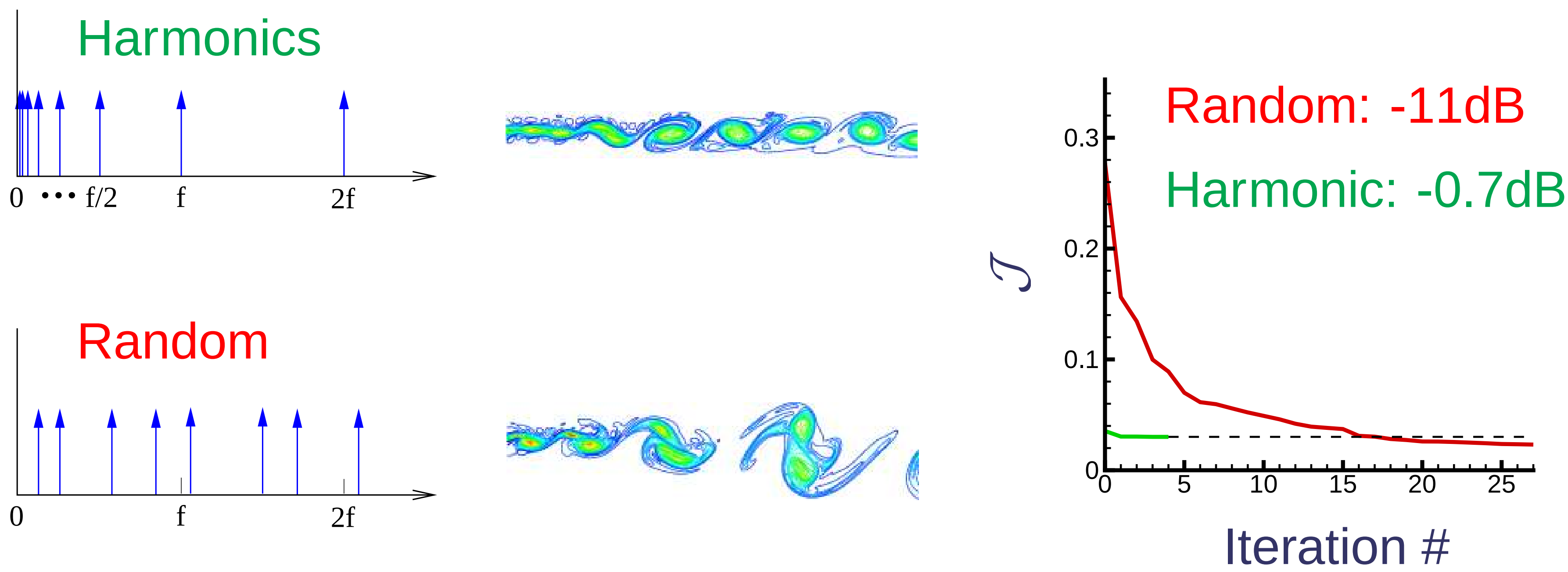

- Is there an underlying order induced in controlled case?
- Use empirical eigenfunctions (pod) as surrogates for Fourier modes in streamwise direction

Empirical Eigenfunctions

$$\vec{q}(\mathbf{x}, t) = \sum_i a_i(t) \vec{\psi}_i(\mathbf{x})$$

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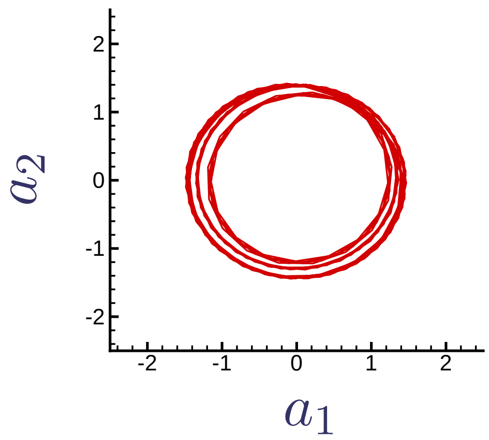

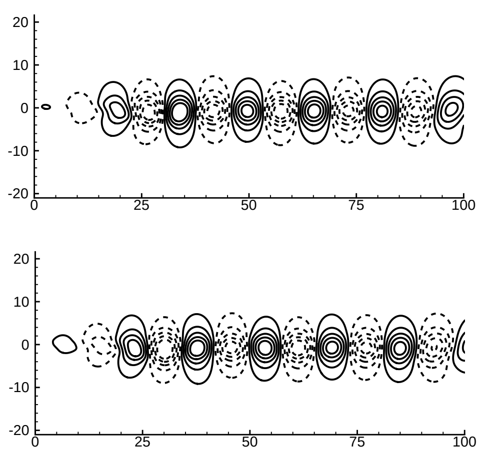

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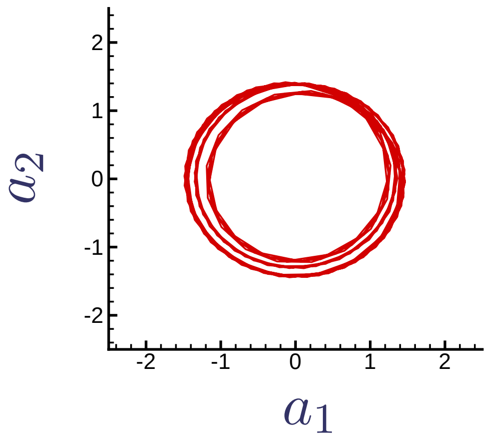
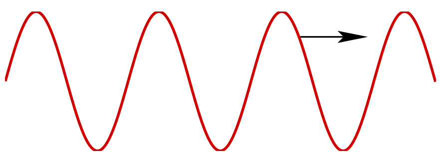
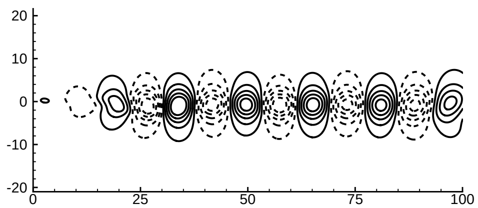
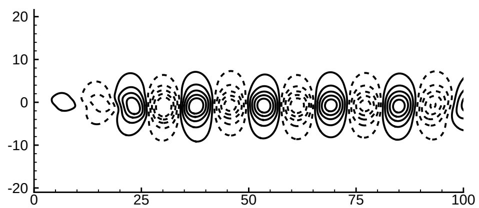


$$q(x,t) = a_1(t)\cos kx + a_2(t)\sin kx$$



- Circles in $a_1(t)$–$a_2(t)$ advect the wave
- Smooth advection
  - small radiation capable component
  - supersonic phase from envelope
  - acoustically inefficient





