1º COBEF
Congresso Brasileiro de Engenharia de Fabricação
02 a 04 de abril de 2001 • Curitiba - Paraná - Brasil

TEMPERATURE FIELD DETERMINATION FOR ORTHOGONAL METAL CUTTING

João Batista de Aguiar
Universidade de São Paulo, Depto de Eng. Mecatrônica e de Sistemas Mecânicos
jbaguiar@usp.br – São Paulo, SP, Brasil
Melchor R. Madrigal[1]
(1)Universidade ISPJAE, Depto de TCM. Fac. Ingenieria Mecánica
mrm@usp.br. Havana. Cuba.
Gilmar F. Batalha[2]
(2) gfbatalha@usp.br – São Paulo, SP, Brasil
Universidade de São Paulo, Depto de Eng. Mecatrônica e de Sistemas Mecânicos

Abstract. *Principle of virtual temperatures as applied to the problem of orthogonal cutting, included the dissipation terms associated with mechanical work, is used to computed the temperature field resulting from the interation between tool and workpiece. The problem is formulated, discretized and the resulting equations solved with the finite element method. As an application the temperature field is computed for the case of machining AISI 1020. The stress field is also shown in order to present the coupled problem. Material dependence with temperature, strain rate and strains is allowed in the model used. In general good agreement is obtained with results presented in the literature.*

Keywords: *Orthogonal Cutting, Temperature Field, Finite Element Method*

1. PRELIMINARIES

During orthogonal cutting of steel, high temperatures are generated in the region of the tool cutting edge. These temperatures have a controlling influence on the rate of wear of the cutting tool and on the friction between the chip and tool.

As the chip is formed, in the contact interface with the cutting tool, two regions of contact are observed. One region in a stick contact condition, where no dissipation occurs. And another complementary region, the slip contact region, where dissipation due to friction occurs. The heat generated in this process appears as a source of temperature rise for the chip and workpiece. Another source is provided by the energy of deformation. When a material is deformed to an elastoplastic condition, only a small part of the energy is stored as elastic, and therefore, recoverable, energy. The gross part of the energy is used to cause permanent deformations, and most of this energy is converted into heat.

In metal cutting the material is subjected to extremely high strains in two principal regions: The shear zone or primary deformation zone, and another zone, known as secondary deformation zone. In general, friction between tool and the new workpiece is considered an important source of friction, as well as in the contacting surfaces of the crack being opened by the cutting process.

In this work the problem of determining the temperature distribution in the orthogonal cutting is studied. Plane strain conditions are considered, and quasi-static conditions assumed, so as to disregard dynamic effects in the problem. An Updated Lagrangian formulation is considered for the coupled thermo-mechanical problem. For the thermal problem studied here, input from the mechanical problem is considered in the standard form, and the problem equated thereafter.

2. FORMULATION

2.1 Model

The chip formation model to be analysed has the geometry shown in figure 1. In it we may distinguish two regions separated by a division line AB, the cutting line, marking the positions separation is going to take place. In this line, two sets of nodes are made to coincide up to the moment of fracture and chip separation. The precise moment of separation is determined from a fracture energy criterion. Once separated, the upper region includes the chip and the lower the machined material.

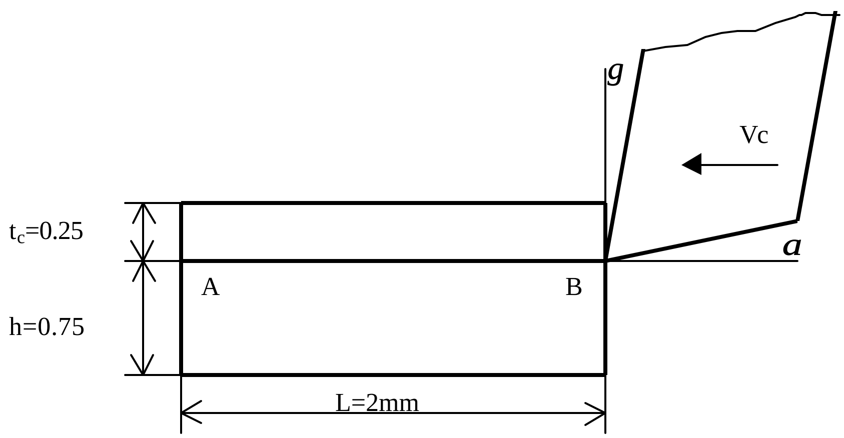


Figure 1. Ship formation model.

