Dynamic Behavior of a DICAS FPSO and Shuttle Vessel under the Action of Wind, Current and Waves

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ABSTRACT

Floating, Production, Storage and Offloading (FPSO) systems consist of a large tanker moored to the seabed operating as an offshore production facility. During the offloading operation, another tanker (the shuttle vessel) is attached to the FPSO through a flexible hawser to allow the safe connection of a hose for cargo transfer. In previous works we have applied a nonlinear dynamical systems approach to investigate the behavior of these systems under the action of wind and current. Both Differentiated Compliant Anchoring System DICAS and turret FPSO mooring arrangements were considered. For the present study we have taken a DICAS system, and performed a numerical investigation of the influence of the action of sea waves in addition to wind and current. Suitable spectra were employed, and second-order effects such as slow drift forces, as well as wave-current interaction corrections to mean drift forces, namely wave drift damping, were calculated. The predicted dynamics for the system were calculated for relevant ranges of environmental parameters such as wind and current speeds, wave spectra, and respective angles of incidence. The influence of the draft of the vessels was also inspected. Results were summarized in bifurcation diagrams displaying the evolution of steady-state responses as a function of parameters characterizing the environmental state. The stability of steady-state responses was investigated using time-domain simulations. The results were also employed to illustrate the production of diagrams in which inadequate (or unacceptable) dynamic solutions are identified.

KEY WORDS: FPSO; shuttle vessel; offloading; bifurcations.

INTRODUCTION

Floating, Production, Storage and Offloading (FPSO) systems represent an important engineering solution for the exploitation of deep-water oil and gas fields, and they have played a substantial role in the Brazilian offshore basins. An FPSO consists of a large converted tanker moored to the seabed, and as part of its operation the main vessel has to offload its cargo periodically to a shuttle vessel that takes the oil to onshore installations. For the offloading, the vessels are connected to each other through a polyester cable (a hawser) so that a flexible pipe can be used to transfer the oil. The offloading operations therefore require for their safe carrying out that adequate relative positioning between the two vessels be kept, even under the action of environmental forces such as wind, waves and currents.

It is thus necessary to have knowledge of the dynamic behavior of these large, expensive systems as an important aspect of their design. The FPSO-only problem has been investigated elsewhere, and results have shown the system’s response to be considerably complex, with a variety of features being observed, including bifurcations of static equilibria, limit cycles, and chaotic motions (Leite, Aranha, Umeda and de Conti, 1998; Garza-Rios and Bernitsas, 1996). The present work investigates a two-body problem where the main ship, whilst moored in a DICAS arrangement, is attached to a shuttle vessel for the offloading operation. It furthers previous studies on the modeling and dynamics of both of single-body and two-body FPSO systems under wind and current (Morishita and Cornet, 1998, Souza Junior, Morishita, Fernandes and Cornet, 2000; Morishita and Souza Junior, 2001, Morishita, Souza Junior and Fernandes, 2001; Morishita, Souza Junior and Cornet, 2001). These recent investigations have revealed that the coupled system can exhibit complex behavior that starts with a multiplicity of static equilibrium solutions whose number and stability properties vary according to wind and current relative speeds and angles. Some fixed points were shown to be unstable or undesirable (or both), involving physically impossible (overlapping) relative positions or implying in dangerous proximity between vessels.

The influence of ocean waves had not been considered in those previous efforts and represents the main focus of the present work. First-order wave effects are not relevant to the analysis of the overall, long-term behavior of the system’s motions in the horizontal plane. The effect of irregular seas is represented by second-order forces and moments, namely mean and slow drift forces, as well as by wave-current interaction terms (wave drift damping). The inclusion of wave forces into the analyses helps completing the picture of the dynamics of these systems under the marine environment, now constituted of wind, current and wave effects.