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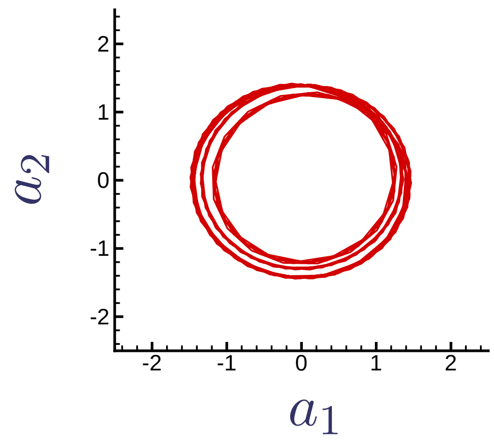
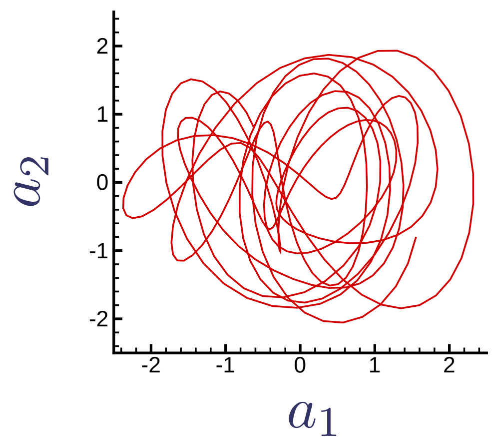
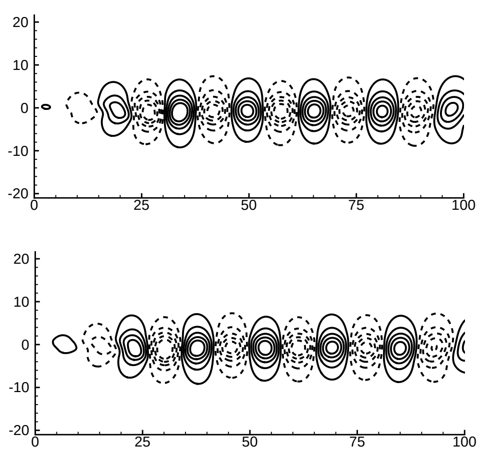
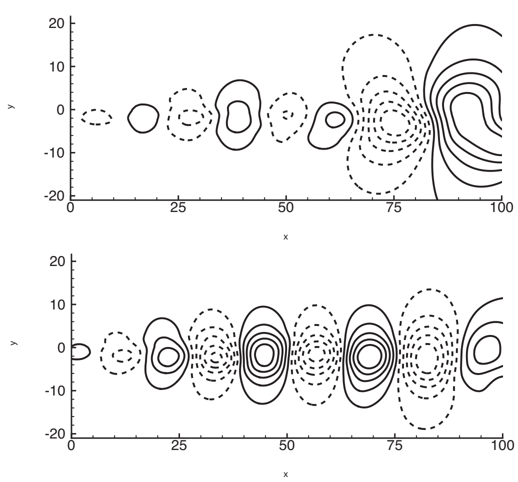


Random







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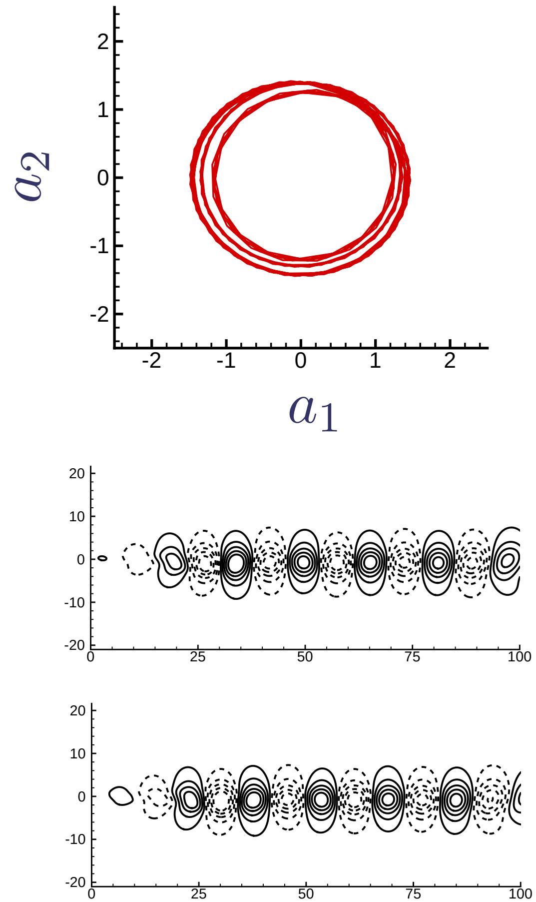
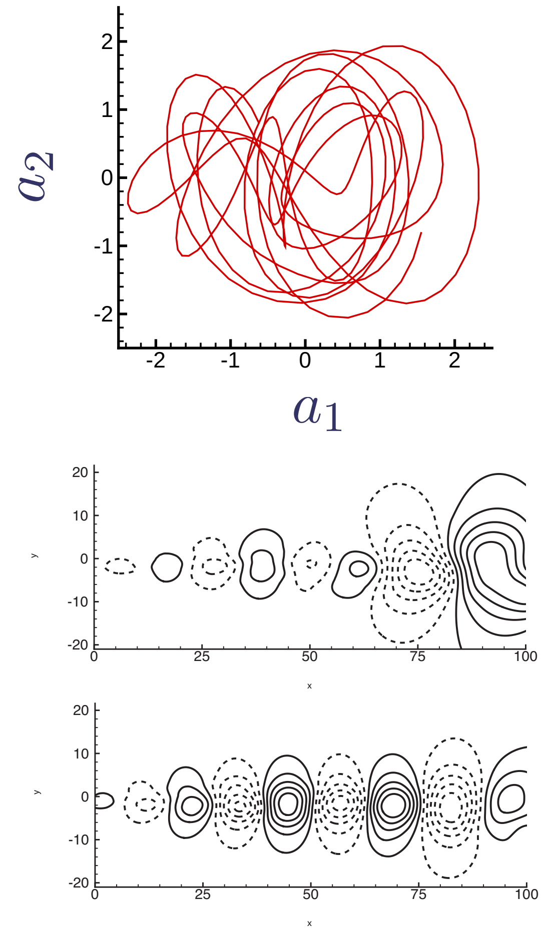
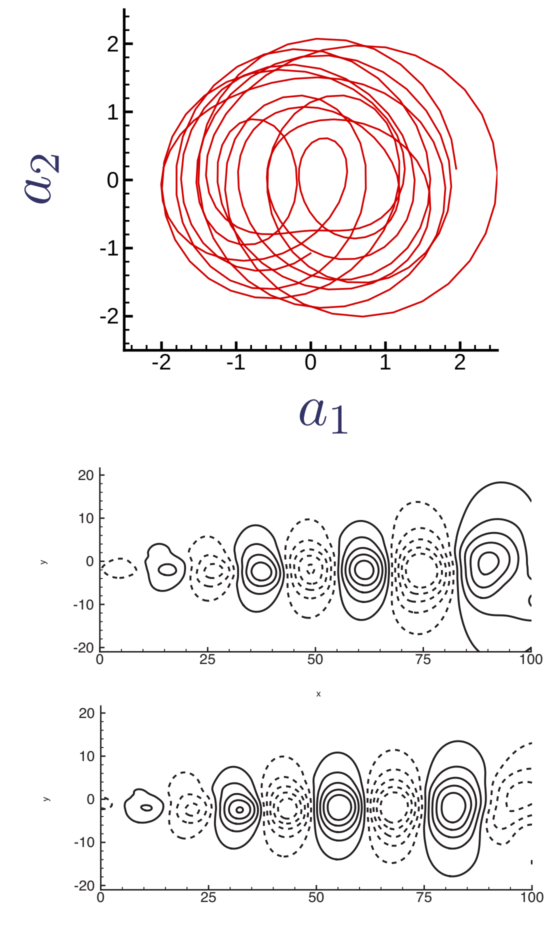


