

A Qualitative-Probabilistic Approach to Autonomous Mobile Robot Self Localisation and Self Vision Calibration

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Abstract—Typically, the spatial features of a robot’s environment are specified using metric coordinates, and well-known mobile robot localisation techniques are used to track the exact robot position. In this paper, a qualitative-probabilistic approach is proposed to address the problem of mobile robot localisation. This approach combines a recently proposed logic theory called Perceptual Qualitative Reasoning about Shadows (PQRS) with a Bayesian filter. The approach herein proposed was systematically evaluated through experiments using a mobile robot in a real environment, where the sequential prediction and measurement steps of the Bayesian filter are used to both self-localisation and self-calibration of the robot’s vision system. The results demonstrate that the qualitative-probabilistic approach effectively improves the accuracy of robot localisation, keeping the vision system well calibrated so that shadows can be properly detected.

Keywords-Qualitative Spatial Reasoning; Bayesian Filtering; Mobile Robot; Self-localisation.

I. INTRODUCTION

Navigation is one of the challenges for autonomous mobile robots. Four distinct skills compose navigation: perception, localisation, cognition, and motion control [17]. Although a significant amount of work on mobile robot navigation has been done, systems that can handle uncertainty are still a challenge today. The information gathered by a robot is invariably incomplete and uncertain, usually due to partial observability or noisy sensors.

In this paper, the goal is to perform localisation under uncertainty, by reasoning with information from images that contain shadows. The localisation problem is characterised as position tracking assuming that an initial qualitative robot pose is given. The environment is static, with a fixed light source, and the mobile robot is able to move and get frames from its camera. Previous studies have addressed the problem of qualitative region localisation using information from shadows. We address the localisation problem using a combination of qualitative and probabilistic techniques.

We employ the theory of Perceptual Qualitative Relations about Shadows (PQRS), proposed in Ref. [15], to reason about relations between cast shadow and objects. This particular formalisation allows the robot to infer its qualitative position with respect to the fixed light source and the object. Although the formal theory can describe

all possible locations the robot might be, a major issue arises when the vision system extracts the shadow from the scenes: the camera is rather noisy and therefore a threshold parameter must be fine-tuned in order to correctly detect the shadows. A method to overcome this issue was proposed in [7], where beliefs about the robot’s previous location were used.

The present paper extends that previous work by combining the qualitative beliefs defined by PQRS with a Bayesian filter. In this approach, the qualitative beliefs represent the prediction, whilst the extracted object and shadow observed in the image represent the measurement update. Additionally, the filter is able to make predictions not only about the robot’s localisation, but also about the best threshold value to be used with the next image to be captured by the robot. Our empirical results clearly demonstrate that the proposed approach considerably enhances the accuracy in localisation when compared to [7].

This paper is organised as follows. Section II outlines related work, discussing the state of the art. Then, the PQRS approach used in robot self localisation is briefly introduced in Section III. Next, the proposed probabilistic approach to PQRS with a Bayesian filter approach is detailed in Section IV and V. Finally, Section VI describes the experiments performed, results and discussions, while Section VII concludes this paper and presents possible future directions.

II. RELATED WORK

Renaissance painters have always exploited the effect of shadows in the human depth perception [3]. However, only on the 21st century researchers begun investigating the cognitive processes involved in utilising shadows as cues for external world perception [5], [12]. Shadows carry information about presence, location, as well as intensity of the light source. Moreover, shadows provide information about the caster (object), its shape and texture. Besides, the distance between caster and screen can be hypothesised given whether or not the caster and the shadow appear to be in contact with each other. The information about shadows enhances detection techniques in Geographic Information Systems, even improving the detection of buildings [2]. In Ref. [4], a system is described that determines the 3D shape

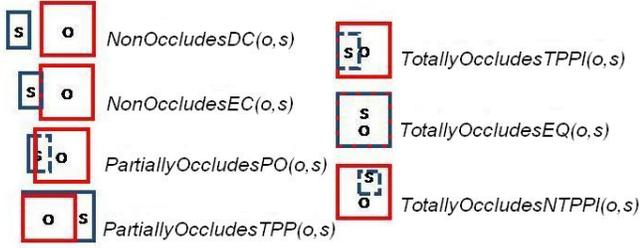


Figure 1: Representation of PQRS relations from several points of view: an object o and its shadow s assume different relations.

of known vertical objects using information from moving shadows cast on the ground. The work presented in Ref. [1] shows that using a single shadow from a fixed light source can provide a similar disambiguation effect as using additional cameras for human pose recognition. In robotics, cast shadows have been used as a cue in trajectory control [6], and in tracking the position of a robot's arm [8]. In [15], a mobile robot employs a qualitative self localisation routine using a representation of cast shadows in terms of a spatial formalism based on occlusion relations.

