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SYSTEMATIC INVESTIGATION OF THE DYNAMICS OF A TURRET FPSO UNIT IN SINGLE AND TANDEM CONFIGURATION

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ABSTRACT

This work considers nonlinear dynamical aspects of a turret FPSO unit plus shuttle vessel. Firstly, the dynamics of the main FPSO unit alone is investigated under the combined action of wind and current. Several environmental conditions as defined by a systematic variation of relative angles and speeds are considered. The effect of the longitudinal position of the turret is also assessed. The relative importance of wind and current forces also depends on the draft of the vessel, and therefore the analyses are performed for two load conditions. As a first step in characterizing the most relevant aspects of the problem, static equilibrium solutions are calculated. Next, their stability properties are studied. The system displays a variety of different regimes of solutions in which both their number and their stability may change as one or more parameters are varied. The mathematical model of the forces due to the hydrodynamic action of currents is based on a theory of low aspect ratio wings that includes experimentally verified heuristic terms. Wind action is modeled by drag forces using experimental coefficients. The tensions in the mooring lines are approximated by linear spring formulae. Due to the great complexity of the mathematical models involved, solutions are obtained numerically, and their stability is studied via time-domain simulations. Results are summarized in a series of bifurcation diagrams covering the influence of all relevant parameters involved, namely, wind to current speed ratio, wind and current relative angles of incidence, position of the turret, and vessel draft.

A potentially more complex situation arises during the offloading operation when a shuttle vessel is attached to the FPSO unit through a hawser. The two vessels form a coupled system for which new equilibrium positions exist with their own stability properties. The shuttle vessel is shown to exhibit at least two equilibrium solutions for the whole range of parameters under investigation. For each equilibrium solution of the shuttle vessel the FPSO unit can have two or more equilibrium positions. The study of the FPSO-shuttle vessel tandem configuration can therefore be quite complex. In this work preliminary results about the dynamics of the two-body floating system are discussed.

NOMENCLATURE

A_L Wind lateral area of the vessel.
C_{ic}, $C_{iw}, i = 1,2,6$ Current and wind coefficients, see the Appendix.
I_z Moment of inertia about the GZ axis.
m Mass of the vehicle.
$m_{i,j}, i, j = 1,2,6$ Added mass in surge, sway and yaw, respectively.
u, v, r Surge, sway, and yaw velocities of the vehicle, respec-

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tively.

u_c, v_c Current speeds relative to GX and GY directions, respectively.

V_c, ψ_c Velocity and direction of the current, respectively.

V_w, ψ_w Velocity and direction of the wind, respectively.

X, Y, N Total external forces and moments in surge, sway and yaw directions, respectively.

x_g Co-ordinate of the vessel's center of gravity along the GX axis.

$\dot{x}$ Time derivative of x.

$\dot{x}_0$, $\dot{y}_0$ Components of the vessel's speed in the OX and OY axes, respectively.

α_T Turret longitudinal position ($\alpha_T = 0$ at midship, $\alpha_T = 0.5$ at the bow).

ψ_1, ψ_2 FPSO and shuttle vessel heading, respectively.

ρ, ρ_a Mass densities of water and air, respectively.

σ Wind to current speed ratio: $\sigma = V_w/V_c$.

INTRODUCTION

The dynamic behavior of Floating Production Storage and Offloading (FPSO) systems is an important aspect of their design, and has been the subject of numerous recent studies, (Bernitsas et al., 1999), (Morishita and Cornet, 1998), (Morishita and Souza Junior, 2001), (Souza Junior et al., 2000). As in any engineering problem, there are conflicting features of the system that have to be balanced. For instance, layout and pitch motion considerations might suggest that the turret should be positioned near midship, but such location is undesirable from the point of view of the dynamics in the horizontal plane. From the latter standpoint the turret would be best located as far ahead as possible. Similar compromises may have to be reached involving other parameters such as hawser length and mooring system stiffness. In other cases, as with the draft of the vessels, and predominant directions and intensities of environmental agents, variations are inevitable and their consequences must be properly assessed within reasonable ranges. In the present work a systematic investigation of the influence of some of these parameters on the static and dynamic behavior of the system is performed.

Even for a single-vessel problem in which sea current alone is considered, its complex hydrodynamic action upon the ship can give rise to unexpected features such as bifurcations of equilibria as the position of the turret is changed, (Leite et al., 1998). Wind forces can also play an important role in determining the behavior of the ship, and their inclusion brings additional complexity to the problem. The vessel's draft affects wind action markedly: for full draft wind has a stabilizing ('self-aligning') effect, whereas for partial draft such effect can be destabilizing. When both wind and current forces are present the draft of the vessel also acts changing the relative strength of wind to current forces and moments. The interplay of wind and current actions will be shown here to produce situations in which the system can exhibit several equilibrium positions. The number of possible equilibria is further increased when considering the shuttle vessel in tandem with the FPSO. An example is shown in this paper where the coupled system displays twelve equilibrium positions.