The understanding of the influence of the several environmental agents considered here combined with the possible variations in the value of significant parameters – such as strength and angle of incidence of
wind and current, and ship draft – requires careful analysis. A systematic study was carried out in which the consequences of each agent or parameter are assessed separately. An initial analysis is restricted to the changes in equilibrium positions caused by the inclusion of a mean wave drift force component. To that purpose a set of diagrams is produced, each displaying the static equilibrium positions of the system (represented by the heading of the shuttle vessel) for a range of wind speeds and a given angle of wind incidence. These diagrams, the reference for the mean drift force study, can then be compared with previous results without waves. Next, a zero-wind scenario is considered. The influence of current strength or speed is also investigated. To conclude the study of the mean drift force influence on static equilibria the effect of shuttle vessel loading is considered by a comparison of two different draft conditions.

The inclusion of slow drift wave forces, a time periodic component, evidently precludes the existence of static equilibria. The simplest possible solutions are now periodic responses, i.e. limit cycles, which have to be analyzed in terms of their amplitudes around their mean positions. This new scenario is compared with the mean drift only results.

The final part of the present work investigates the stability of responses and their engineering suitability. These studies were conducted considering both a mean wave drift and a full (mean plus slow drift forces) second order wave model.

MATHEMATICAL MODEL

Motions of the vessels in the horizontal plane are expressed in three orthogonal co-ordinate reference systems as shown in Fig. 1. The first system, OXYZ, is earth-fixed; the second and third ones, G1xG2yG3zG, are body-fixed in the center of gravity of the FPSO and shuttle ship, respectively. The axes of each body-fixed co-ordinate system coincide with the principal axes of inertia of the vessel. The low frequency horizontal motions of each vessel are then given by:

\[
(m + m_{11}) \ddot{u} = (m+m_{22})v - (mx_x + m_{30})\dot{v}^2 + (m_{11} - m_{12}) \dot{v} + r + X
\]  
\[
(m + m_{22}) \ddot{v} = (m_{11} + m)ur - (mx_x + m_{26})\dot{v} + (m_{11} - m_{12})u + r + Y
\]  
\[
(I_{c} + m_{66}) \ddot{r} = -(mx_y + m_{26})(\ddot{u} + ru) + N
\]

where \(m\) is the mass of the vehicle; \(m_{ij}\), \(i,j = 1, 2, 6\) are the added mass coefficients in surge, sway and yaw, respectively; \(u\) and \(v\) are the surge and sway velocities of the vehicle, respectively; \(u\) and \(v\) are current speeds related to GX and GY directions, respectively; \(r\) is the yaw rate; \(I_{c}\) is the moment of inertia about the GZ axis; \(X, Y\) and \(N\) represent the total external forces and moments in surge, sway and yaw directions, respectively; \(x_{c}\) is the co-ordinate of the vessel’s center of gravity along the GX axis and the dot means time derivative of the variable. The position and heading of each vessel related to the earth-fixed co-ordinate system are obtained from the following equations:

\[
\dot{x}_0 = u \cos \psi - v \sin \psi
\]  
\[
\dot{y}_0 = v \sin \psi + u \cos \psi
\]  
\[
\dot{\psi} = r
\]

where \(x_{0}\) and \(y_{0}\) are the components of the vessel’s speed in the OX and OY axes, respectively, and \(\psi\) is the vehicle heading. The components \(u\) and \(v\) of the current are calculated as:

\[
u_c = V_c \sin(\psi_c - \psi)
\]  
\[
u_c = V_c \cos(\psi_c - \psi)
\]

where \(V_c\) and \(\psi_c\), are the velocity and direction of the current, respectively.

The forces \(X\) and \(Y\), and the moment \(N\) considered in this paper are due to current, wind, waves, hawser, yaw hydrodynamic damping and, in the case of the FPSO, mooring lines. Forces due to current are determined through a heuristic model based on a low aspect ratio wing theory with experimental validation (Leite, Aranha, Umeda and de Conti, 1998) and the wind forces are calculated employing coefficients suggested by OCIMF (OCIMF, 1994). Forces due to waves are usually split in low and high frequency terms. In this paper only the former are taken into account, and those can be considered as the sum of slow and mean drift forces. In particular, calculation of the mean drift forces considers corrections due to wave and current interaction. These forces are calculated based on unidirectional sea spectra, and their parameters, namely, significant height and period are defined from the wind speed. The aero- and hydrodynamic interactions between the two vessels are not considered in this work. The forces produced by mooring lines and the hawser are calculated with catenary equations. Details of the mathematical models employed for the determination of external forces \(X, Y,\) and \(N\) are displayed in Appendix A.

