DYNAMIC BEHAVIOUR OF A TURRET FPSO IN SINGLE AND TANDEM CONFIGURATIONS IN REALISTIC SEA ENVIRONMENTS

Jessé R. de Souza Junior
Helio M. Morishita

Department of Naval Architecture and Ocean Engineering
University of São Paulo
Av. Prof. Mello Moraes, 2231 - Cidade Universitária
São Paulo, São Paulo, Brazil, 05508-900
Emails: jsouza@usp.br, hmmorish@usp.br

ABSTRACT
A comprehensive analysis of the dynamics of ship-type Floating, Production, Storage and Offloading systems anchored through a turret system (Turret-FPSO) is presented. Firstly, the behavior of the FPSO unit alone is examined under the combined action of wind, current and waves. Wave forces are represented by second-order effects such as slow drift forces, as well as wave-current interaction corrections to mean drift forces, namely wave drift damping. The influence of the vessel’s draft and of the longitudinal position of the turret is also assessed. The study of the dynamics of the system is performed by calculating static equilibrium solutions and evaluating their stability properties. Relevant ranges of environmental parameters such as wind and current speeds, wave spectra, and respective angles of incidence are investigated. Numerical results are obtained and shown in bifurcation diagrams depicting the evolution of steady-state responses as a function of environmental parameters. The system displays a variety of different regimes of solutions in which both their number and their stability may change as one or more parameters are varied. The stability analysis of the Turret-FPSO system is extended to include the tandem configuration with a shuttle vessel. The main features of this more intricate scenario are summarized in 3D bifurcation diagrams.

NOMENCLATURE

$A$ Wind lateral area of the vessel.
$C_{lc}, C_{iw}, i = 1, 2, 6$ Current and wind coefficients, see the Appendix.
$d_i, i = 1, 2, 6$ mean drift force in regular incident wave
$g$ acceleration of the gravity
$H_s, T_m$ significant height and mean period of the waves
$I_z$ Moment of inertia about the GZ axis.
$L$ Length between perpendicular of the vessel
$m$ Mass of the vehicle.
$m_{i,j,i,j} = 1, 2, 6$ Added mass in surge, sway and yaw, respectively.
$T$ draft of the vessel
$u, v, r$ Surge, sway, and yaw velocities of the vehicle, respectively.
$u_c, v_c$ Current speeds relative to GX and GY directions, respectively.
$V_c, \psi_c$ Velocity and direction of the current, respectively.
$V_w, \psi_w$ Velocity and direction of the wind, respectively.
$X, Y, N$ Total external forces and moments in surge, sway and yaw directions, respectively.
$x_g$ Co-ordinate of the vessel’s center of gravity along the GX axis.
$\dot{x}$ Time derivative of $x$.

* Address all correspondence to this author.
\(\dot{x}_0, \dot{y}_0\) Components of the vessel’s speed in the OX and OY axes, respectively.

\(\alpha_T\) Turret longitudinal position (\(\alpha_T = 0\) at midship, \(\alpha_T = 0.5\) at the bow).

\(\psi\) Vessel heading, \(\psi_1\) and \(\psi_2\) are the FPSO and shuttle vessel headings, respectively.

\(\psi_s\) direction of the waves

\(\rho, \rho_a\) Mass densities of water and air, respectively.

**INTRODUCTION**

The dynamic behavior of DICAS and Turret Floating Production Storage and Offloading (FPSO) systems has been investigated in several different aspects in recent years, both in single and in tandem configurations, (Bernitsas et al., 1999), (Morishita and Cornet, 1998), (Souza Junior et al., 2000), (Morishita and Souza Junior, 2001), (Morishita et al., 2001a). A number of design parameters can have marked influence upon the behavior that the vessels will exhibit when exposed to the action of wind, current, and waves. Among the most important parameters for a turret system are the longitudinal position of the turret, mooring stiffness, superstructure layout, and hawser length. Other parameters that inevitably suffer variations as a natural consequence of operational conditions (such as vessels drafts) or environmental changes (such as intensity and direction of wind, current, and waves) also play an important role in determining the relative and absolute positioning and motions of the vessels. The selection of the best parameter values in design is clearly related to the behavior they afford under critical environmental conditions. In the present work a systematic investigation of the influence of some of these parameters on the static and dynamic behavior of the system is performed.