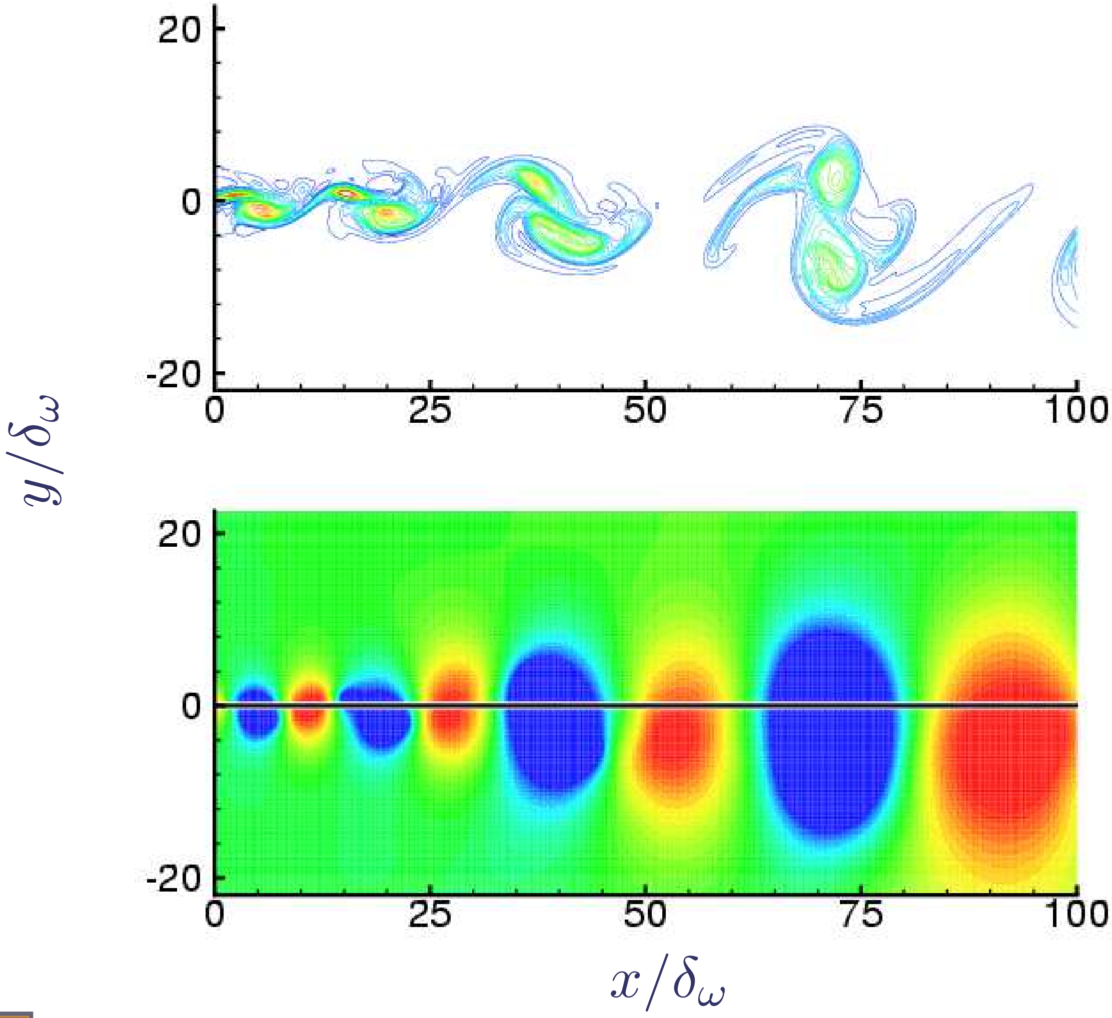
Random



Random w/Control



Small Changes in Convection



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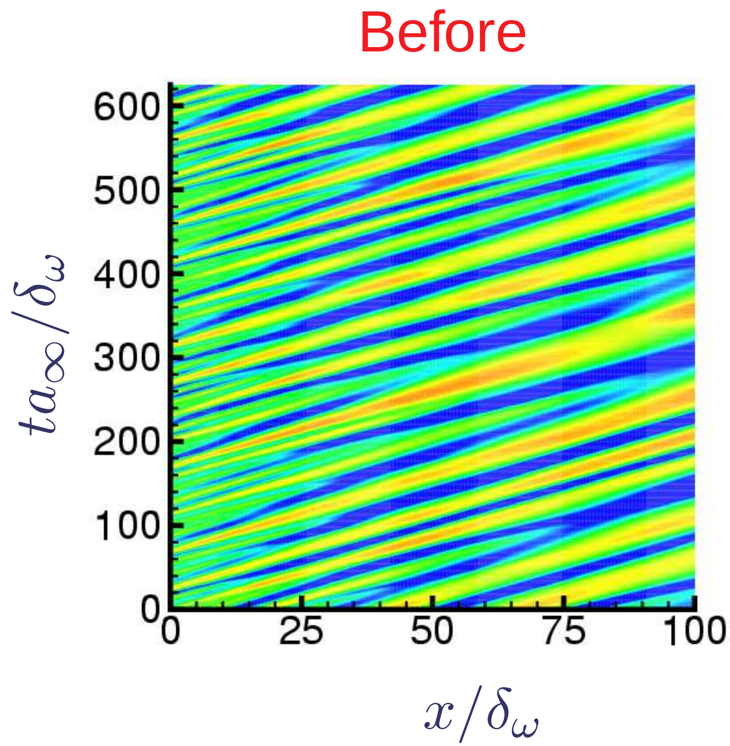