The first idea of qualitative self-localisation was presented by Ref. [11], where a topological map was formed by spaces bounded by sets of lines connecting pairs of pointwise landmarks. Later, several efforts [9], [16], [19] developed spatial representation behind this idea. In particular, Fogliaroni et al. [9] considered extended convex objects to be handled as landmarks and the decomposition of navigable space is based on the model of occlusion and visibility. Following similar ideas, we have in previous work [7], [15] proposed a qualitative spatial theory named Perceptual Qualitative Relations about Shadows (PQRS), that has been used in the task of robot self-localisation. As the present work builds upon this theory, it is presented in more detail in the next section.

III. PERCEPTUAL QUALITATIVE RELATIONS ABOUT SHADOWS (PQRS)

The PQRS theory [7], [15] provides a formalism that can be used to reason about shadows from a particular viewpoint. The theory relates shadows with occlusion using seven occlusion relations from the Region Occlusion Calculus (ROC) [14], here defined with respect to the light source, the caster and its shadow. The set of relations for a caster o , its shadow s and a viewpoint ν defined by PQRS includes the Region Connection Calculus (RCC) [13] and a subset of ROC, as follows:

$$\begin{aligned} & NonOccludesDC(o, s, \nu) \\ & NonOccludesEC(o, s, \nu) \\ & PartillyOccludesPO(o, s, \nu) \end{aligned}$$

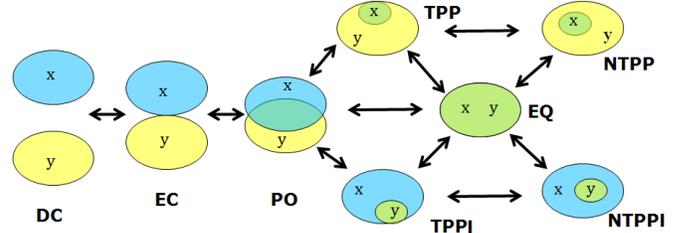


Figure 2: The Conceptual Neighbourhood Diagram of RCC8 relations representation.

$$\begin{aligned} & PartillyOccludesTPP(o, s, \nu) \\ & TotallyOccludesTPPI(o, s, \nu) \\ & TotallyOccludesEQ(o, s, \nu) \\ & TotallyOccludesNTPPI(o, s, \nu) \end{aligned}$$

A graphical representation of these relations is shown in Figure 1.

Besides relations about occlusion, PQRS includes one relation about shadows, $Shadow(s, o, Scr, L)$, that represents that a shadow s is cast by a caster o , from the light source L , on the screen Scr . The axiom constraining the $Shadow/4$ relation is represented by the following formula:

$$\begin{aligned} & Shadow(s, o, Scr, L) \leftrightarrow \neg \exists o' (o \neq o') \wedge \\ & Occludes(o', o, L) \wedge PO(region(s), region(Scr)) \wedge \\ & TotallyOccludes(o, s, L). \end{aligned} \quad (1)$$

Formula 1 states that the shadow of a caster o is the region in a screen Scr that is totally occluded by o from the light source viewpoint L (if there is no other object o' occluding o from L).

Informally, there are three relations between shadows and objects. The first relation, *free of occlusions* (NonOccludes), is defined when the viewer ν can see the whole shadow cast, which can be seen in the image as being disconnected (DC) from or externally connected (EC) to the caster. The second, *partial occlusion* (PartiallyOccludes), is defined when the caster covers part of shadow cast and hence the shadow cast appears in the image as overlapped by the caster. Finally, the third relation, *total occlusion* (TotallyOccludes) is defined when, from the viewpoint ν , only the caster can be observed in the image, because the shadow cast is totally occluded by caster.