The underlying physical phenomena that govern the stability and motions of vessels at sea can be strongly nonlinear, and even a single-vessel turret problem in which sea current alone is considered can yield complex results such as bifurcations of equilibria as the position of the turret is changed, (Leite et al., 1998). The inclusion of wind and wave forces brings additional complexity to the problem. The vessel’s draft affects wind action markedly: for full draft wind has a stabilizing (‘self-aligning’) effect, whereas for partial draft such effect can be destabilizing. The relative strength of wind to current and wave forces and moments is affected by the draft of the vessel. The interplay of wind, wave, and current actions will be shown here to produce situations in which the system can exhibit several equilibrium positions. The number of possible equilibria is further increased when considering the shuttle vessel in tandem with the FPSO.

**MATHEMATICAL MODEL**

Motions of the vessels in the horizontal plane are expressed in three orthogonal co-ordinate reference systems as shown in

![Systems of Reference](image)

Figure 1. The first system, OXYZ, is earth-fixed; the second and third ones, \(G_1x_1y_1z_1\) and \(G_2x_2y_2z_2\), are body-fixed in the center of gravity of the FPSO and shuttle ship, respectively. The axes of each body-fixed co-ordinate system coincide with the principal axes of inertia of the vessel. Based on these assumptions, the low frequency horizontal motions of each vessel are given by:

\[
(m + m_{11})\dot{u} = (m + m_{22})v r - (m x_g + m_{26})r^2 + (m_{11} - m_{12})v_r r + X
\]

\(1\)

\[
(m + m_{22})\dot{v} = -(m + m_{11})u r - (m x_g + m_{26})\dot{r} + (m_{11} - m_{12})u_r r + Y
\]

\(2\)

\[
(m + m_{66})\dot{r} = -(m x_g + m_{26})(\dot{v} + u r) + N
\]

\(3\)

The position and heading of each vessel related to the earth-fixed co-ordinate system are obtained from the following equations:

\[
\dot{x}_0 = u \cos \psi - v \sin \psi
\]

\(4\)

\[
\dot{y}_0 = u \sin \psi + v \cos \psi
\]

\(5\)

\[
\psi = r
\]

\(6\)
The components \( u_c \) and \( v_c \) of the current are calculated as:

\[
\begin{align*}
  u_c &= V_c \cos(\psi_c - \psi) \\
  v_c &= V_c \sin(\psi_c - \psi)
\end{align*}
\] (7) (8)

The forces \( X \) and \( Y \), and the moment \( N \) appearing in (1)-(3) are considered in this paper as due to the action of current, wind, hawser, yaw hydrodynamic damping and, in the case of the FPSO, mooring lines. Forces due to current are determined through a heuristic model based on a low aspect ratio wing theory with experimental validation (Leite et al., 1998) and the wind forces are calculated employing aerodynamic drag expressions with experimental coefficients recently measured in reduced scale tests of an FPSO (Leite and Umeda, 2001). Waves induce high and low frequency motions. In this paper only the latter are considered, and they are caused by the so-called slow and mean drift forces. They are calculated based on unidirectional sea spectra, and their parameters, namely, significant height and period are defined from empirical formulae based on the wind speed. Wave-current interaction corrections are also taken into account to calculate the mean drift forces. The forces produced by mooring lines and the hawser are calculated from catenary formulae.

Details of the mathematical models employed for the determination of external forces \( X \), \( Y \), and \( N \) are displayed in the Appendix.