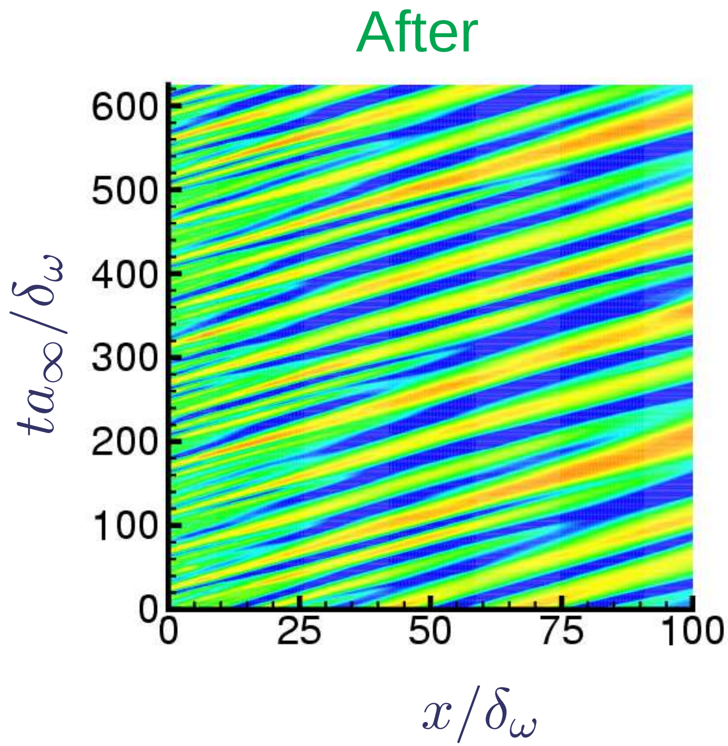

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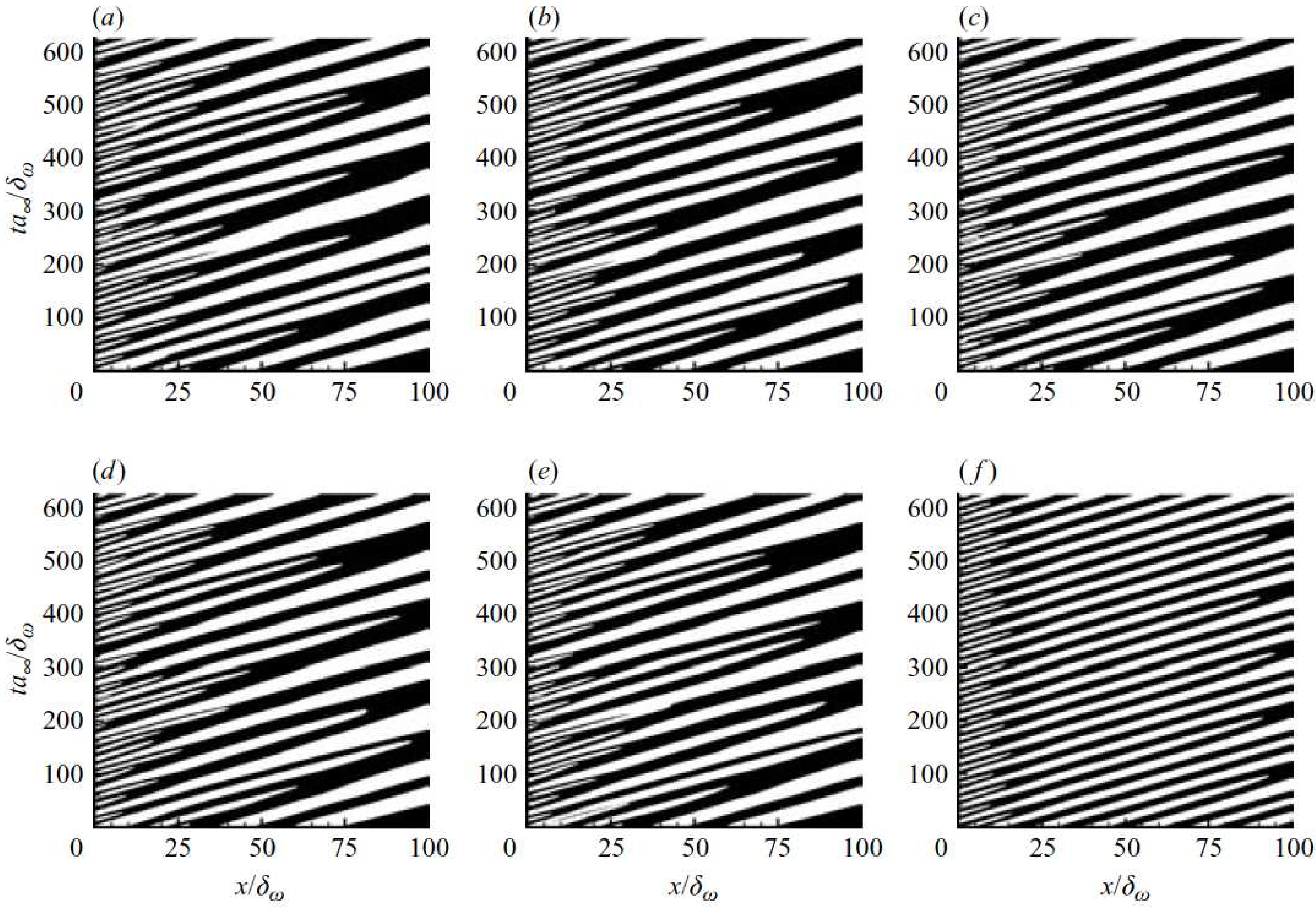
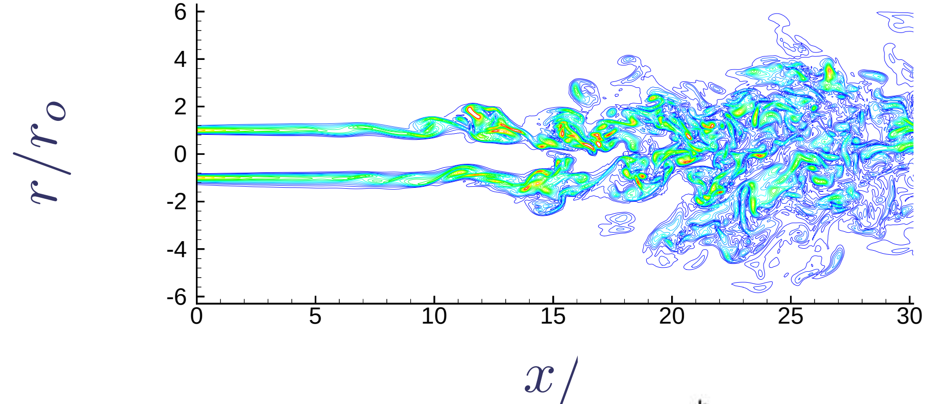


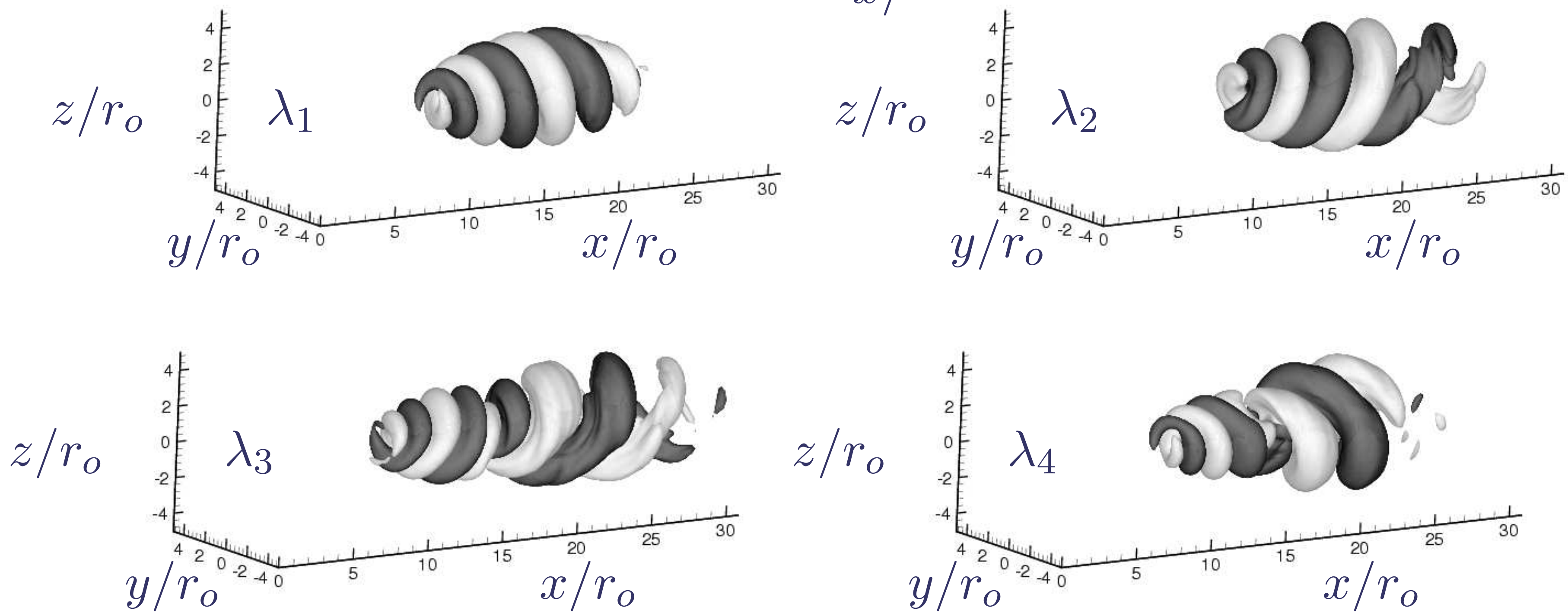
Small Changes in Convection

3-D?