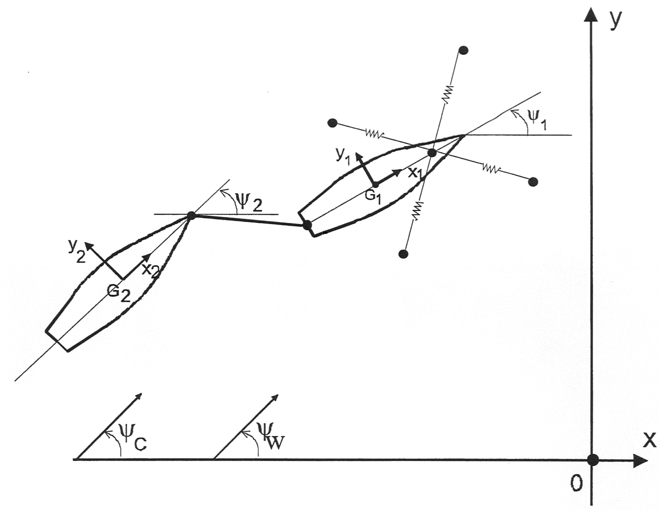

Figure 1. BODY-FIXED AND EARTH-FIXED CO-ORDINATE SYSTEMS.

MATHEMATICAL MODEL

Motions of the vessels in the horizontal plane are expressed in three orthogonal co-ordinate reference systems as shown in Figure 1. The first system, OXYZ, is earth-fixed; the second and third ones, G1XYZ and G2XYZ, are body-fixed in the center of gravity of the FPSO and shuttle ship, respectively. The axes of each body-fixed co-ordinate system coincide with the principal axes of inertia of the vessel. Based on these assumptions, the low frequency horizontal motions of each vessel are given by:

$$(m-m_{11})\dot{u} = (m-m_{22})vr - (mx_g - m_{26})r^2 - (m_{11}-m_{12})v_c r + X \quad (1)$$

$$(m-m_{22})\dot{v} = (m_{11}-m)ur - (mx_g - m_{26})\dot{r} - (m_{11}-m_{12})u_c r + Y \quad (2)$$

$$(I_z - m_{66})\dot{r} = -(mx_g - m_{26})(\dot{v} + ru) + N \quad (3)$$

The position and heading of each vessel related to the earth-fixed co-ordinate system are obtained from the following equations:

$$\dot{x}_0 = u\cos\psi - v\sin\psi \tag{4}$$

$$\dot{y}_0 = u\sin\psi + v\cos\psi \tag{5}$$

$$\dot{\psi} = r \tag{6}$$

The components u_c and v_c of the current are calculated as:

$$u_c = V_c\cos(\psi_c - \psi) \tag{7}$$

$$v_c = V_c\sin(\psi_c - \psi) \tag{8}$$

The forces X and Y, and the moment N appearing in (1)-(3) are considered in this paper as due to the action of current, wind, hawser, yaw hydrodynamic damping and, in the case of the FPSO, mooring lines. Forces due to current are determined through a heuristic model based on a low aspect ratio wing theory with experimental validation (Leite et al., 1998) and the wind forces are calculated employing aerodynamic drag expressions with experimental coefficients recently measured in reduced scale tests of an FPSO (Leite and Umeda, 2001). The forces produced by mooring lines and the hawser are calculated from spring formulae.

Details of the mathematical models employed for the determination of external forces X, Y, and N are displayed in the Appendix.

EQUILIBRIUM SOLUTIONS AND THEIR STABILITY

An essential step to develop a comprehensive view of the dynamics of this system is the investigation of its equilibrium solutions. These not only constitute a first logical step for the study, but in this case include most of the desirable operating conditions for the system, since other solutions (periodic and non-periodic), although possibly acceptable, are in principle less satisfying.

The analysis here will concentrate on the fixed points of the single-vessel (FPSO only) problem. A detailed investigation of fixed points for the shuttle vessel has been performed for a DICAS setting, and reported elsewhere (Morishita and Souza Junior, 2001), (Morishita et al., 2001). It turns out that the equilibria of the shuttle vessel can be calculated without taking the FPSO into account. For each shuttle vessel equilibrium condition thus determined the corresponding equilibrium position(s) for the FPSO can then be calculated. Of course, the results of the latter step will depend on the FPSO mooring system, and that is where the DICAS and turret configurations will differ. In particular, it can be anticipated that a turret system will display a larger number of equilibrium solutions due to its inherently greater freedom of movement. An example calculation is presented for the system here under study (see below).

Turret FPSO Configuration

Equilibrium (or static) solutions are linear and angular displacements of the vessel(s) such that the resultant forces and moments are null. Even in a single-vessel scenario, the complexity of the governing equations precludes the analytical determination of fixed points, and numerical schemes must be used. It can be shown that the equilibrium headings of the turret FPSO can be determined by combining the sway and yaw static equations which, when taking into account the current and wind force and moment expressions shown in the Appendix, yield:

$$\frac{\rho_a A_L}{\rho T L}\sigma^2 \quad [C_{6w}(\psi_w - \psi_1) - \tfrac{1}{2}C_{2w}(\psi_w - \psi_1)\alpha_T] \\ + C_{6c}(\psi_c - \psi_1) - \tfrac{1}{2}C_{2c}(\psi_c - \psi_1)\alpha_T = 0 \tag{9}$$