Proceeding with the discription of the model, the tool is supposed to be rigid, but capable of absorbing of heat and approaching the workpiece with a velocity v. Geometry of tool depends on the rake angle γ and clearance angle α.

2.2 Energy balance

The uncoupled heat transfer analysis is designed to accompany the mechanical problem, from where many of the required inputs are required. The starting point is the basic energy balance (Rebelo & Kobayashi, 1980a):

$$\int_{V_{t+\partial t}} \rho \dot{U} dV = \int_{V_{t+\partial t}} \eta \dot{W}_p dV + \int_{S^{csl}_{t+\partial t}} \dot{W}_f dS + \int_{S^{q}_{t+\partial t}} q_n dS \qquad (1)$$

where $V_{t+\partial t}$ is the volume of the region of analysis composed by the chip-workpiece at time t+∂t, step n+1, with a contact interface with the tool $S^c_{t+\partial t}$, and a heat transfer surface $S^q_{t+\partial t}$.The density of the material is ρ, the internal energy is U, here included in the material time rate form, η measures the part of the plastic dissipation converted into heat, q_n is the normal heat flux per unit area of the body, flowing into the body. It is assumed that the thermal and mechanical problems are uncoupled in the sense that U is function of the temperature θ, and that q_n does not depend on strains or displacements in the body.

In writing Eq. (1) already part of the mechanical work appearing as dissipation is considered in the volume, plastic dissipation $\dot{W}_p$, and surface, frictional $\dot{W}_f$, terms. These terms may be written as :

$$
\begin{aligned}
&\dot{W}_p = \bar{\sigma}\,\dot{\bar{\varepsilon}}^p \qquad \bar{\sigma} = \sqrt{\frac{3}{2}\,\sigma' : \sigma'} \qquad \dot{\bar{\varepsilon}}^p = \sqrt{\frac{2}{3}\,\mathbf{D}^p : \mathbf{D}^p} \\
&\dot{W}_f = \mu\,\mathbf{t}_c \cdot \dot{\mathbf{u}}
\end{aligned}
\tag{2}
$$

where the deviatoric part of the Cauchy stress tensor is given by :

$$
\sigma' = \sigma - \frac{1}{3}\mathrm{tr}\,\sigma\,\mathbf{I} \qquad \mathrm{tr}\,\sigma = \sigma : \mathbf{I} \tag{3}
$$

and:

$$
\mathbf{D} = \frac{1}{2}(\mathbf{L} + \mathbf{L}^T);\ \mathbf{L} = \frac{\partial \dot{\mathbf{u}}}{\partial \mathbf{x}};\quad \mathbf{D}^p = \mathbf{D} - \mathbf{D}^e - \mathbf{D}^t;\quad \mathbf{D}^e = \mathbf{C}^{e\,-1} : \overset{\nabla}{\sigma};\quad \mathbf{D}^t = \frac{\dot{(\alpha\theta)}}{1 + \alpha\theta}\mathbf{I} \tag{4}
$$

where α is the coefficient of thermal expansion, $\dot{\mathbf{u}}$:the velocity and θ the temperature. Moreover, the other elements are calculated from the expressions :