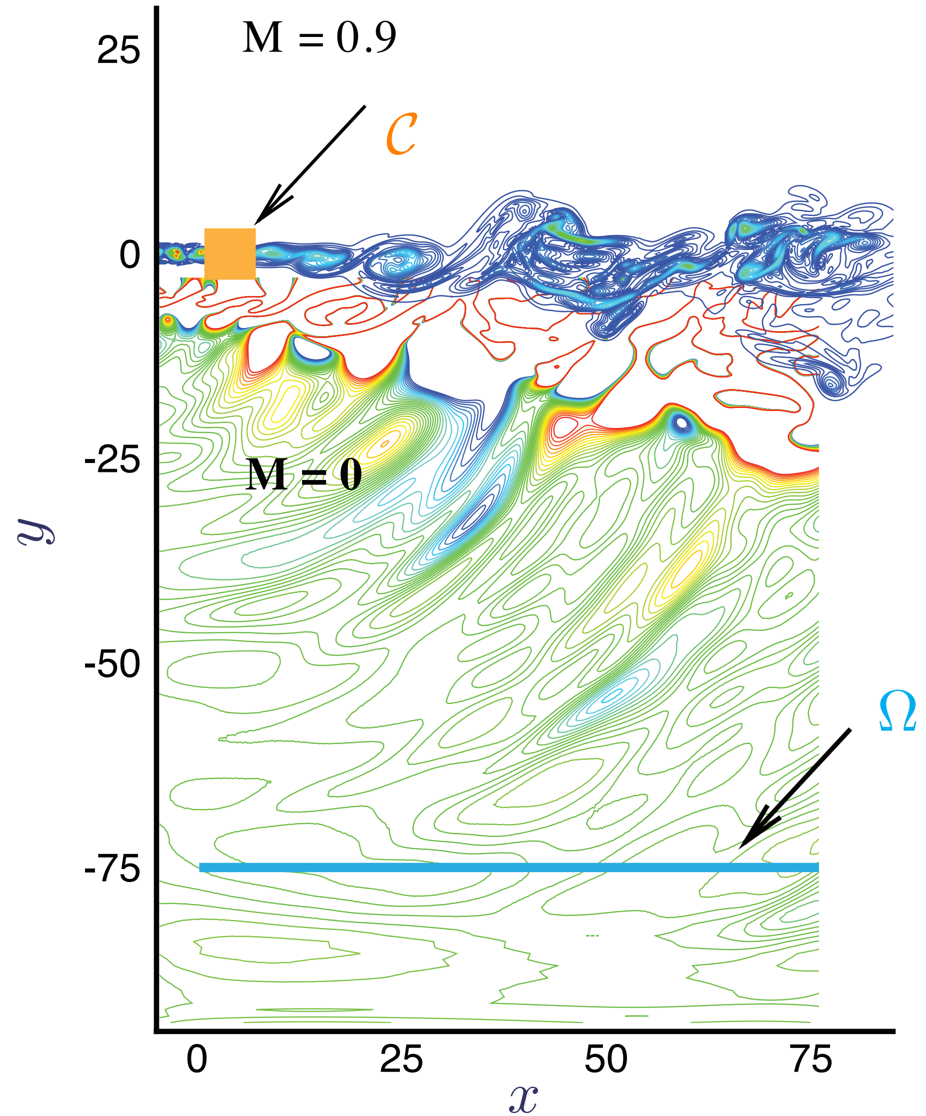
Empirical Eigenfunctions

Freund (2001); Freund & Colonius (2009)



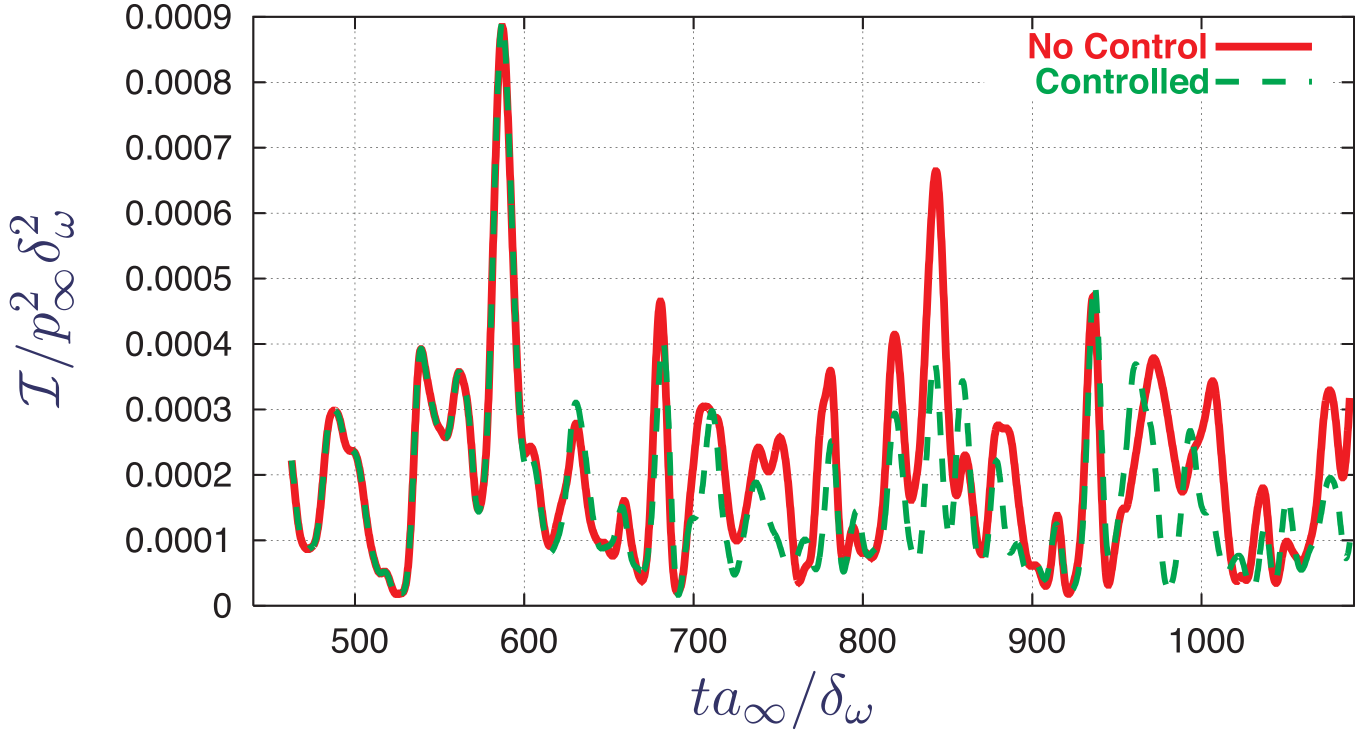


Turbulent Mixing Layer



Initial 5 Line Searches

$$\mathcal{J} = \int_{t_0}^{t_1} \mathcal{I}(t)\, dt$$



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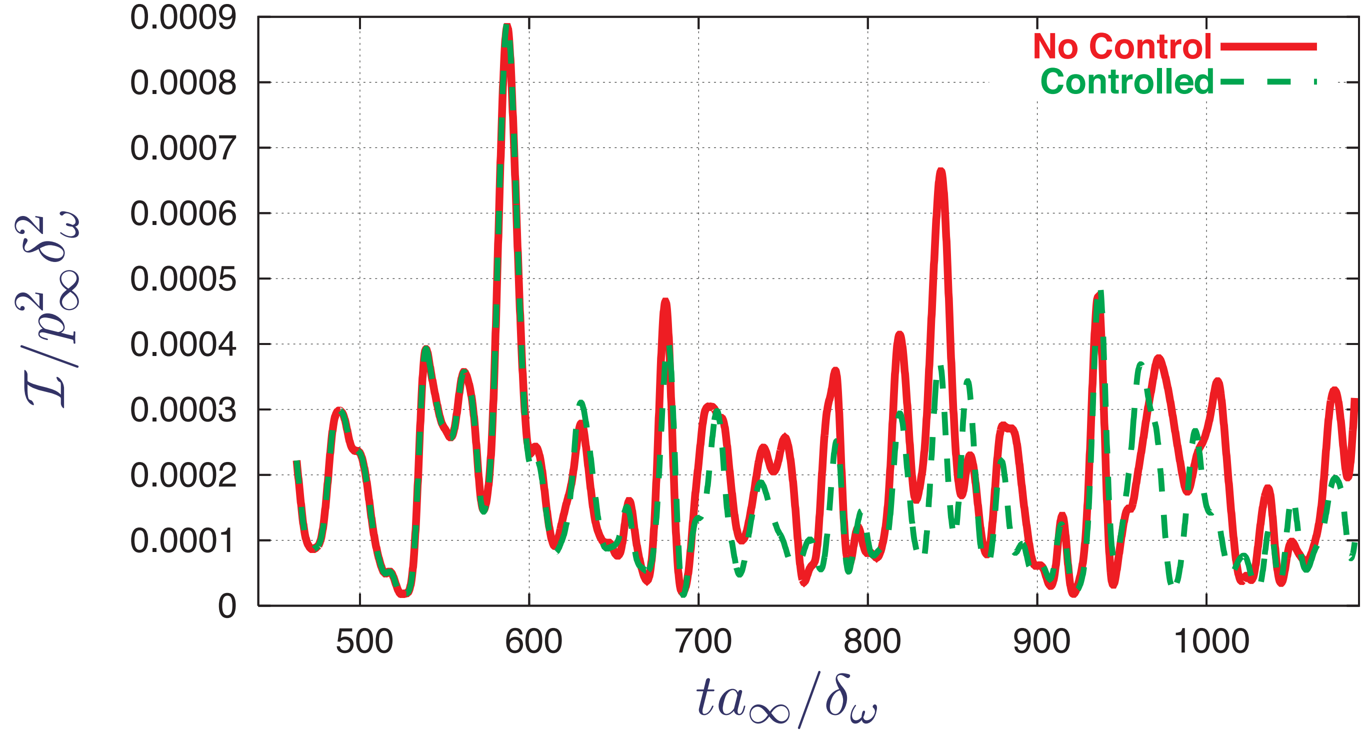
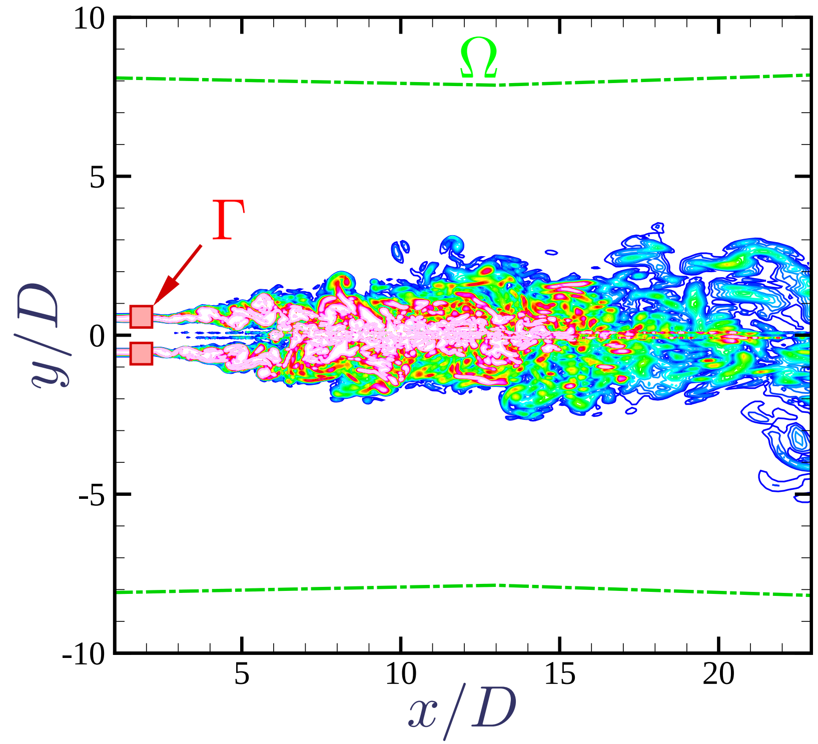