Equation (9) shows that, for a given FPSO, once ψ_c and ψ_w are chosen the equilibrium headings ψ_1^* depend **only** on the ratio of wind to current speed σ. Moreover, if one of the environmental forces is not taken into account, for example $V_c = 0$ (respectively, $V_w = 0$), the equilibrium headings of the FPSO depend only on the aerodynamical (respectively, hydrodynamical) characteristics of the ship in the sway and yaw directions. They will **not** depend, for instance, on the absolute magnitude of the wind or current speeds. Of course, in a preliminary design stage the position of the turret is still a parameter under investigation, and it actually has considerable influence upon the number and features of equilibria. Taking the full draft condition as an example, it has been found that under current alone four equilibrium positions exist (two stable, two unstable) for $0.0 < \alpha_T < 0.35$, whereas two solutions (one stable, one unstable) exist for $0.35 < \alpha_T < 0.5$. Similarly, the full draft condition under wind alone displays four equilibrium positions (two stable, two unstable) for $0.0 < \alpha_T < 0.17$, and two solutions (one stable, one unstable) for $0.17 < \alpha_T < 0.5$.

When considering the broader scenario of simultaneous action of wind and current careful judgement has to be exercised to limit computations to an acceptable level. Equation (9) has to be solved numerically, and the number of possible combinations of system parameters can be quite large. A significant

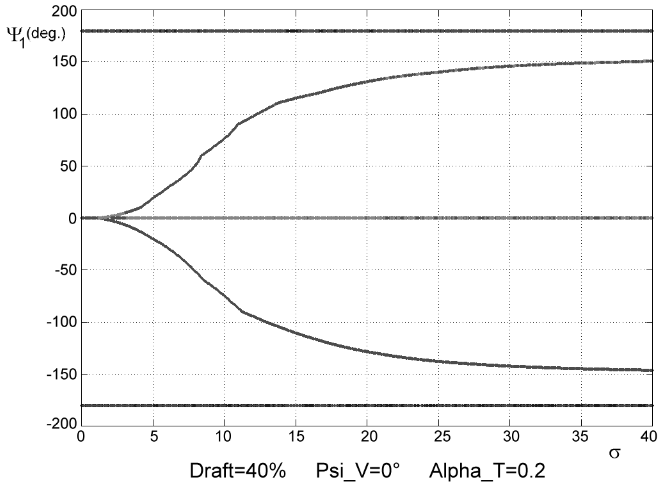

Figure 2. BIFURCATION OF EQUILIBRIA FOR FPSO SYSTEM.

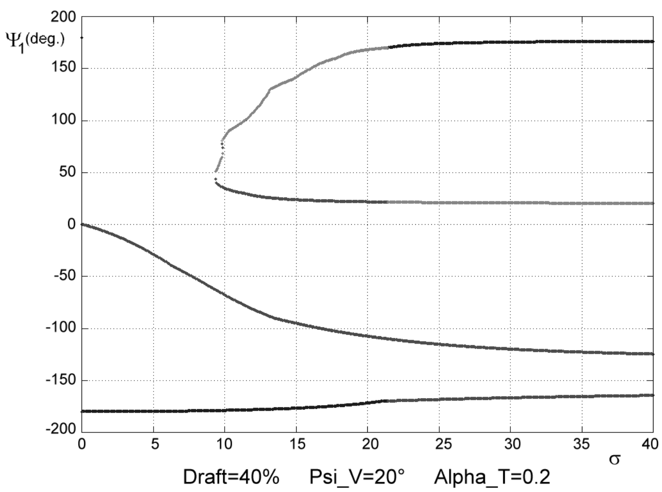

Figure 3. BIFURCATION OF EQUILIBRIA FOR FPSO SYSTEM.

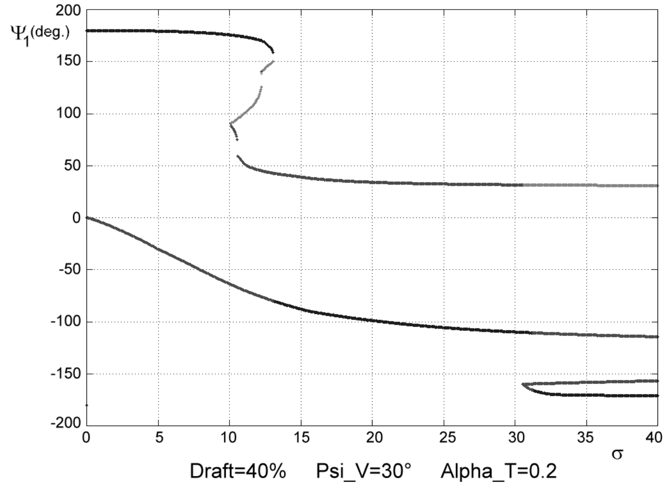

Figure 4. BIFURCATION OF EQUILIBRIA FOR FPSO SYSTEM.