Figure 1 – Body-fixed and earth-fixed co-ordinate systems

EQUILIBRIUM SOLUTIONS AND BIFURCATION DIAGRAMS

As shown in previous works (see for instance Morishita, Souza Junior and Fernandes, 2001), the net effect of the hydrodynamic action of the current is destabilizing, in the sense of making the vessel steer away from the direction aligned with the current (head or following), irrespective of the vessel’s draft: under the sole influence of current forces the vessel tends to stabilize sideways to them. The effect of the wind can depend on the vessel’s draft: following winds are
destabilizing, but for head winds the yaw moment tends to align the vessel with the incoming wind in conditions of full displacement whereas it is destabilizing for a 40% partial draft. As for the mean drift force, the overall effect on the vessel is qualitatively the same as for the current, although quantitatively less pronounced, especially for the full displacement condition, see Fig. 2. Of course, the action of the hawser will always be stabilizing in the sense above.

The balance of forces and moments on each vessel determines static equilibrium positions. Due to the complex, nonlinear character of forces and moments involved here – from wind, current, and waves—the number of resulting equilibria as well as their features must be found numerically. This analysis was performed initially obtaining the solution of equations (1)-(6) vanishing all time derivative terms. The scheme employed assumed that for each equilibrium position of the shuttle vessel there is only one corresponding equilibrium position for the FPSO. Such hypothesis is justified in this case where the mooring used is of the DICAS type. Therefore, attention was focused on the linear and angular equilibrium positions of the shuttle vessel.

It is not feasible to summarize here a complete investigation of all possible relevant combinations of magnitude and angle of incidence of wind, current and waves, and also of the effect of different drafts. Two main simplifications were adopted: the angle of incidence of the current \( \psi_c \) was set to \( \pi \) (head current), and the magnitude and angle of incidence of wave forces were linked directly to those of the wind. It is worth noting that in contrast to what was shown for the wind-current problem (see Morishita, Souza Junior and Cornet, 2001), equilibrium points are no longer dependent only on the ratio between wind and current speeds: the inclusion of wave forces causes solutions to depend on the absolute value of each environmental force. Representative values for other parameters such as the FPSO vessel displacement and hawser nominal length were chosen and kept constant throughout the present analysis (see Appendix B for a more complete list of parameters). The FPSO and the shuttle vessel are 330,000 ton dwt and 130,000 ton dwt vessels, respectively. Unless otherwise stated, the draft of the shuttle vessel was kept at a value corresponding to 40% of its full displacement, and the current speed was 1 m/s.

The Effect of Mean Drift Forces

Before going into the detailed discussion of the effect of each particular parameter, it is perhaps useful to recall here that, as remarked in the previous works already mentioned, complex behavior of static solutions can be broadly attributed to a certain balance of angle-dependent forces and moments, through which their combined potential well – as given by the integral of forces and moments – become relatively flat. In such flat regions of the wells, small variations in the net sum of forces and moments due to wind, current, and now waves, produce local maxima and minima – the equilibrium positions – whose number can be rather large.

To illustrate the main result of introducing a mean drift wave component, a comparison was made between bifurcation diagrams obtained with and without such component. Figure 3, which contains the reference diagrams for the mean drift force study, shows equilibrium positions, represented by the static heading of the shuttle vessel \( \psi_2 \) for a range of wind speeds, and two wind angles of incidence. These diagrams are to be contrasted with those of Fig. 4, which are reproduced here from our previous investigations of the wind-current scenario (see Morishita, Souza Junior and Fernandes, 2001). In order to systematize the present analysis, two aspects related to the complexity of the response are here introduced, namely, degree and range of complexity. Degree of complexity is related to the number and character of equilibrium solutions that coexist for a specific combination of parameters. Therefore, if for a given situation the system has eight equilibria (and possibly some limit cycles), this condition is said to have a higher degree of complexity than a situation where only two equilibria are found (say, one stable and one unstable fixed points). But also of relevance is the range of complexity, defined by the range of parameter values (say wind speed or angle of incidence) within which behavior of a high degree of complexity arises. As will be shown shortly the two aspects of complexity do not always go hand in hand.