**STATIC EQUILIBRIUM ANALYSIS FOR THE FPSO VESSEL**

A first picture of the dynamics of the system can be assembled by investigating its equilibrium solutions, which are obtained by setting all time derivative terms of the equations (1)-(3) to zero (time-varying terms related to slow drift forces are also set to zero here). The solutions are the linear and angular positions of the vessel calculated for every set of independent parameters. These sets are defined by combining angles of incidence and speed of current and wind, significant height and period of waves and draft of the ship. The number of possible combinations can be quite large even considering a specific range for each parameter. However, because of its weathervane characteristic, a typical portrait for a Turret-FPSO system can be drawn taking into account only the relative angles of incidence of current, wind and wave. Furthermore, it is reasonable to assume that waves are wind-dependent. Bearing in mind those aspects, three simplifications were adopted here: the angle of incidence of the current was set to \( \pi \), wind and wave were assumed to have the same angles of incidence, and the significant height and period of the waves were assumed to be functions of wind speed only (see the Appendix for the actual formulae employed).

The different equilibrium solutions encountered as system parameters are varied can be best depicted in bifurcation diagrams in which one system variable (is this case the heading of the FPSO vessel) is chosen to represent the equilibrium solution, and one of the system parameters (the wind speed) is systematically varied. Each bifurcation diagram thus obtained is valid for a certain combination of the remaining parameters: wind and wave angle of incidence, vessels drafts, and turret position.

**The Influence of Mean Drift Forces**

The first step in this analysis is to assess the effects that the inclusion of mean wave drift forces and moment have upon the set of equilibrium solutions displayed by the system. For that purpose, a specific configuration in which wind and waves are in close opposition to the current was chosen to exemplify the kind of overall effect observed. The reference picture here is the no-wave scenario depicted in Figure 2, which can be compared with the results shown in Figure 3. Note that, due to reduced forces in the no-wave situation when compared to the scenario that includes the effect of waves (see Figure 3), and in order to display a similar range of bifurcation phenomena, wind speeds in Figure 2 have to be extended up to \( 100m/s \). It can be seen that one effect of the action of mean drift forces is that of reducing the wind speed at which a fold (saddle-node) bifurcation occurs that doubles the number of static solutions. In fact, by reducing this wind speed from \( \sim 40m/s \) to \( \sim 12m/s \), the waves bring such complex behaviour into the range of wind speeds of usual interest for design. A second effect observed in Figures 2 and 3 is that the range of values of wind speed within which more than two equilibria are found is increased from \( \sim 40 – 52m/s \) to \( \sim 12 – 37m/s \). Numerical simulations from the vicinity of these equilibria show that for any vertical line corresponding to a fixed value of wind speed, stable and unstable solutions alternate starting from the stable lower branch shown in Figure 3.

A physical explanation for the two main effects described above can be derived from the analysis of the behavior of the curves of yaw moment applied to the vessel by wind, waves, and current as seen in Figure 4. The first fact to be noted here is that the moment curves for the wind and mean wave drift reinforce each other: for this reason the curves in Figure 4 show the net sum of wind and wave moments. Recalling that it has been assumed that wind and waves are co-linear, and that wave strength increases with wind speed, it can be envisaged that roughly the same effects produced by wind alone will be produced by the sum of wind and waves at a lower value of wind speed. The curves shown in Figure 4 also help to understand another interesting feature of the bifurcation diagrams of Figure 3: the appearance of four equilibrium solutions at a wind speed \( \sim 12m/s \).
Equilibrium solutions are associated to zeroes of the total moment curve. It can be seen in Figure 4 that the changing shapes of the total moment curve as wind speed is varied first produce only two zeroes for low wind speed. At \( \sim 12 \text{ m/s} \) two new zeroes appear until, after \( \sim 40 \text{ m/s} \), the total moment curve again crosses the horizontal axis only twice (the actual value is closer to \( 37 \text{ m/s} \)). As remarked in previous works, see for instance (Morishita et al., 2001b), (Morishita et al., 2001a), the larger number of equilibrium solutions can be broadly attributed to a balance between the effects of wind (and now waves) and current. As far as these forces are dependent on the angular position of the vessel, they can be seen as “potential” forces to which “potential wells” can be associated (equilibria are the critical points of such potentials). From this perspective, the complexity of a large number of equilibria is a consequence of the geometry of those “potential wells”, which acquire a number of weakly defined maxima and minima, in contrast to the relatively simple situation in which only one maximum and one minimum (sharply defined) exist.