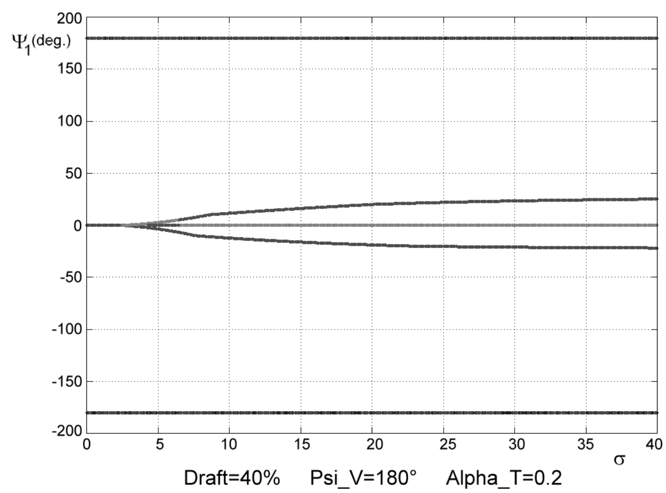

Figure 5. BIFURCATION OF EQUILIBRIA FOR FPSO SYSTEM.

simplification is achieved observing that, as the turret FPSO can turn freely around a vertical axis, a typical portrait of the system can be obtained by fixing the angle of incidence and the speed of the current and varying the speed and angle of incidence of the wind. This approach is reasonable since current direction changes slowly when compared to wind characteristics. In this paper the angle of incidence of the current was set to $\psi_c = \pi$, and the speed of current was fixed at $V_c = 1m/s$. The FPSO is a 330,000 ton dwt tanker. A more complete list of the parameters of the system considered in this paper is shown in the Appendix.

Figures (2)-(17) depict selected results of a systematic study for the FPSO-only system under various conditions. Thus, Figures (2) to (9) refer to the 40% draft condition, and Figures (10) to (17) are for the 100% draft condition. In each set of eight figures the first four relate to $\alpha_T = 0.2$ while the remaining four are for $\alpha_T = 0.3$. Finally, each set of four figures sweep a range of angles of incidence of the wind given by $\psi_V = 0^o$, $\psi_V = 20^o$, $\psi_V = 30^o$, and $\psi_V = 180^o$.

The main conclusions to be drawn from the inspection of these bifurcation diagrams can be summarized as follows:

The number and location of equilibrium positions can fol-

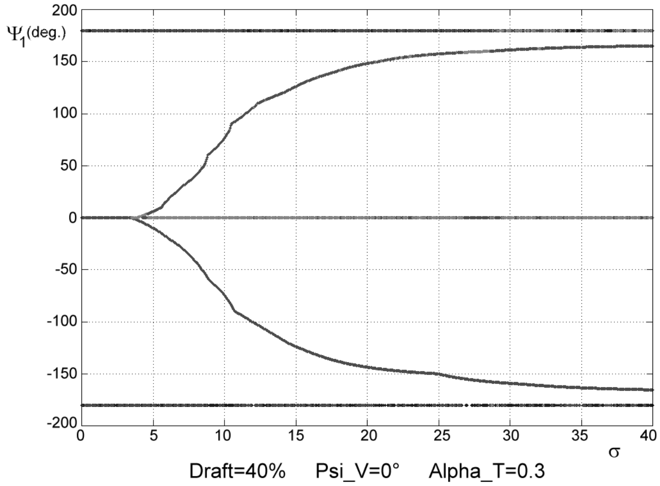

Figure 6. BIFURCATION OF EQUILIBRIA FOR FPSO SYSTEM.

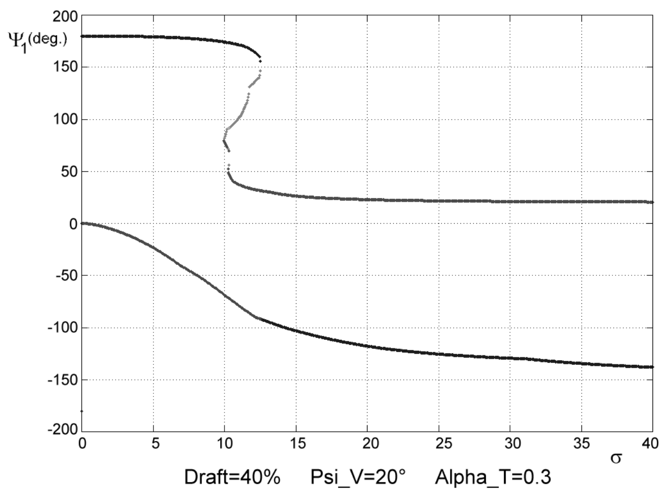

Figure 7. BIFURCATION OF EQUILIBRIA FOR FPSO SYSTEM.

low intricate patterns. Bifurcations of equilibria (especially saddle-node bifurcations) can occur, giving rise to several co-existing fixed points whose position and stability have to be determined numerically.