The PQRS theory inherits a central concept from the RCC8 calculus [13]: the continuous transitions between relations are represented by the Conceptual Neighbourhood Diagram (CND), depicted in Figure 2 [10]. Continuous transitions are represented by arrows between neighbour relations; only connected regions can assume neighbour relations. For instance, two disconnected regions x and y ($DC(x, y)$) will never overlap without having previously been externally connected ($EC(x, y)$).

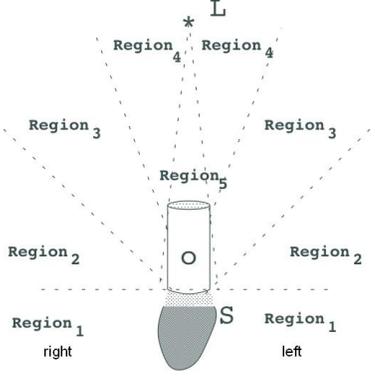


Figure 3: The representation of the qualitative map with five distinct regions, showing both sides (right, left) and regions (1, 2, 3, 4 and 5). The lines of sight between the light L , the caster O and its shadows define boundaries between regions

Relations expressed by PQRS divide the space into five distinct regions obtained from the distinctions of the perceived relations between a caster and its shadow's top. This qualitative map is shown on Figure 3, in which for each region the robot perceives a distinct relation between object-shadow. Hence, any viewpoint ν located in $Region_1$ observes the shadow's top $top(s)$ and the object caster o as *NonOccludesDC*($o, top(s), \nu$). Similarly, in $Region_2$, ν perceives object-shadow as *NonOccludesEC*($o, top(s), \nu$). When the ν is located on $Region_3$ it perceives *PartiallyOccludesPO*($o, top(s), \nu$), while on $Region_4$ the shadow appears as a strip on the side of its caster, so it is perceived as *TotallyOccludesTPPI*($o, top(s), \nu$). Finally, in $Region_5$ the shadow is behind the object, thus, ν perceives *TotallyOccludesNTPPI*($o, top(s), \nu$).

As we can observe, the motion throughout the environment represented by the qualitative map follows the continuous transition in the Conceptual Neighbourhood Diagram. As the robot navigates from $Region_1$ to $Region_5$, it observes the relation sequence EC, DC, PO, TPPI, NTPPI between the object and its shadow on the image. This continuous change on perception (due to robot motion) is called *qualitative motion*. The changes in perception occur constantly while the observer moves. In the boundary between regions the features perceived are similar. This implies that at their borders, it is hard to distinguish between two regions. This problem is reflected in our results as we shall see further in this paper.

IV. PROBABILISTIC INFERENCE

PQRS is defined on a first-order logic language, and therefore it is not able to handle sensor uncertainty. On the other hand, the robot's vision system, besides being noisy in general, is dependent on the threshold which is a key parameter to object-shadow segmentation. Thus, inferences within PQRS are dependent on the image segmentation. In

order to achieve a good segmentation, it is necessarily to find appropriate thresholds as the robot changes its position. In this work, we extend the PQRS theory combining it with a Bayesian filter. This filter is used to provide an estimation for the robot's location and the best shadow-caster segmentation threshold, given the robot's current state (i.e., the robot's location wrt a region on the map shown in Figure 3).

We refer to *belief distributions* as the posterior probabilities over state variables, conditioned on the available data. We use the notation $bel(s_t) := P(s_t|e_{0:t})$; that is, the belief $bel(s_t)$ is the probability of a state (s) given all evidence (e) until an instant (t). In this work, the state s_t indicates the region where the robot is located and (e) represents evidence obtained from the images. This evidence is denoted in terms of PQRS relations. Thus, we need to find the value of s_t , and also of Th (threshold used to extract shadows from images) that maximises $bel(s_t)$.

In order to compute the beliefs, we use:

$$\begin{aligned} bel(s_t) &= P(s_t, Th|e_{0:t}) \\ &= P(s_t, Th|e_{0:t-1}, e_t) \\ &= \eta P(e_t, Th|s_t, e_{0:t-1}) P(s_t|e_{0:t-1}), \end{aligned}$$

where η is a normalisation constant. Using the Markov hypothesis, the next state is independent of earlier measurement $e_{0:t-1}$; hence:

$$bel(s_t) = \eta P(e_t, Th|s_t) P(s_t|e_{0:t-1}).$$

This expression is the *measurement update* [18]. The term $P(e_t, Th|s_t)$ is, basically, the image model. The posterior belief $P(s_t|e_{0:t-1}) = \bar{bel}(s_t)$ should be calculated before incorporating the evidence e_t . The term $P(e_t, Th|s_t)$ incorporates the new evidence (e_t).