Figure 5 shows the effect of a larger angle of incidence of wind and waves. A more transversal direction produces larger yaw moments, and the balance mentioned above is maintained for a narrower range of wind speeds. The resulting behavior is consequently less complex. Incidentally, the trend of reducing complexity as wind and waves do not oppose the current is a prevalent feature of this system. For this reason, the present study concentrates on the more intricate situations where wind and waves roughly oppose the current.
Figure 5. EFFECT OF WIND/WAVE ANGLE OF INCIDENCE.

The Effect of Turret Position

The results shown in Figure 4 were obtained for a turret position of 0.5L, i.e. for the turret located at the bow of the ship. The effect of locating the turret closer to midship is that of an overall reduction in the value of yaw moments. This fact can be easily understood by considering that a ship with perfect bow-stern symmetry is subjected to zero net moment when the turret is located at midship. It can therefore be expected that, although the vessel under study is not perfectly symmetric, less pronounced local maxima and minima of the potential curves associated to yaw moments will lead to more complex behavior (larger number of equilibria) for the turret located at 0.2L. This reasoning is confirmed by the curves shown in Figures 6 and 7, which reveal that four and even six equilibrium solutions can now be found for most of the range of wind speeds. In this scenario, a second type of bifurcation (a cusp) is also brought within the range of wind speeds of interest. This codimension-2 bifurcation, generic under control of two parameters such as wind speed and angle of incidence, occurs for a wind speed ~ 12 m/s at an angle of incidence between those corresponding to Figures 6 and 7.

The Zero-Wind Scenario

The line of reasoning conducted in the present analysis can be further explored by considering the case of a zero-wind situation, in which the effect of a mean drift wave force is isolated from the contribution of wind. Since the formulation employed here couples wave significant height and period to wind speed (see the Appendix), the contribution of mean drift was here calculated assuming a hypothetical wind speed whose direct influence was then not considered in the equations. The result, Figure 8, is a diagram for a mean drift and current scenario. The main feature of this diagram is to show that, because of the absence of wind forces, the onset of the wind-dominated regime (marked by the second fold bifurcation at ~ 37 m/s in Figure 3) is delayed to the extent that it falls beyond the upper limit of wind speed under study here. Consequently, the more complex scenario with four equilibria persists even for high wind speeds, producing an overall increase in complexity.

The Effect of Current Speed

The current speed used throughout this study (1 m/s) is close to the upper limit of realistic values. It is of interest to evaluate the situation found for a lower value of current speed, say 0.5 m/s. This scenario is represented by Figure 9. A reduced current speed changes the balance of forces necessary to produce several equilibria, and the result is that, in addition to moving the first fold bifurcation (that marks the end of the current-dominated scenario) to lower values of wind speed, it also nar-
rows the range of complex behavior by moving the second fold bifurcation closer to the first. The wind-dominated regime settles in rather early and the overall picture is one of reduced complexity.

The Effect of FPSO Vessel Draft

To assess the influence of the vessel draft, a partial draft (40%) condition was considered. The first aspect to be noted here is that a smaller draft for the vessel decreases the wetted surface of the ship (thereby reducing the area exposed to current and wave forces) while increasing the area subjected to wind. The second fact to be kept in mind is that, whereas the wind is “stabilizing” or “self-aligning” for full draft, it becomes “destabilizing” in partial draft. As far as the former aspect is concerned, it turns out that the net effect is similar to a reduction in current speed, as depicted in Figure 10 (compare with Figure 9). However, because the wind effect is itself destabilizing, the wind-dominated scenario may not be as simple here as it is case for full draft. This feature is perhaps more clearly seen in Figure 11, which shows the situation for $\alpha_T = 0.2$. It can be seen that, although the wind-dominated regime settles in at $\sim 7 m/s$, this regime is itself more complex, with four equilibria persisting throughout.