Most of the complexity observed can be attributed to an interplay of wind and current forces and moments (see comments above about the wind-only and current-only cases). Therefore, the more complicated scenarios happen in situations where the influence of those two agents is approximately balanced. For the system under study this corresponds to winds of moderate to strong intensity (say, $5 <$

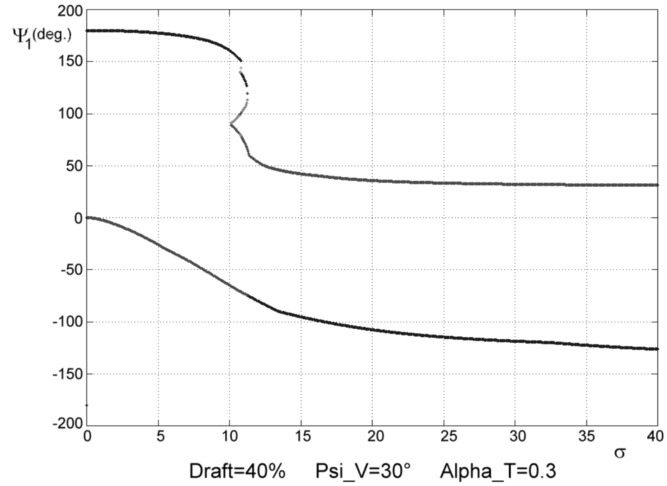

Figure 8. BIFURCATION OF EQUILIBRIA FOR FPSO SYSTEM.

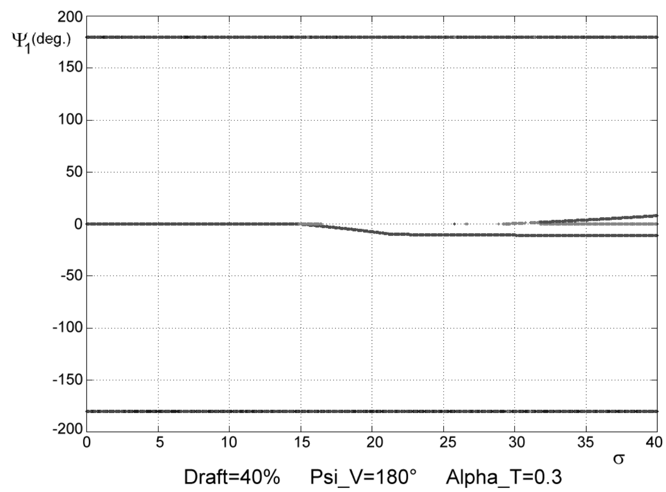

Figure 9. BIFURCATION OF EQUILIBRIA FOR FPSO SYSTEM.

$\sigma < 15$), opposing the current with a small (but non-zero) angle (say, $20^o < \psi_V < 30^o$), partial draft, and with the turret located fairly near midship (say, α_T up to 0.3). Representative examples of complex behavior can be seen in Figures 3 and 4.

The bifurcation diagram for α_T (see Figure 18) displays similar structure with fold bifurcations (saddle-nodes). Again, bifurcational structure is more complex for partial draft.

As a general rule, the complexity of behavior tends to disappear as α_T moves towards the bow, particularly for $\alpha_T > 0.4$.

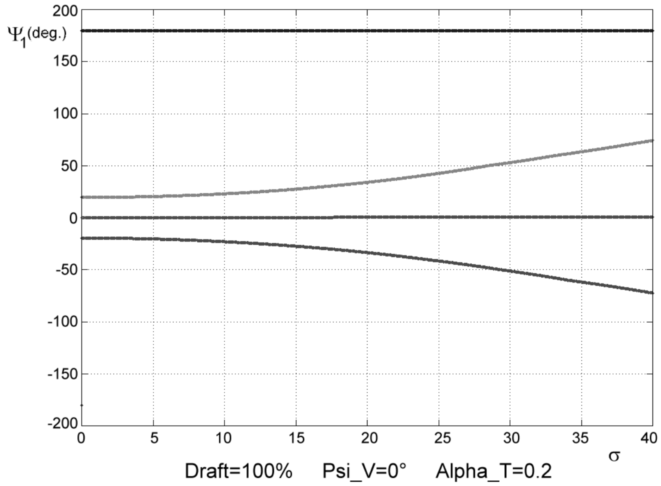

Figure 10. BIFURCATION OF EQUILIBRIA FOR FPSO SYSTEM.

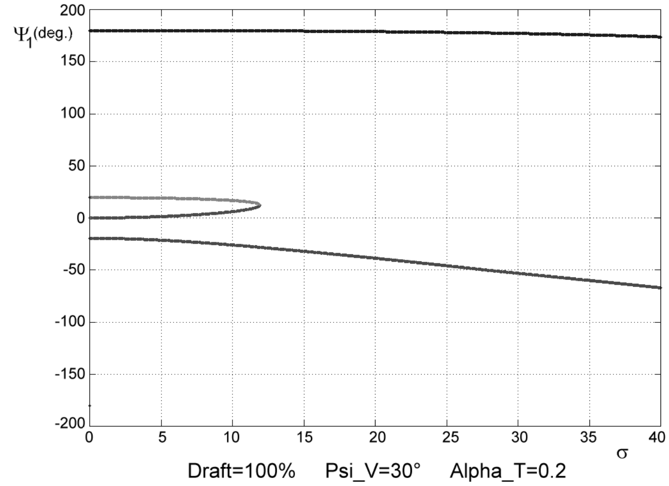

Figure 12. BIFURCATION OF EQUILIBRIA FOR FPSO SYSTEM.