It should perhaps be remarked here that, as with previous studies of these vessels, the range of angles of incidence of wind (and now also waves) that allow the possibility of complex behavior is limited to small to moderate angles of up to, say, 30° to 40°. Therefore, the present study is also focused on the more adverse situations where wind and waves roughly oppose the direction of the current.

Inspection of Figs. 3 and 4 reveals that the overall effect of the introduction of a mean drift wave component is one of reducing the complexity of response. Such conclusion can be reached by observing

**Figure 2—Yaw moments from current, wind, and mean wave drift forces: (a) Shuttle vessel partial loading condition (draft is 40% of design draft), (b) Shuttle vessel full loading condition (draft is 100% of design draft).**
that the range of complex behavior is reduced both in terms of wind speeds and of wind angles of incidences in which complex behavior is found. The maximum degree of complexity observed is also smaller after the inclusion of a mean drift force component. A physical explanation is suggested from the analysis of Fig. 2(a). Firstly, it must be recalled that in the present study the mean drift force has always the same angle of incidence of the wind. One can, therefore, think of their yaw moments as forming a combined effect for each incidence. A close inspection of Fig. 2 shows that for most angles of incidence the mean drift force moment adds to that of the wind, magnifying their absolute value.

Consequently, the potential wells associated to (the integral of) these moments are “deeper” than those corresponding to the wind alone. Now, complex behavior for this system is typically related to “shallow” potential wells (see for instance Morishita, Souza Junior and Fernandes, 2001), whose small undulations give rise to several equilibria. It is therefore to be expected that mean drift forces, which cause yaw moments to vary more sharply with wind speed or angle of incidence, tend to decrease the complexity of response.

Unless stated otherwise, all results presented here include the effect of mean drift forces.

The Zero-Wind Scenario

Since wind forces, when added to wave effects, are found to have a complexity-reducing effect, it is of interest to study the solutions of equilibrium equations in the absence of wind. To that effect, a zero-wind scenario was studied in which mean drift forces due to waves generated by a certain wind speed were employed, but the wind effect itself was not included. Figure 5 shows the equilibrium points for the system in such situation. The increase in the range of parameters for which there are more than two equilibria is clearly seen. This can be understood as being caused by slower changes in the forces produced by wave drift alone as parameters change when compared to the changes produced by the combined action of wind and waves.
This scenario has also revealed (or rather, brought into the range investigated here) the existence of a codimension-2 cusp-like bifurcation represented by the simultaneous occurrence of two saddle-node bifurcations. This bifurcation, more closely inspected in Figs. 5(c), helps clarifying the dynamic mechanism responsible for the reduction in the number of static solutions observed for higher values of wind speed; it is after the occurrence of this bifurcation that the solution branches in the upper right corner of Fig. 5(c) move away to the right (see Fig. 5(b)) and eventually leave the range of wind speeds of interest.

![Figure 5 - Bifurcation diagrams with mean drift wave forces: zero-wind scenario, $V_c = 1 m/s$, 40% draft: (a) $\psi_w = 10^\circ$, (b) $\psi_w = 20^\circ$, (c) $\psi_w = 13.708^\circ$]

![Figure 6 - Bifurcation diagrams with mean drift wave forces, $V_c = 0.5 m/s$, 40% draft: (a) $\psi_w = 10^\circ$, (b) $\psi_w = 20^\circ$]

**The Effect of Current Speed**

The range of engineering relevant current speeds is not wide. For that reason, the present analysis is limited to a single extra current speed ($V_c = 0.5 m/s$) in addition to the value used for the previous studies ($V_c = 1.0 m/s$). Results are displayed in Fig. 6, and show that for this lower value of current speed complex behavior is predictably brought to lower values of wind speed, when compared to Fig. 3. Also, although the wind speed range of complex response is now slightly narrower, a
rather large number of equilibrium solutions can be seen to occur within it. Lastly, the range of angles of incidence for complex behavior is also slightly decreased for low current strength. Because the wind and wave forces are unaltered, the reduction of current strength decreases the range in which balance between them occurs. This fact justifies the reduction in complexity observed in these situations.