EQUILIBRIA OF THE SYSTEM IN TANDEM

From time to time a shuttle vessel is connected to the Turret-FPSO through a hawser to perform the offloading operations. It is therefore of interest to analyze the dynamic behaviour of the system comprised of both ships in tandem under the action of
current, wind and waves. In practice, the shuttle vessel is kept away from the FPSO by the action of its dynamic positioning system (DPS) or of a tugboat to avoid collision between the vessels. However, in this paper only forces due to environmental agents are taken into account. This approach is justified by the need to know about the uncontrolled dynamics of a system as a preliminary stage in the definition of its control strategy.

The procedure employed here to obtain the equilibrium solutions for the system in tandem is based on the fact that the equilibrium headings (not linear positions) for the shuttle vessel are not affected by the FPSO, although the reciprocal is not true. The fixed points of the complete system were obtained in two steps. Firstly, the equilibrium headings of the shuttle vessel were determined and then for every solution the equilibrium headings of the Turret-FPSO and linear positions of both ships were calculated.

It is not feasible in this paper to summarize all analyzes considering several combinations of the independent parameters, but the approach would be the same adopted in the case of the shuttle vessel alone. As an example of the kind of portrait that is produced in this study, the following condition was selected: shuttle vessel draft of 40%, FPSO vessel draft of 100%, wind angle of $10^\circ$, and current speed of 1 m/s. Results are shown in a 3-D plot in Figure 12, which has as axes the heading of the FPSO, heading of the shuttle vessel, and wind speed. The complexity of the picture can be traced to the bifurcations of the headings of both the shuttle vessel and Turret-FPSO. Figures 13 and 14 show the projections of the curves displayed in Figure 12 onto the $(\psi_2, V_w)$ and $(\psi_1, V_w)$ planes, respectively. In Figure 13 the equilibrium headings of the shuttle vessel were sorted and individually identified with a symbol (star,circle,plus sign,square). The same symbols are then used in Figure 14 to identify how many equilibrium headings of the FPSO are to be found for each equilibrium heading of the shuttle vessel. Of course, there are at least two equilibria of the FPSO for each equilibrium of the shuttle vessel. For some ranges of wind speed, however, there can be up to eight distinct equilibria of the FPSO vessel (see for instance the range between $V_w = 11.5 m/s$ and $V_w = 25.0 m/s$).

Broadly speaking the two branches displayed in bifurcation diagrams for the FPSO alone are changed into four branches with the inclusion of the shuttle vessel. Furthermore, the force applied to the FPSO by the shuttle vessel has the overall effect of reducing the range of complex behavior.

**STABILITY ANALYSIS WITH SLOW DRIFT FORCES**

All the results that include the action of waves presented so far were obtained considering only the time-independent compo-
iments of wave forces, namely mean wave drift forces and wave-current interaction terms. A realistic sea spectrum also imposes forces and moments that vary with time. In the present analysis these are represented by the so-called slow drift forces. The first aspect to be noted here is that no static equilibrium solution can now be found since the system is driven by time-varying terms.

Time domain simulations show that in general terms the system tends to oscillate around the previously stable equilibria. From an engineering point of view, the interesting issues brought by slow drift forces are related to: what are the amplitudes of oscillation?; which oscillations lead to dangerous proximity or collision of the vessels?; what are the average and oscillatory forces on the hawser and mooring lines? These issues were not considered in the present work and will be the object of further studies.

CONCLUSION

A systematic investigation of the dynamical behavior of a turret FPSO system in single-ship and tandem configurations was carried out. A realistic sea environment was considered for the studies, consisting of wind, current, and wave forces. The latter were represented mainly by mean drift forces and wave-current interaction terms. A number of relevant parameters such as vessel’s drafts, relative intensities and angles of incidence of wind/wave and current, and turret position were assessed. Equilibrium positions of the vessels were analyzed in detail, and it has been shown that the system displays bifurcations of equilibria leading to intricate scenarios where several fixed points co-exist.

In summary, the system has shown complex behavior when wind and waves are roughly in opposite direction to current and/or when the turret position is moved towards midship. The creation or extinction of new equilibria are governed by bifurcations (typically folds) that can be seen as delimiting three different regimes of response dominated by current, wind/waves or characterized by a balance between them. The latter regimes usually correspond to the more complex situations. In some cases (typically for the turret positioned near midship) the wind/waves and current dominated scenarios tend also to be complex.