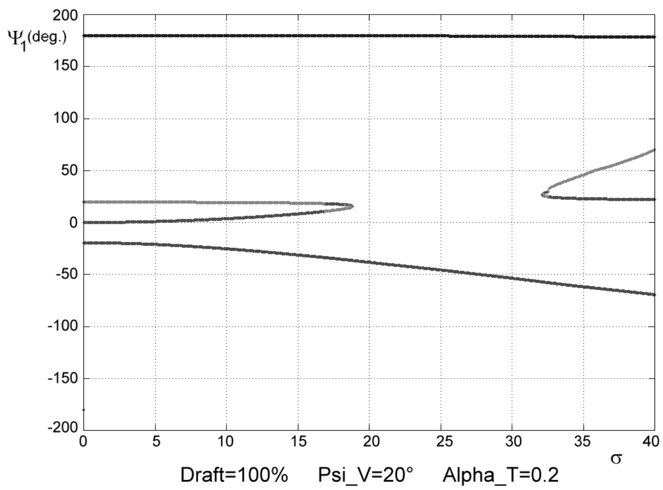

Figure 11. BIFURCATION OF EQUILIBRIA FOR FPSO SYSTEM.

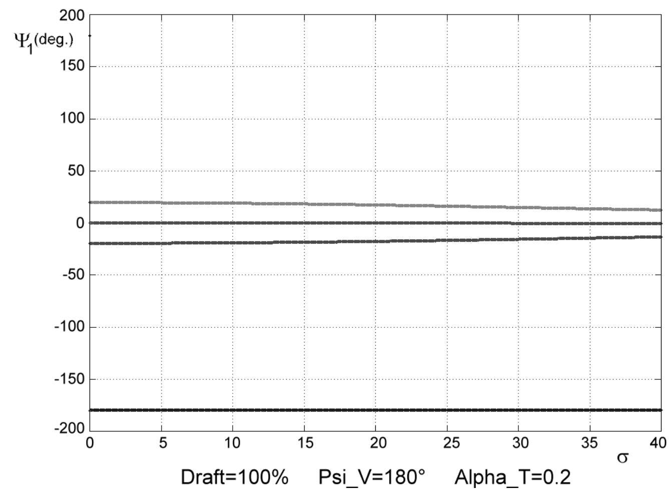

Figure 13. BIFURCATION OF EQUILIBRIA FOR FPSO SYSTEM.

Tandem Configuration

When regarding the two-ship configuration with the shuttle vessel attached to the FPSO through a hawser, it is useful to observe, as noted above, that the equilibrium heading(s) for the shuttle vessel do **not** depend on the FPSO. This is a consequence of the fact that the heading equilibrium equation for the shuttle vessel can be solved without reference to any variable from the FPSO problem. Therefore, once the equilibrium headings for the shuttle vessel are determined, the corresponding equilibrium headings (and position) for the FPSO can be calculated, from which the complete position of the shuttle can then be easily computed. The final positions must then be verified in terms of their physical feasibility (non-overlapping of hulls).

The compounded number of distinct equilibrium positions for the system in tandem can be large. Moreover, attracting periodic (stable limit cycle) solutions tend to occur for a considerable range of parameters, see (Morishita and Souza Junior, 2001) and (Morishita et al., 2001). Table 1 shows an example of the kind of results that can be obtained for the two-ship problem.

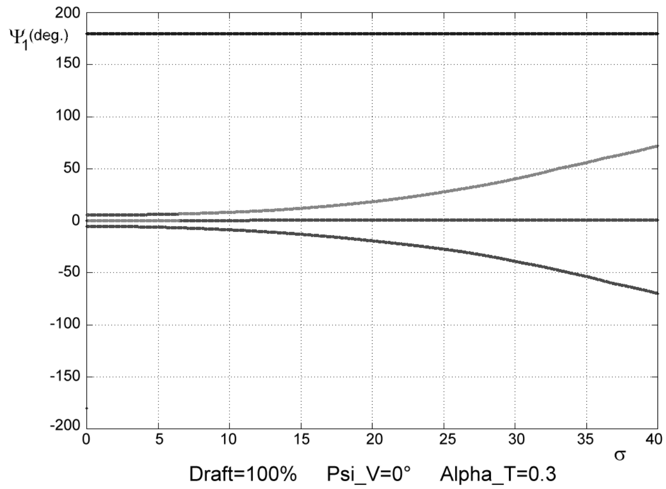

Figure 14. BIFURCATION OF EQUILIBRIA FOR FPSO SYSTEM.

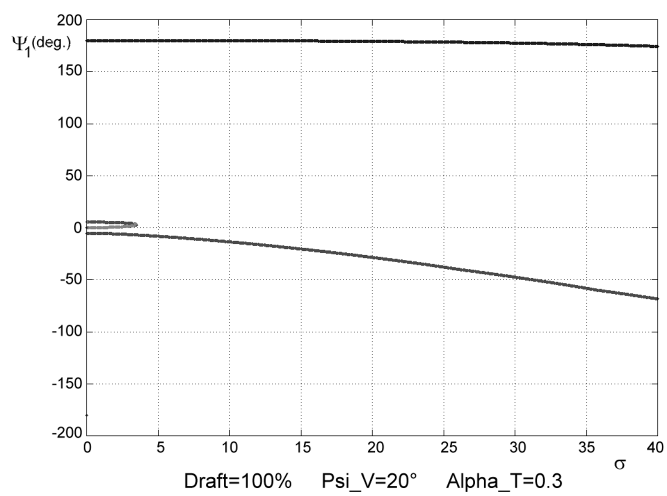

Figure 15. BIFURCATION OF EQUILIBRIA FOR FPSO SYSTEM.