**The Effect of the Shuttle Vessel’s Draft**

The effect of different loading conditions of the shuttle vessel was assessed here by comparing the results of Fig. 3 with the diagrams obtained in a full loading condition, see Fig. 7. While the full draft condition of the vessel does not have a pronounced influence on the magnitude of wave forces, it alters substantially the wind-current balance, decreasing the effect of the former and increasing that of the latter. It is therefore predictable that the range of complex response is shifted towards higher values of wind speed, as shown in Fig. 7.

![Figure 7 - Bifurcation diagrams with mean drift wave forces, $V_c = 1 m/s$, 40% draft: (a) $\psi_w = 10^\circ$, (b) $\psi_w = 20^\circ$](image)

The degree of complexity observed is similar to that seen for the 40% draft condition (see Fig. 3), but the range of complex response – both in terms of wind speed and of wind/wave angles of incidence – is actually larger for the full draft condition. This increase in the range of complex behavior can be seen as derived from a lower sensitivity of the system to changes in wind strength and direction when operating with full draft.

**Stability and Engineering Assessment of Dynamical Behavior**

The results presented so far concentrate on the equilibrium positions of the shuttle vessel attached to the FPSO. Although the equilibrium positions for a DICAS FPSO alone are unique and tend to be stable (limit cycles are a relatively rare occurrence), a stability analysis of the system in tandem must, to be rigorous, include the effect of the FPSO. Such study can also be complemented by an engineering assessment of steady-state responses. In the present work, such assessment included the effect of time-varying slow drift wave forces. The stability of the fixed points encountered for each configuration of system and environmental parameters was tested by numerically integrating equations of motion from a suitable vicinity of the point. As an illustration of the main results of such analysis, Fig. 8 shows the same diagram of Fig. 3 (mean drift forces only), now complemented with marker points corresponding to notable changes in stability and/or engineering suitability. The engineering evaluation of responses was made by considering the actual relative positioning of the vessels for each equilibrium or periodic response (see also Morishita, Souza Junior and Fernandes, 2001).

![Figure 8 - Stability and engineering assessment of responses with mean drift wave forces, $V_c = 1 m/s$, 40% draft: (a) $\psi_w = 10^\circ$](image)

The scenario represented by this picture is representative of the complex dynamical behavior the system is capable of, and can be summarized as follows. The upper branch of the diagram (A-E) consists mainly of unstable equilibria, with the exception of a short length (B-C) of stable fixed points, which are, however, undesirable from an engineering viewpoint. At C a supercritical Hopf bifurcation gives rise to a stable limit cycle (periodic response), also undesirable, that dies out roughly at D. The lower branch (F-I) starts with stable, acceptable limit cycles for low wind and wave forces (F-G). After a supercritical Hopf bifurcation at G, an attracting equilibrium results that will persist for all higher values of wind and wave forces (G-I). However, as those forces become stronger, the shuttle vessel is pushed towards the main FPSO until, approximately at H, the stable solution is no longer acceptable.
Broadly speaking, when wind and waves oppose currents, acceptable behavior results for low to moderate wind and wave strengths (F–H). Within this range, the desirable solution will be the only attracting response, with the exception of a relatively short range (say, from 8 to 9 m/s) in which an unacceptable attracting limit cycle coexists with the desirable fixed point. For strong wind and wave forces no acceptable solutions will exist.

The Effect of Slow Drift Wave Forces

The complete mathematical model for this study includes the effect of slow drift wave forces. From a dynamical point of view, the inclusion of a periodic term such as the slow drift force will generally preclude the existence of static equilibria. The amplitude of time varying, slow drift forces will, however, depend on the sea state, which, according to the model employed here, is directly related to wind speed. Consequently, there can be responses obtained with the full model that depart only slightly from those shown in the previous pictures. This should be particularly the case when there is a strongly attracting underlying equilibrium.