- Sound reduced ($\sim 30\%$), simulation terminated, DNS $\rightarrow$ LES

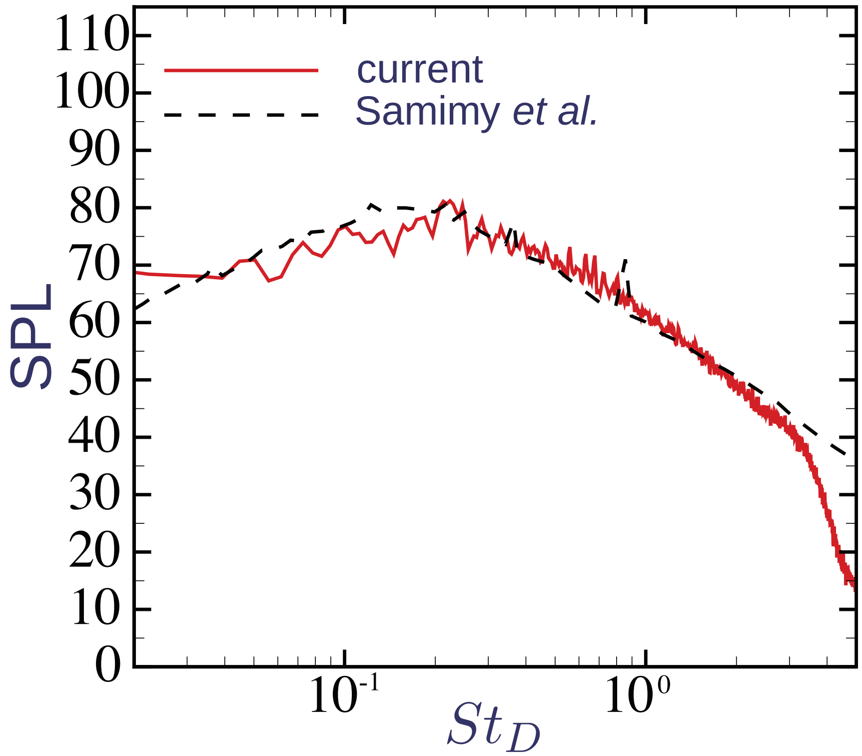
Jet LES



- Matches OSU Samimy *et al.* plasma actuated jet
- Mean inflow:
  CFD of OSU nozzle
- Inflow perturbation: random linear instability modes
- $M = 1.3$
- $Re = 1.1 \times 10^6$
- Mesh: 2.8×10^6 points
- Control: r.h.s. thermal source
- High-order, overset meshes

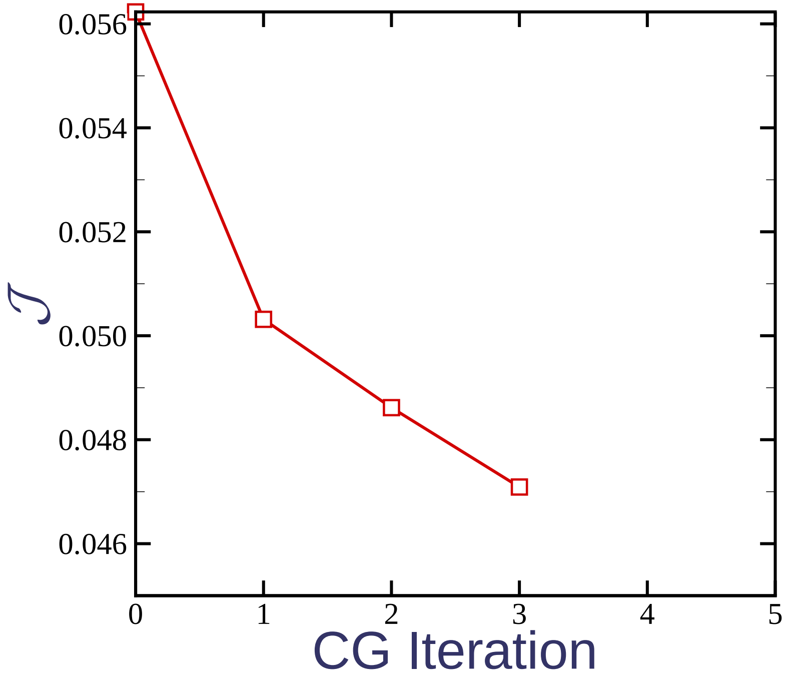
Far-field Sound Spectrum 30°

- Narrow-band spectra at $80D$
- $\mathrm{SPL} = \mathrm{SPL}_{\mathrm{measured}} - 10\log_{10}(R_{\mathrm{norm}}/d)^2 - 10\log_{10}(\Delta f)$
- Overall sound pressure level (OASPL) matches within 1dB



Jet Noise Reduction

$$\mathcal{J} = \int_0^T \mathcal{I}(t)\, dt = \int_0^T \int_{\mathbf{x}} W(\mathbf{x})[p'(\mathbf{x},t)]^2\, d\mathbf{x}dt$$