$$
\begin{aligned}
&\overset{\nabla}{\sigma} = \mathbf{C}^{ep} : (\mathbf{D} - \mathbf{D}^t) + \left(\frac{\partial R}{\partial \dot{\bar{\varepsilon}}}\dot{\bar{\varepsilon}} + \frac{\partial R}{\partial \theta}\dot{\theta}\right)\mathbf{S};\quad \mathbf{S} = \frac{\mathbf{C}^e : \mathbf{N}}{H' + \mathbf{N} : \mathbf{C}^e : \mathbf{N}}\mathbf{N} \\
&\mathbf{N} = \frac{\partial f}{\partial \sigma};\qquad H' = \frac{\partial R}{\partial \bar{\varepsilon}^p};\qquad f = \bar{\sigma} - R;\qquad \bar{\varepsilon}^p = \int_0^t \dot{\bar{\varepsilon}}^p\,dt \\
&\mathbf{C}^{ep} = \mathbf{C}^e - \frac{\mathbf{C}^e : \mathbf{N}\mathbf{N} : \mathbf{C}^e}{H' + \mathbf{N} : \mathbf{C}^e : \mathbf{N}};\qquad \mathbf{C}^e = 2G\mathbf{J}' + K\,\mathbf{I}\,\mathbf{I};\quad \mathbf{J}' = \mathbf{J} - \frac{1}{3}\mathbf{I}\,\mathbf{I}
\end{aligned}
\tag{5}
$$

where R is a function prescribed for each material and G and K are elastic parameters of the material. The above set of equations allows us to find the plastic work term (Madrigal, Batalha and de Aguiar, 2000).

Overall heat transfer occurs in possibly three forms, the conduction q_k, the convection q_c and the radiation q_r, components so that

$$
q_n = q_n^k + q_n^c + q_n^r \tag{6}
$$

being :

$$
q_n^k = -\mathbf{k}\frac{\partial \theta}{\partial \mathbf{x}} \cdot \mathbf{n};\quad q_n^c = h(\theta_c - \theta);\qquad q_n^r = A(\theta^4 - \theta_r^4) \tag{7}
$$

being θ_c and θ_r the convection and radiation reference temperatures, h and A coefficients dependent on the presence of lubricant and tool geometry, among others. Tractions $\mathbf{t_c}$ are determined from the solution of the contact problem, which assumes a Coulomb-type friction model with parameter μ and determines W_f (Idesman & Levitas, 1995).

2.3 Discretization

A variational statement of energy balance, the principle of virtual temperatures can be obtained directly from the standard Galerkin approach as

$$\int_{V_{t+\Delta t}} \delta\theta \rho \dot{U} dV + \int_{V_{t+\Delta t}} \frac{\partial \delta\theta}{\partial \mathbf{x}} \mathbf{k} \frac{\partial \theta}{\partial \mathbf{x}} dV = \int_{V_{t+\Delta t}} \delta\theta \eta \bar{\sigma} \dot{\bar{\varepsilon}}^{p} dV + \int_{S^{csl}_{t+\Delta t}} \delta\theta \mu \mathbf{t}_c \cdot \dot{\mathbf{u}} dS + \int_{S^{q}_{t+\Delta t}} \delta\theta q_n dS \quad (8)$$

where $\delta\theta$ is an arbitrary variational field, time independent, satisfying the essencial boundary conditions. In order to approximate geometrically the body, an interpolation with finite elements is performed (the same mesh of the mechanical problem), so that temperatures are interpolated by

$$\theta = \mathbf{N}\Theta; \qquad \dot{\theta} = \mathbf{N}\dot{\Theta} \quad (9)$$

where $\mathbf{N(x)}$ is the interpolation function for the temperatures, and Θ are the nodal temperatures corresponding to the n nodes used as variables. The function $\mathbf{N}$ is in general composed by polynomials. Assuming that the variational field may be interpolated by the same functions leads us to the system :

$$\mathbf{F(N)} = \int_{V_{t+\Delta t}} \mathbf{N}^T \rho c \dot{\theta} dV + \int_{V_{t+\Delta t}} \frac{\partial \mathbf{N}^T}{\partial \mathbf{x}} \mathbf{k} \frac{\partial \theta}{\partial \mathbf{x}} dV - \int_{V_{t+\Delta t}} \mathbf{N}^T \eta \dot{W}_p dV - \int_{S^{csl}_{t+\Delta t}} \mathbf{N}^T \dot{W}_f dS - \int_{S^{q}_{t+\Delta t}} \mathbf{N}^T q_n dS \quad (10)$$

which sums zero, for any possible solution.