Figure 9 illustrates the behavior of the system with the inclusion of a slow drift wave force component. The remaining parameters for this diagram are the same as for Fig. 8. The main features of this diagram can be summarized as follows. The upper branch (A–D) consists largely of unstable responses, with the exception of a short range (B–C) where a stable but undesirable periodic response occurs. The lower branch is entirely made of attracting periodic responses. For low wind and wave forces (E–F) the amplitude of oscillations is relatively large but still acceptable, decreasing in amplitude towards F. Between F and G the response is almost indiscernible from the underlying point attractor. After G the amplitudes of response grow and the horizontal position of the shuttle vessel also changes as stronger wind and wave forces push the vessel towards the FPSO, until roughly after H the responses are no longer acceptable.

\[ F_{ci}(\beta, V) = \frac{1}{2} \rho TL^2 c_{i\omega}(\beta) \left| V_c \right|^2, i = 1, 2, 6, \]

\[ p = 1 \text{ for } i = 1, 2, p = 2 \text{ for } i = 6 \]

where the hydrodynamic coefficients are given by:

\[ C_{i\omega}(\beta) = \left[ \frac{0.09375}{\log(Re) - 2} \frac{S}{TL} \right] \cos(\beta) \]

\[ + \frac{\pi T}{8L} \cos(3\beta) \cos(\beta) \]

\[ C_{i\omega}(\beta) = \left[ C_T - \frac{\pi T}{2L} \sin(\beta) \sin(\beta) \right] - \frac{\pi T}{2L} \sin(\beta) \]

\[ + \frac{\pi T}{L} \left[ 1 + 0.4 \frac{C_B}{T} \right] \sin(\beta) \cos(\beta) \]

\[ C_{i\omega}(\beta) = \frac{\pi T}{L} \left[ \frac{1}{2} \left( \frac{1}{2} \frac{\pi T}{2L} \sin(\beta) \sin(\beta) \right) - \frac{\pi T}{2L} \sin(\beta) \cos(\beta) \right] \]

where \( B \) and \( T \) the breadth and draft of the ship respectively; \( C_B \) is the block coefficient; \( C_T \) is the lateral force coefficient in transversally steady current; \( Re \) is the Reynold’s number (based on the length \( L \)); \( l_g \) measures the longitudinal distance between the hull’s centre of mass and the midship section; \( \beta \) is the angle of attack defined as:

\[ \beta = \arctan(\frac{u - u_s}{v - v_c}) \]
Damping Due to Yaw

The damping due to yaw is also calculated based on low aspect ratio wing theory and is given by:

\[ X_D = -\frac{1}{4} \rho \pi T^2 L v_T - \frac{1}{16} \rho \pi T^2 L^2 \left[ \frac{u_r}{u_r} \right] r^2 \] (14)

\[ Y_D = \frac{1}{2} \rho T L^2 C_{D,2} u_r - 0.035 \rho T L^2 v_r \] (15)

\[ -0.007 \rho T L^2 \left[ \frac{v_r}{v_r} \right] r \]

\[ N_D = -\frac{1}{2} \rho T L^2 C_{D,6} \left[ \frac{v_r}{v_r} \right] r - \frac{3}{20} \rho T L^2 C_{Y} \left[ v_r \right] r \]

\[ \frac{1}{32} \rho T L^2 C_{Y} \left[ v_r \right] r \]

\[ u_r = u - u_c \] (17)

\[ v_r = v - v_c \] (18)

\[ C_{D,2} = \frac{\pi T}{2L} \left( 1 - 4.4 \frac{B}{L} + 0.16 \frac{B}{T} \right) \] (19)

\[ C_{D,6} = \frac{\pi T}{4L} \left( 1 + 0.16 \frac{B}{T} - 2.2 \frac{B}{L} \right) \] (20)

Wind

The wind forces are determined by the following equations:

\[ F_w = \frac{1}{2} C_{c_w} (\psi_w) \rho_w V_w^2 A L B \] (21)

\[ F_p = 0 \] for \( i = 1,2, \) \( p = 1 \) for \( i = 6 \)

\[ \psi_{w} = \psi - \psi \] (22)

where the \( C_{c_w} \) are coefficients determined experimentally; \( V_w \) is the wind speed; \( A \) is the corresponding projected area of the vessel and \( \psi \) is the direction of the wind.