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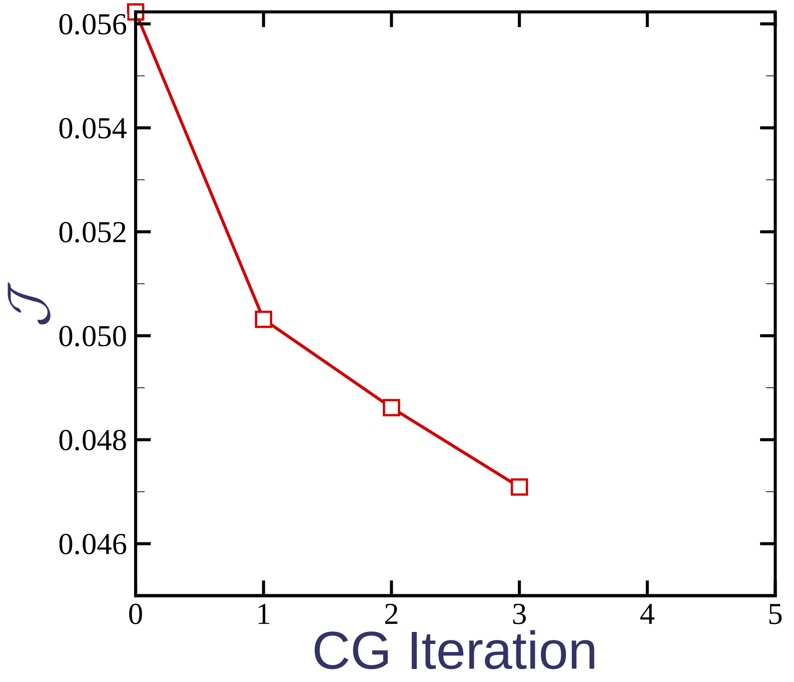
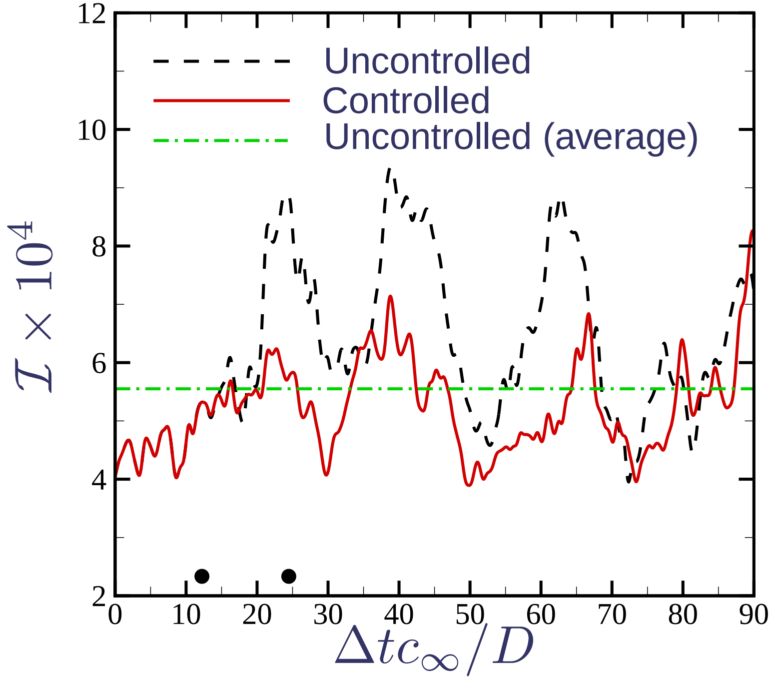
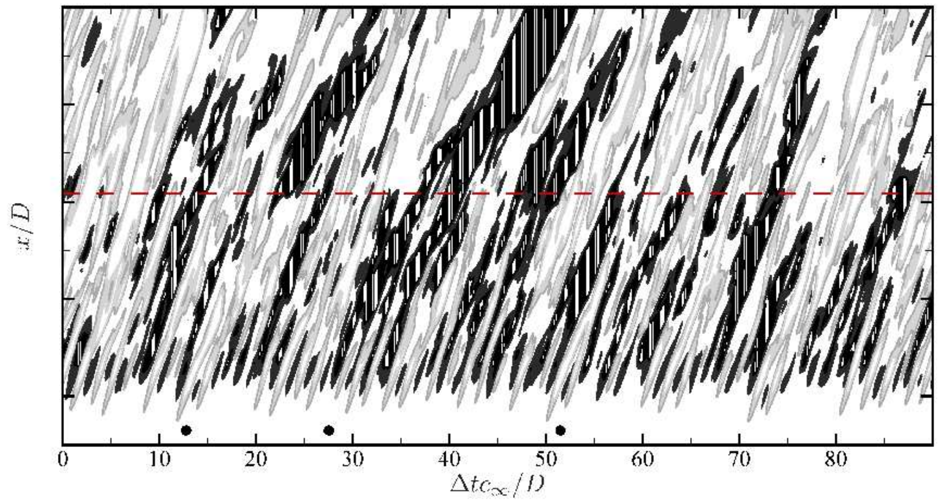
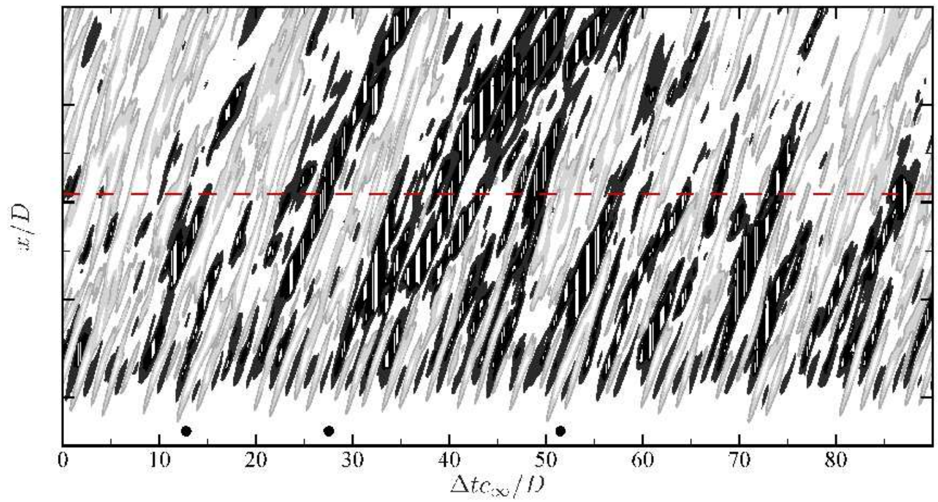