The posterior belief $P(s_t|e_{0:t-1})$ represents a one-step prediction of the next state s_t , obtained by conditioning on the previous state s_{t-1} . Consequently,

$$\begin{aligned} \bar{bel}(s_t) &= \sum_{s_{t-1}} P(s_t|s_{t-1}, e_{0:t-1}) P(s_{t-1}|e_{0:t-1}) \\ &= \sum_{s_{t-1}} P(s_t|s_{t-1}) P(s_{t-1}|e_{0:t-1}). \end{aligned}$$

In the previous expression, the term $P(s_t|s_{t-1})$ is the state transition probability. In this work mov_t is a qualitative motion throughout the five regions represented in Figure 3 and it could be either clockwise or counterclockwise. The state s_t is a $Region_i$ in which $i = 1, 2, 3, 4, 5$. The term $P(s_t|s_{t-1}, mov_t)$ is defined on Table I and represents the transition model $P(s_t|s_{t-1})$. We then obtain $P(s_{t-1}|e_{0:t-1})$ by conditioning on Th :

$$P(s_{t-1}|e_{0:t-1}) = \sum_{Th} P(s_{t-1}, Th|e_{0:t-1}).$$

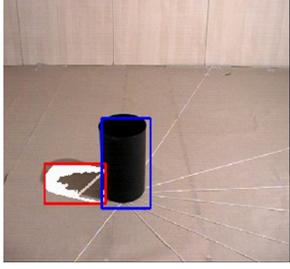


Figure 4: Example of shadow-caster segmentation.

Therefore, the posterior belief (or prediction) is the combination of previous expressions:

$$\overline{bel}(s_t) = \sum_{s_{t-1}} P(s_t|s_{t-1}) \sum_{Th} P(s_{t-1}, Th|e_{0:t-1}).$$

Finally, the belief is given by:

$$bel(s_t) = \eta P(e_t, Th|s_t) * \overline{bel}(s_t).$$

V. THE SELF-LOCALISATION, SELF-CALIBRATION ALGORITHM

The robot estimates its relative location using a monocular colour camera. From its viewpoint, the robot perceives a relation between shadow and object. The PQRS theory then gives the robot the ability to infer its position.

A threshold on light intensity segments the image into object and shadow. We use a morphological operator and saturation values to perform segmentation; after that, the algorithm looks for the shadow connected to the object's base. If more than one shadow is connected to object, the nearest shadow of light are discarded. Figure 4 shows an output of this segmentation procedure.

Algorithm 1 initialises the system with an uniform uncertainty about robot position (line 1). Line 2 runs a threshold calibration, assuming a good shadow-caster recognition on first frames. The frame sequence is represented by t .

Then, Algorithm 1 calls repeatedly Algorithm 2. Algorithm 2 returns the evidence with maximum probability from $P(E)$. In Algorithm 2, PQRS relations between shadow-caster are evaluated by some detected features, such as the degree of connectivity between the side shadow's top and the caster, the degree of connectivity between shadow-object, the shadow's bounding-box sizes and the relative position to the object's bounding box. It returns the relation with the greatest number of features perceived. The evidence can be DC , EC , PO , $TPPI$ or $NTPPI$. If the object has no shadow cast, only the relation $NTPPI$ is analysed. Then, given the evidences, line 5 in Algorithm 1 calculates beliefs for all five regions (here represented by S_t) and a threshold range ($[Th - 5, Th + 5]$). Line 6 gives the results about the current region (s_t) and the threshold (Th) to be used in the next image. The two last lines in Algorithm 1 run a Bayesian prediction step, calculated by line 8, in which current beliefs

Algorithm 1 THRESHOLD – AND – POSITION()