In order to solve the above system, we may consider an increment of temperature taking place between two configurations, all other parameters kept fixed, with the heat dissipated instantaneously at the end of the interval:

$$\int_{V_{t+\Delta t}} \mathbf{N}^T \rho \Delta(c\dot{\theta}) dV + \int_{V_{t+\Delta t}} \frac{\partial \mathbf{N}^T}{\partial \mathbf{x}} \Delta(\mathbf{k} \frac{\partial \theta}{\partial \mathbf{x}}) dV - \int_{V_{t+\Delta t}} \mathbf{N}^T \eta \Delta \dot{W}_p dV - \int_{S^{csl}_{t+\Delta t}} \mathbf{N}^T \Delta \dot{W}_f dV - \int_{S^{q}_{t+\Delta t}} \mathbf{N}^T \Delta q_n dS = \mathbf{0} \quad (11)$$

and upon defining the matrices :

$$\mathbf{K}_k = \int_{V_{t+\Delta t}} \mathbf{B}^T \mathbf{k} \mathbf{B} dV \qquad \mathbf{B} = \frac{\partial \mathbf{N}}{\partial \mathbf{x}} \quad (12)$$

$$\mathbf{K}_c = \int_{S^{q}_{t+\Delta t}} \mathbf{N}^T h \mathbf{N} dS \quad (13)$$

$$\mathbf{K}_r = \int_{S^{q}_{t+\Delta t}} \mathbf{N}^T (4A\theta^3) \mathbf{N} dS \quad (14)$$

$$\mathbf{C} = \int_{V_{tt}} \mathbf{N}^T c \mathbf{N} dV \tag{15}$$

and the vectors :

$$\partial \mathbf{Q}_p = \int_{V_{t+\partial}} \mathbf{N}^T \eta \partial \dot{W}_p dV \tag{16}$$

$$\partial \mathbf{Q}_f = \int_{S^{csl}_{t+\partial t}} \mathbf{N}^T \partial \dot{W}_f dS \tag{17}$$

$$\partial \mathbf{Q}_k = - \int_{V_{t+\partial}} \mathbf{B}^T \partial \mathbf{k} \frac{\partial \theta}{\partial \mathbf{x}} dV + \int_{S^q_{t+\partial}} \mathbf{N}^T \partial q^k_n dS \tag{18}$$

$$\partial \mathbf{Q}_r = \int_{S^q_{t+\partial}} \mathbf{N}^T \partial A(\theta^4 - \theta_r^4) dS \tag{19}$$

$$\partial \mathbf{Q}_c = \int_{S^q_{t+\partial}} \mathbf{N}^T \partial h(\theta - \theta_c) dS \tag{20}$$

$$\partial \mathbf{Q}_u = \int_{S^q_{t+\partial t}} \mathbf{N}^T \rho \partial c \dot{\theta} dV \tag{21}$$

so that :

$$\mathbf{C}\partial\dot{\Theta} + (\mathbf{K}_k + \mathbf{K}_c + \mathbf{K}_r)\partial\Theta = \partial\mathbf{Q}_p + \partial\mathbf{Q}_f + \partial\mathbf{Q}_k + \partial\mathbf{Q}_\mathbf{c} + \partial\mathbf{Q}_r + \partial\mathbf{Q}_u \tag{22}$$

which determines a form of solution for the heat transfer problem. The matrices **K** and **C** are determined, in a forward Euler approach for the heat problem, from the solution at step n for the temperature field, and from step n+1 for the mechanical problem. Part of the loading vectors are also known at this same stage, because their input comes from the solution of the mechanical problem, while the others come from an iterative process to find the heat transfer related part.