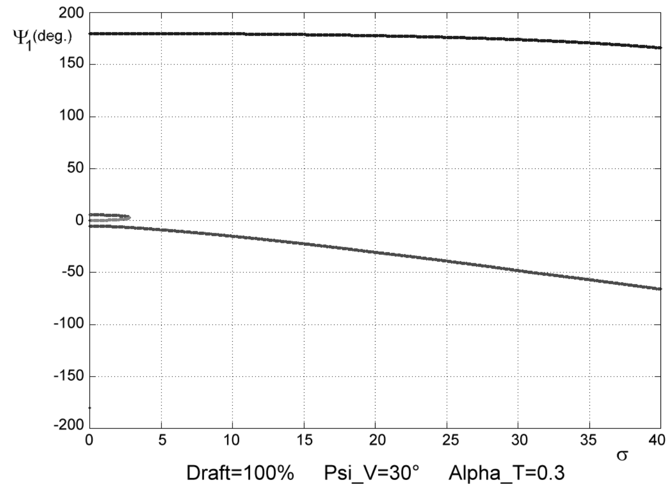

Figure 16. BIFURCATION OF EQUILIBRIA FOR FPSO SYSTEM.

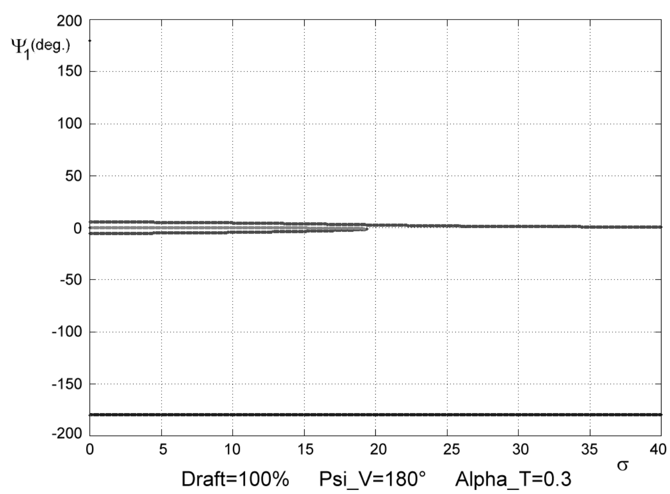

Figure 17. BIFURCATION OF EQUILIBRIA FOR FPSO SYSTEM.

CONCLUSION

A systematic investigation of the dynamical behavior of a turret FPSO system in single-ship and tandem configurations was performed. Wind and current forces were considered, and a number of other relevant parameters, such as vessel's drafts, relative intensities and angles of incidence of wind and current, and turret position were assessed. Emphasis was given to the equilibrium positions of the vessels, and it has been shown that the system can display bifurcations of equilibria leading to intricate scenarios where several fixed points co-exist. Similar bifurcations have been shown to occur for wind intensity and turret position slow variations.

It should be stressed that from a design or operational point of view the complexity uncovered here can be problematic. Situations can be envisaged where an apparently well-behaved system will, after a change in, say, wind direction, suddenly display new undesirable attracting equilibria (for example, involving collision of the vessels). Even if vessels are not led to collide with each other, the fact that the system may possess more than one attracting equilibrium is uncomfortable: in practice the vessels would tend to oscillate between stable equilibria, giving the impression of unstable behavior. The detailed numerical results

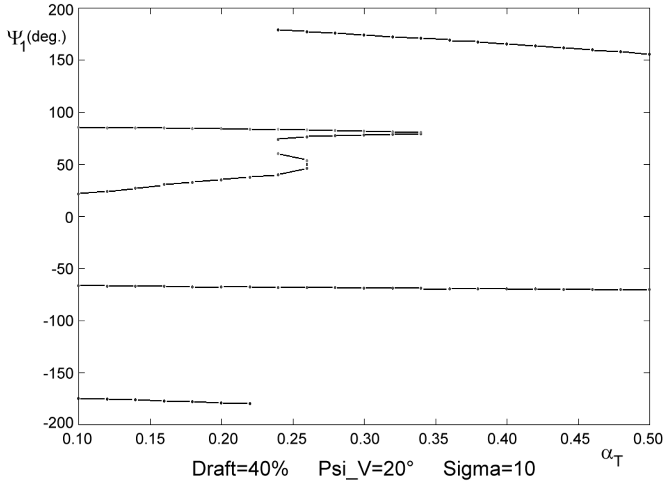

Figure 18. BIFURCATION OF EQUILIBRIA FOR FPSO SYSTEM.

Table 1. EXAMPLE OF EQUILIBRIUM POSITIONS FOR THE SYSTEM IN TANDEM: FPSO DRAFT = 100%, SHUTTLE VESSEL DRAFT = 40%, $\psi_V = 20^o$, $\alpha_T = 0.2$, $\sigma = 13$.