- 1: $\overline{bel}(S_1) = P(S_0) = [0.2, 0.2, 0.2, 0.2, 0.2]$
 - 2: Calibration(th, Scene, ν)
 - 3: **while** (1) **do**
 - 4: $e_t \leftarrow \max PERCEPTION - ACTION(th, image, \nu)$
 - 5: $bel(S_t) = P(S_t, Th|e_{0:t}) = P(e_t, Th|S_t) * \overline{bel}(S_t)$
 - 6: $(s_t, th) \leftarrow \arg \max_{s_t, th} P(S_t, Th|e_{0:t})$
 - 7: $P(S_t|e_t) \leftarrow \sum_{Th} P(S_t, Th|e_{0:t})$
 - 8: $\overline{bel}(S_{t+1}) = \sum_{s_t} P(S_{t+1}|s_t) P(s_t|e_t)$
 - 9: **end while**
-

Algorithm 2 PERCEPTION – ACTION($th, Scene, \nu$)

Segment Scene using the threshold th to obtain a caster O and the top of its shadow $top(S)$.

- 1: **if** caster and shadow were found **then**
 - 2: $P(e = DC) \leftarrow NODC(O, top(S), \nu)$
 - 3: $P(e = EC) \leftarrow NOEC(O, top(S), \nu)$
 - 4: $P(e = PO) \leftarrow POPO(O, top(S), \nu)$
 - 5: $P(e = TPPI) \leftarrow TTPPI(O, top(S), \nu)$
 - 6: **else**
 - 7: **if** only caster was found **then**
 - 8: $P(e = NTPPI) \leftarrow TNTPPI(O, top(S), \nu)$
 - 9: **else**
 - 10: **return** e_{t-1} ;
 - 11: **end if**
 - 12: **end if**
 - 13: **return** $\arg \max_e P(E)$;
-

for all states (given the evidence) was calculated in line 7. The probability of the next states (given the current state) is the motion model given by Table 1 (as we shall see further in this paper).

In Algorithm 1 we have abbreviated the PQRS relations, as follows the $NODC(O, top(S), \nu)$ represents $NonOccludesDC(o, s, \nu)$, $NOEC(O, top(S), \nu)$ represents $NonOccludesEC(o, s, \nu)$, $POPO(O, top(S), \nu)$ represents $PartillyOccludesPO(o, s, \nu)$, $TTPPI(O, top(S), \nu)$ represents $TotallyOccludesTPPI(o, s, \nu)$ and, $TNTPPI(O, top(S))$ represents $TotallyOccludesNTPPI(o, s, \nu)$.

The Bayesian filter handles two probability models, the sensor model and the transition model. In Algorithm 1 our sensor model is the image model (line 5). The motion model represents the transition between regions in the map and is used on the posterior belief's calculation (line 8). Now we describe how the image and motion models were designed.

A. The image model

The image model is represented by $P(e_t, Th|s_t)$, that indicates what we should expected from sensor data when in a give state. In this work, each evidence are linked to a region and this idea is formalised within the PQRS theory.

However, the evidences are dependent of the vision system threshold. In this way, we need a model that indicates the frequency that a threshold gives the right evidence to each region. That was obtained from a set of images. A set of images captured by the robot was obtained for each region where the robot could located. On these sets the target object-shadow were always on the scenes observed. This guarantees that evidence was always available to be perceived by vision system, during the evaluation of the image model. For each threshold value, the sets of images were analysed. The results form a matrix with information about the percentage of right answer found *per* threshold.

B. The motion model

The transition or motion model represents the changes between the robot’s states, given a moving action. Considering the qualitative map (Figure 3), and steady-speed clockwise motion around the target object-shadow, the probability of the changes between regions are mostly dependent on the regions’ sizes. Therefore, the larger the region, the higher is the probability that the robot stays on it. This is represented by the numbers shown on Table I. The multiplying factor w represents perceived shadow features that modulate the probability for a state change. This can be understood by taking into account that, in this work, the robot’s motion is inferred from image changes with respect to shadow features. When the robot approximates a borderline region, the detection of shadow features related to the next region should increase the probability of a state change.

VI. EXPERIMENTS AND RESULTS

In our experiments, a Pioneer Peoplebot mobile robot collected snapshots in an office-like environment. In order to compare results, we used the same data as in Ref. [7]. Images were captured by the camera into a HSV colour space. The shadow-object detection was performed with a threshold filter on V.