The results for the tandem configuration reveal additional complexity because for each equilibrium of the shuttle vessel two or more equilibria of the FPSO vessel can exist. The further issues brought by the inclusion of slow drift forces were briefly commented.

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REFERENCES


Appendix: Details of the Mathematical Models

This Appendix contains the main formulae detailing the mathematical models employed throughout this study, as well as the main numerical parameters defining the vessels, Table 1.

Current

The forces and moment due to current are given by the following equations, (Leite et al., 1998):

\[ F_C = \rho CV^2 \frac{d}{2} \left( 1 - \frac{d}{D} \right) \text{ and } M_C = \rho CV^2 \frac{d}{2} \frac{d}{D} \text{ for a } \frac{d}{D} < 0.5 \]

\[ F_C = -\rho CV^2 \frac{d}{2} \left( 1 - \frac{d}{D} \right) \text{ and } M_C = -\rho CV^2 \frac{d}{2} \frac{d}{D} \text{ for a } \frac{d}{D} > 0.5 \]
where the hydrodynamic coefficients are given by:

\[ C_{1c}(\beta) = \frac{0.09375}{(\log(Re)-2)^2} \frac{S}{TL} \cos(\beta) + \frac{1}{8} \beta \left( \cos(3\beta) - \cos(\beta) \right) \]  
(10)

\[ C_{2c}(\beta) = \left[ C_Y - \frac{\pi T}{2} \right] \sin(\beta) \sin(\beta) + \frac{\pi T}{2} \sin^3(\beta) + \frac{\pi T}{2} \left[ 1 + 0.4 \frac{c_Y}{T} \right] \sin(\beta) \cos(\beta) \]  
(11)

\[ C_{6c}(\beta) = -\frac{\pi T}{2} \left[ C_Y - \frac{\pi T}{2} \right] \sin(\beta) \sin(\beta) - \frac{\pi T}{2} \sin(\beta) \cos(\beta) - \left[ \frac{\pi T}{2} \right] \frac{C_Y}{2} \left[ \frac{1}{2} - 2.4 \frac{T}{2} \right] \sin(\beta) \cos(\beta) \]  
(12)

where \( B \) and \( T \) the breadth and draft of the ship respectively; \( C_B \) is the block coefficient; \( C_Y \) is the lateral force coefficient in transversal steady current; \( Re \) is the Reynolds number (based on the length \( L \)); \( l_c \) measures the longitudinal distance between the hull’s centre of mass and the midship section; \( \beta \) is the angle of attack defined as:

\[ \beta = \tan^{-1} \left( \frac{v - v_c}{u - u_c} \right) \]  
(13)

Damping due to yaw

The damping due to yaw is also calculated based on low aspect ratio wing theory and is given by:

\[ X_D = -\frac{1}{4} \rho T^2 L v_r - \frac{1}{16} \rho T^2 L^2 \frac{u_r}{|u_r|} r^2 \]  
(14)

\[ Y_D = \frac{1}{2} \rho T^2 C_{D,2} u_r - 0.035 \rho T^2 v_r \]  
(15)

\[ N_D = -\frac{1}{4} \rho T^2 C_{D,6} |u_r| r - \frac{3}{2} \rho \beta T^2 C_Y |v_r| r - \frac{1}{4} \rho T^2 C_{Y,1} |v_r| r \]  
(16)

\[ u_c = u - u_c \]  
(17)

\[ v_r = v - v_c \]  
(17)

\[ C_{D,2} = \frac{\pi T}{4L} \left( 1 - 4.4 \frac{B}{L} + 0.16 \frac{B}{T} \right) \]  
(18)

\[ C_{D,6} = \frac{\pi T}{4L} \left( 1 + 0.16 \frac{B}{T} - 2.2 \frac{B}{L} \right) \]  
(19)

\[ F_w = \frac{1}{2} C_{iw} (\psi_{iw}) \rho_w V_w^2 A_{BP} \]  
(20)