Waves

The second order wave forces are resulting of the sum of mean and slow drift forces. Taking into account wave-current interaction corrections the mean drift forces can be calculated by the following equations (Aranha, 1994)

\[ F_{md_1}(\psi_r, \mu) = \int_0^{\infty} S(\omega) d_1(\psi_r, \omega) d\omega \] (23)

\[ \psi = \psi - \psi \] (24)

where \( S(\omega) \) is the sea spectrum, \( \omega \) is the frequency, \( \psi_r \) is the direction of the wave and \( d_1 \) is the mean drift force in regular incident wave taking into account the wave drift damping. Those terms can be defined shortly using matrix notation. Let \( D \) the vector of the components \( d_1 \), ie,

\[ D = \begin{bmatrix} d_1 & d_2 & d_3 \end{bmatrix} \] (25)

The vector \( D \) is calculated as:

\[ D = D^0 + B \left[ u_c - u \ v_c - v \ r \right]^T \] (26)

\[ B = \begin{bmatrix} 3x2 & B_{3x1} \end{bmatrix} \] (27)

The elements \( b_{i,j} \) are calculated using the following equation:

\[ b_{i,j} = \begin{bmatrix} b_{w1} & b_{w2} \\ b_{w3} & b_{w4} \end{bmatrix} \] (28)

\[ b_{i,j} = \begin{bmatrix} b_{w1} & b_{w2} \cos(\psi) \sin(\psi) \\ b_{w3} & b_{w4} \sin(\psi) \cos(\psi) \end{bmatrix} \]

The elements \( b_{i,j} \) are calculated using the following equation:

\[ b_{i,j} = \begin{bmatrix} b_{w1} \sin(\psi) & b_{w2} \sin(\psi) \end{bmatrix} \] (29)

where

\[ D^0 = \begin{bmatrix} d_1^0 & d_2^0 & d_3^0 \end{bmatrix} \] is the mean wave force in regular incident waves;

\[ b_{w1} = 4 d_1(\omega, \psi_r) + \omega \frac{\partial d_1(\omega, \psi_r)}{\partial \omega} \]

\[ b_{w2} = -2 \frac{\partial d_1(\omega, \psi_r)}{\partial \omega} \]

The slow drift forces are determined as time series from their spectra. Aranha and Fernandes (1995) have shown that these spectra correspond to white noise for low frequency and can be obtained as:

\[ S_{w_2}(\omega, \mu) = 8 \int_0^{\infty} S^2(\omega) d_1^0(\omega, \psi_r) d\omega \] (27)

where \( \mu \) is the difference between two wave frequencies.

The Pierson-Moskowitz spectrum was taking into account in order to calculate the second order wave forces. It requires the significant height and a mean period. The former one can be obtained from the wind speed as (Fossen, 1994):

\[ H_s = 0.21 V_w^2 / g \] (28)

The mean period is calculated from the following relationship:

\[ H_s(2\pi)^2 = 0.24 \] (29)

where \( g \) is the acceleration of the gravity.

Mooring Lines and Hawser

The forces due to mooring lines and hawser are modeled considering conventional catenary equations.
APPENDIX B

This Appendix contains the main parameters defining the vessels employed in this study.

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<th>PARAMETERS</th>
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<th>SHUTTLE VESSEL</th>
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</table>

The coefficients $d_i$ were calculated running WAMIT software and were supplied by Petrobras.

ACKNOWLEDGEMENTS

The authors wish to acknowledge the support given to this work by FAPESP, FINEP/RECOPE, and Petrobras.

REFERENCES