ψ_2 (deg.)	ψ_1 (deg.)			
135.98	176.11	-42.80	—	—
-82.35	179.40	-33.19	7.76	21.25
36.82	175.46	-38.74	—	—
110.79	179.18	-34.17	9.21	20.39

shown here exemplify the kind of study that can be performed to delimit useful operational parameters in a preliminary design stage.

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Appendix: Details of the Mathematical Models

This Appendix contains the main formulae detailing the mathematical models employed throughout this study, as well as the main numerical parameters defining the vessels, Table 2.

Current

The forces and moment due to current are given by the following equations, (Leite et al., 1998):

$$F_c(\beta, V) = \frac{1}{2}\rho T L^p C_{ic}(\beta)|V_c|^2, \qquad (10)$$
$$i = 1,2,6, \quad p = 1 \quad \text{for} \quad i = 1,2, \quad p = 2 \quad \text{for} \quad i = 6$$

where the hydrodynamic coefficients are given by:

$$C_{1c}(\beta) = \left[\frac{0.09375}{(\log(Re)-2)^2}\frac{S}{TL}\right]\cos(\beta) + \frac{1}{8}\frac{\pi T}{L}\left(\cos(3\beta) - \cos(\beta)\right) \qquad (11)$$

$$C_{2c}(\beta) = \left[C_Y - \frac{\pi T}{2L}\right]\sin(\beta)|\sin(\beta)| + \frac{\pi T}{2L}\sin^3(\beta) + \frac{\pi T}{L}\left[1 + 0.4\frac{C_B B}{T}\right]\sin(\beta)|\cos(\beta)| \qquad (12)$$

$$C_{6c}(\beta) = -\frac{l_g}{L}\left[C_Y - \frac{\pi T}{2L}\right]\sin(\beta)|\sin(\beta)| - \frac{\pi T}{L}\sin(\beta)\cos(\beta)$$

$$-\left[\frac{1+|\cos(\beta)|}{2}\right]^2 \frac{\pi T}{L}\left[\frac{1}{2}-2.4\frac{T}{L}\right]\sin(\beta)|\cos(\beta)| \quad (13)$$

where B and T the breadth and draft of the ship respectively; C_B is the block coefficient; C_Y is the lateral force coefficient in transversal steady current; Re is the Reynold's number (based on the length L); l_g measures the longitudinal distance between the hull's centre of mass and the midship section; β is the angle of attack defined as:

$$\beta = \tan^{-1}\left(\frac{v-v_c}{u-u_c}\right) \quad (14)$$

Damping due to yaw

The damping due to yaw is also calculated based on low aspect ratio wing theory and is given by:

$$X_D = -\frac{1}{4}\rho\pi T^2 L v_r r - \frac{1}{16}\rho\pi T^2 L^2 \frac{u_r}{|u_r|} r^2 \quad (15)$$

$$Y_D = \tfrac{1}{2}\rho T L^2 C_{D,2} u_r r - 0.035\rho T L^2 v_r r - 0.007\rho T L^3 |r| r \quad (16)$$

$$N_D = -\tfrac{1}{2}\rho T L^3 C_{D,6}|u_r| r - \tfrac{3}{20}\rho T L^3 C_Y |v_r| r - \tfrac{1}{32}\rho T L^4 C_Y |r| r \quad (17)$$

$$u_r = u - u_c \quad (18)$$

$$v_r = v - v_c \quad (19)$$

$$C_{D,2} = \frac{\pi T}{2L}\left(1 - 4.4\frac{B}{L} + 0.16\frac{B}{T}\right) \quad (20)$$

$$C_{D,6} = \frac{\pi T}{4L}\left(1 + 0.16\frac{B}{T} - 2.2\frac{B}{L}\right) \quad (21)$$

Table 2. MAIN SYSTEM PARAMETERS.

VESSEL FEATURE	VESSELS	
	FPSO	Shuttle Vessel
Length (m)	327.0	260.0
Beam (m)	54.5	44.5
Draft (m)	21.6	6.5
Block Coefficient	0.83	0.77
Wetted Surface (m^2)	27500.0	11745.0
Mass (kg)	312.8E6	57.3E6
Moment of Inertia $(kg.m^2)$	4.12E12	5.22E11
Wind Transversal Area (m^2)	1304.0	1339.0
Wind Lateral Area (m^2)	3893.0	4819.0
HAWSER FEATURE	VALUE	
Length (m)	170.0	
EA (N)	1.0E7	
Linear Density (N/m)	60.0	

Wind

The wind forces are determined by the following equations:

$$F_w = \tfrac{1}{2}C_{iw}(\psi_{rw})\rho_w V_w^2 A L_{BP}^p, \quad (22)$$
$$i = 1,2,6, \quad p = 0 \quad \text{for} \quad i = 1,2, \quad p = 1 \quad \text{for} \quad i = 6$$

$$\psi_{rw} = \psi_w - \psi \quad (23)$$

where C_{iw} are coefficients determined experimentally, (Leite and Umeda, 2001); V_w is the wind speed; A is the corresponding projected area of the vessel, and ψ_w is the direction of the wind.

Mooring Lines and Hawser

Forces due to mooring lines and hawser are modeled considering conventional spring and catenary equations, respectively.