Algorithm 1 uses the belief about the robot’s location and what is perceived from the scene. This algorithm gives both the robot state and the threshold. The robot was teleoperated and could start in any of the regions indicated in Figure 3. While the robot navigated around the object, the operator marked the real region (*gold standard*) to compare with the robot answers.

The approach presented in this paper was applied to 587 snapshots of the target object-shadow. The results on the localisation procedure (on the map shown in Figure 3) are presented in a confusion matrix (Table II). Each column represents our system’s outputs while the rows represent the actual region that the robot was on (according to the gold standard).

In Table III, rows refer to the true location for each image (regions 1, 2, 3, 4 and 5). The column #image is the number of images captured for each region. The results from [7] are

Table I: The motion model considering robot clockwise navigation. This model is based on transition regions.

States clockwise		$s_t = 1$	$s_t = 2$	$s_t = 3$	$s_t = 4$	$s_t = 5$
$s_{t-1} = 1$	right	$0,9w$	$0,1w$	0	0	0
$s_{t-1} = 2$	right	$0,01w$	$0,8w$	$0,19w$	0	0
$s_{t-1} = 3$	right	0	$0,01w$	$0,75w$	$0,24w$	0
$s_{t-1} = 4$	right	0	0	$0,01w$	$0,70w$	$0,29w$
$s_{t-1} = 5$		0	0	0	$0,35w$	$0,65w$
$s_{t-1} = 4$	left	0	0	$0,29w$	$0,70w$	$0,01w$
$s_{t-1} = 3$	left	0	$0,24w$	$0,75w$	$0,01w$	0
$s_{t-1} = 2$	left	$0,19w$	$0,8 w$	$0,01 w$	0	0
$s_{t-1} = 1$	left	$0,99 w$	$0,01w$	0	0	0

Table II: The result of the experiments of self localisation using a Bayesian filter and perceptual qualitative reasoning about shadows.

Regions	$Region_1$	$Region_2$	$Region_3$	$Region_4$
$Region_1$	97%	2%	0%	0%
$Region_2$	25%	74%	1%	0%
$Region_3$	9%	21%	66%	4%
$Region_4$	6%	0%	8%	86%
$Region_5$	0%	0%	35%	65%

represented on the column “knowledge based”. The results shown in column “Belief based” refer to the percentage of right regions returned by the framework presented in this paper. The global performance of the current experiments presents an increase from 58% (obtained in Ref. [7]) to 80%. On the boundary between two regions (where the shadow-caster features give equal evidence for two locations) the system chooses the first region as the robot’s location. This choice reflects positively on Regions 1, 2 and 4, but on Regions 3 and 5 the accuracy decreased in comparison to our early experiments.

Table II shows that the highest number of false negatives for $Region_3$ was related to locating the robot on region 2 (21% of the cases). Part of this error is due to the borderline problem explained above. Another issue is that the system presented in this paper never detected when the robot was located on $Region_5$. This poor performance on $Region_5$ is due to the motion model, where the relation *Totally Occludes* was not considered in the composition of the weight w (because the weight is calculated on the existence of shadows).

We are currently investigating possible solutions to the borderline problem and to the issue of detecting $Region_5$.

Table III: Percentage of correct answers from the knowledge-based and our qualitative Bayesian filter, where '# images' is the number of snapshot to region

Regions	# images	Knowledge based (%)	Belief based (%)
<i>Region₁</i>	225	66	97
<i>Region₂</i>	171	34	74
<i>Region₃</i>	138	80	66
<i>Region₄</i>	36	44	86
<i>Region₅</i>	17	59	0
Global	587	58	80

VII. CONCLUSION

In this paper we have presented an approach to robot localisation using information about shadows in images, where we combined a probabilistic Bayesian filter with a qualitative spatial representation. Our method uses an adaptive threshold with a Bayesian filter to produce better results than a purely qualitative localisation approach. Moreover, considering that the dataset used was composed of 30% of images where self-localisation was impossible (since the target object was not present in these data points), our algorithm had a performance of around 80% over all images. This suggests the robustness of a probabilistic approach to infer qualitative facts about space.

Future work shall consider an extension of the method proposed in this paper in order to allow robot self-localisation within an environment with various objects and their shadows.

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