Streamwise Velocity: X–T



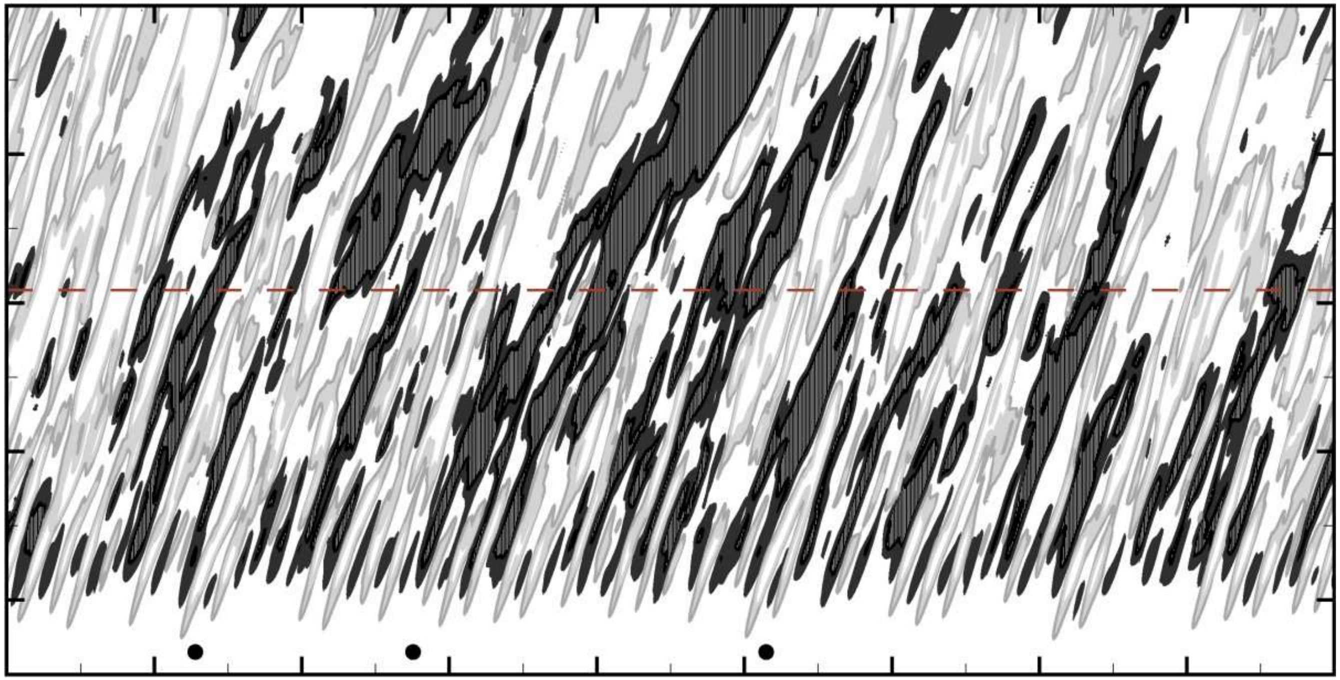
(a) Uncontrolled



(b) Controlled

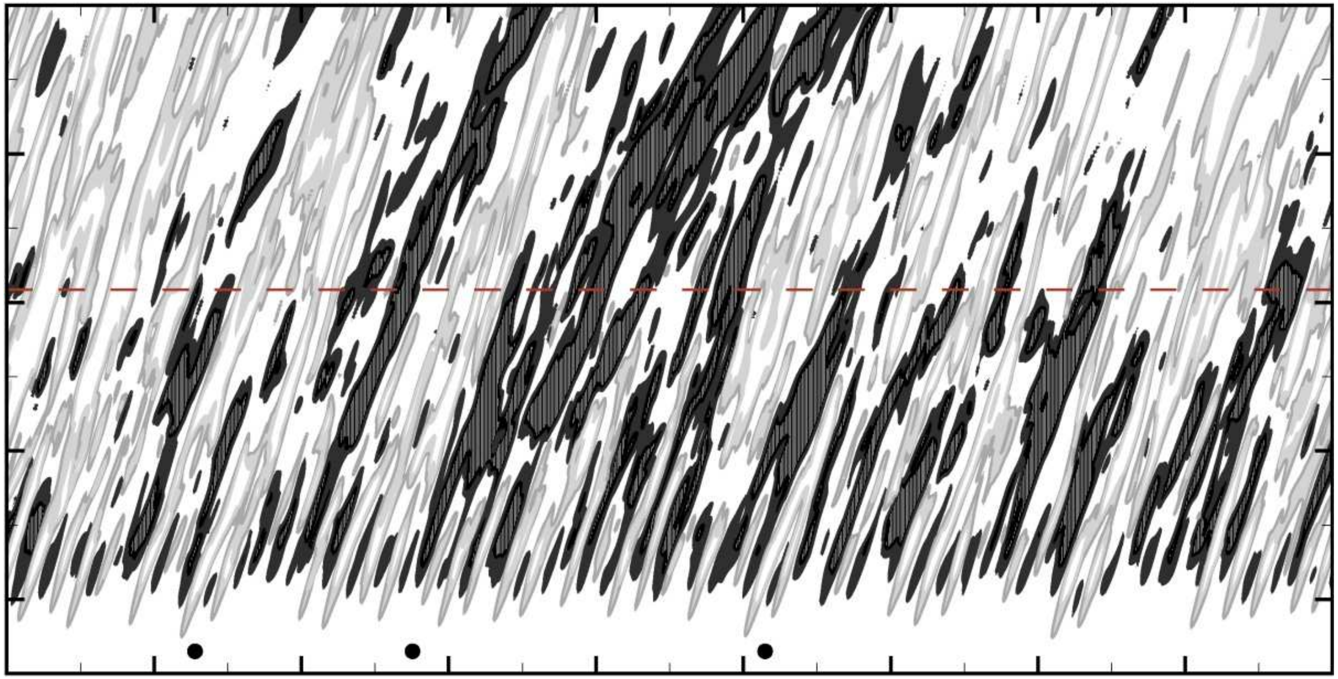
Streamwise Velocity

BEFORE



Streamwise Velocity

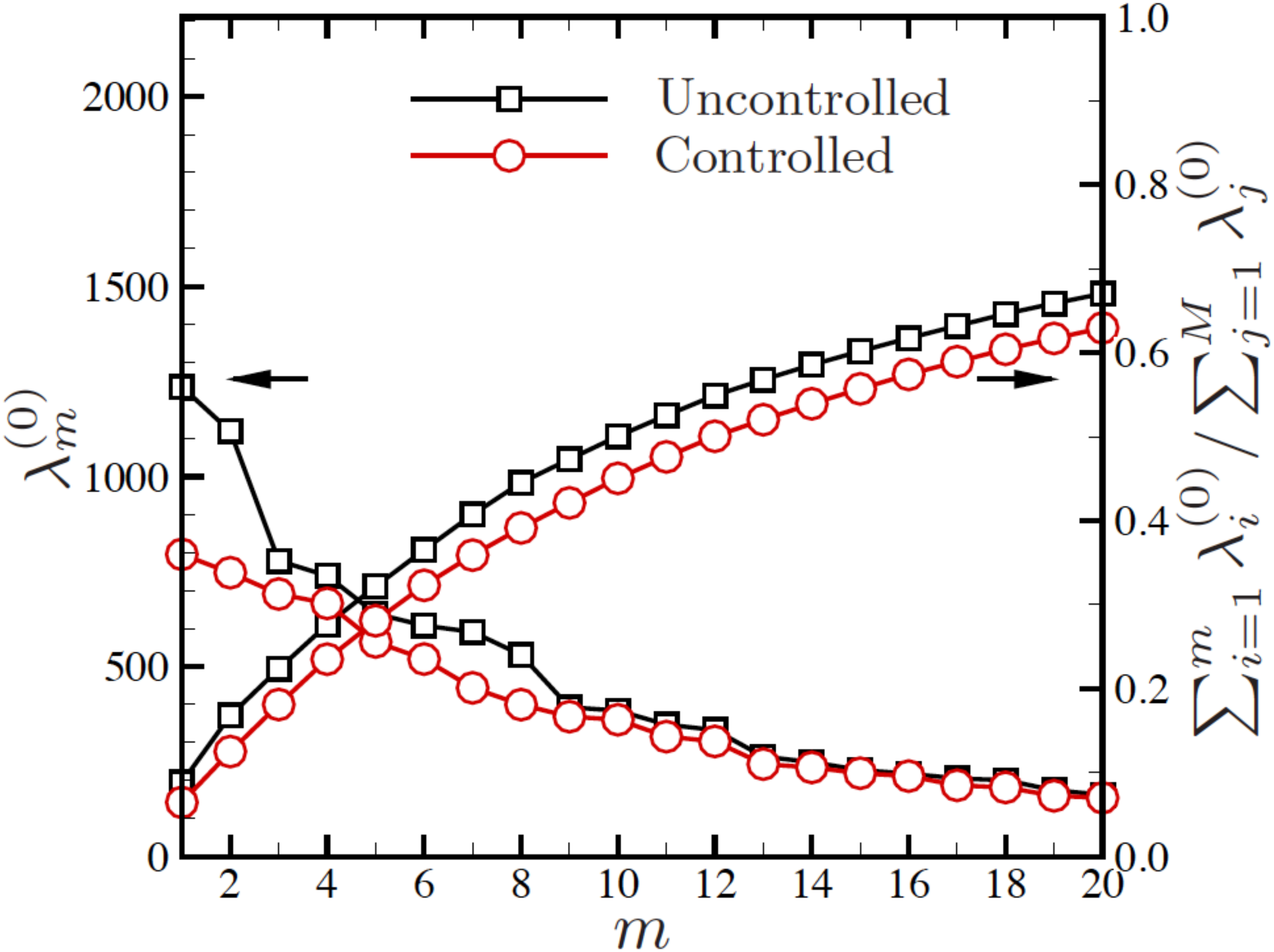
AFTER



Streamwise Velocity

?

Suppression of Axisymmetric Modes



Summary

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- Attractive method for jets
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  - avoids trial-and-error aspect of current reduction strategies

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Summary

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- Adjoint optimization: Jameson (2003), Bewley *et al.* (2001)

- Jets/mixing layers:
  Wei & Freund (2006); Kleinman & Freund (2006); Kim, Bodony, Freund (2010); Freund (2011)