Waves

The second order wave forces result from the sum of mean and slow drift forces. Taking into account wave-current interaction corrections the mean drift forces can be calculated by the following equations, (Aranha and Martins, 2001).

\[ F_{dm}(\psi_s) = 2 \int_0^\infty S(\omega) d_1 (\psi_{rs}, \omega) d\omega \]  
(22)

\[ \psi_{rs} = \psi_s - \psi \]  
(23)

\[ \psi_s = \psi_{iw} - \psi \]  
(21)
where: \( F_{dm}(\psi_i), i = 1, 2, 6 \) are the mean drift forces and moments in the surge, sway, and yaw directions, respectively; \( S(\omega) \) is the sea spectrum; \( \omega \) is the wave frequency component; \( \psi_i \) is the wave direction; the \( d_i(\psi_s, \omega), i = 1, 2, 6 \) are the mean drift force and moment components in surge, sway, and yaw, respectively, for each wave frequency component, and incorporate wave drift damping. Those terms can be defined shortly using matrix notation. Let \( D \) be the vector of components \( d_i \):

\[
D = \begin{bmatrix} d_1 & d_2 & d_6 \end{bmatrix}^T
\]  

(24)

This vector can be calculated as:

\[
D = D^0 + B \left[ u_c - u v_c - v \right]^T
\]

(25)

\[
B = \begin{bmatrix} B_{1_{3 \times 2}} & B_{2_{3 \times 1}} \end{bmatrix}
\]

(26)

The elements \( b_{i,j}, i = 1, 2, 3 \) and \( j = 1, 2 \) are defined as:

\[
B_1 = \begin{bmatrix} b_{w1} & b_{r1} \\ b_{w2} & b_{r2} \\ b_{w6} & r_{r6} \end{bmatrix} \begin{bmatrix} \cos(\psi_{rs}) & \sin(\psi_{rs}) \\ \sin(\psi_{rs}) & \cos(\psi_{rs}) \end{bmatrix}
\]

(27)

The elements \( b_{i,j}, i = 1, 2, 3 \) and \( j = 3 \) are defined as:

\[
B_2 = \begin{bmatrix} b_{3,2} \sin(\psi_{rs}) & b_{3,2} & b_{2,2} \end{bmatrix}
\]

(28)

where:

\[
D_0 = \begin{bmatrix} d_1^0 & d_2^0 & d_6^0 \end{bmatrix}
\]

is the mean wave force in regular incident waves:

\[
b_{wi} = 4 d_i(\omega, \psi_{rs}) + \omega \frac{\partial d_i(\omega, \psi_{rs})}{\partial \omega};
\]

\[
b_{ri} = -2 \frac{\partial d_i(\omega, \psi_{rs})}{\partial \omega}.
\]

The slow drift forces are determined as time series from their spectra. It been shown that these spectra correspond to white noise for low frequency and can be obtained as (Aranha and Fernandes, 1995):

\[
S_{sd_i}(\psi_{rs}, \mu) = 8 \int_0^{\infty} S^2(\omega) d_i^0(\omega, \psi_{rs}) d\omega \quad i = 1, 2, 6
\]

(29)

where \( \mu \) is the difference between two wave frequencies.

### Table 1. MAIN SYSTEM PARAMETERS.

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<th>VESSEL FEATURE</th>
<th>FPSO</th>
<th>Shuttle Vessel</th>
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<tr>
<td>Length (m)</td>
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<tr>
<td>Linear Density (N/m)</td>
<td>60.0</td>
<td></td>
</tr>
</tbody>
</table>

The Pierson-Moskowitz spectrum was used to calculate the second order wave forces. It requires the significant height and a mean period. The former can be obtained from the wind speed as (Fossen, 1994):

\[
H_s = \frac{0.21 V_w^2}{g}
\]

(30)

The mean period is calculated from the following relationship:

\[
\frac{H_s (2\pi)^2}{g T_m^2} = 0.24
\]

(31)

where \( g \) is the acceleration of the gravity.

**Mooring Lines and Hawser**

Forces due to mooring lines and hawser are modeled considering catenary equations. The mooring system for the turret is composed of six equally spaced mooring lines of identical mechanical properties.