2.3 Numerical approach

The jacobian matrix of this problem, may be obtained from the above form, and results to be :

$$\mathbf{J} = \mathbf{C}\frac{\partial\dot{\Theta}}{\partial\Theta} + \mathbf{K}_k + \mathbf{K}_c + \mathbf{K}_\mathbf{r} + \frac{\partial(\mathbf{Q}_p + \mathbf{Q}_f + \mathbf{Q}_k + \mathbf{Q}_c + \mathbf{Q}_r + \mathbf{Q}_u)}{\partial\Theta} \tag{23}$$

which is very hard to evaluate because some of the matrices generated are not symmetric which may cause some numerical problems. Moreover for some terms the dependence with temperature is not exactly put. A procedure considered resorts to an approximate expedient brougth by the Newton Method. In this case, increments of temperature in the field are calculated from ::

$$\mathbf{C}^{(i)}\,\partial\dot{\Theta}^{(i)} + (\mathbf{K}_k + \mathbf{K}_c + \mathbf{K}_r)^{(i-1)}\,\partial\Theta^{(i)} = (\partial Q_p + \partial Q_f)^{(i)} + (\partial Q_k + \partial Q_c + \partial Q_r + \partial Q_u)^{(i-1)} \quad (24)$$

with the temperatures computed with the advective term. Procedure is initialized with the prior knowledge of the configuration (Rebelo & Kobayahi, 1980b)

3. RESULTS

The initial dimensions of the workpiece under consideration, a part of the whole element under machining, are shown in Figure (1). Plane strain conditions are guaranteed with thicknesses five times the cutting depth. Tool is supposed rigid, and non-heat conductive. The cutting velocity is 2 mm/s. Boundary conditions for the problem are as shown in Figure (2). Properties of the material, AISI 1020, appear in Table (1). In particular the material model considers the uni-axial stress-strain relation, for metric units (Shirakashi & Usui, 1983) :

$$\bar{\sigma} = A_0 \bar{\varepsilon}^{0.21} \left(\frac{\dot{\bar{\varepsilon}}}{1000}\right)^{0.0195}; \qquad A_0 = 1394 e^{-0.00118\theta} + 339 e^{-0.0000184\left[\theta - \left(943 + 23.5 \ln\left(\frac{\dot{\bar{\varepsilon}}}{1000}\right)\right)\right]^2} \quad (25)$$

where the temperatures are measured in a absolute scale.

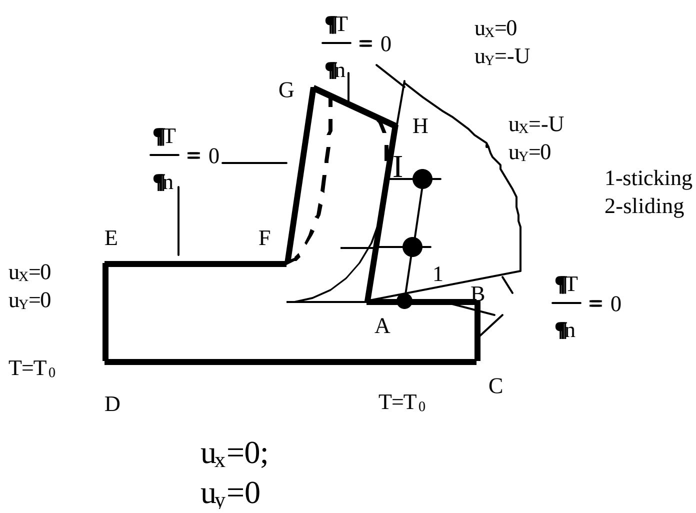


Figure 2 Boundary Conditions of the Model

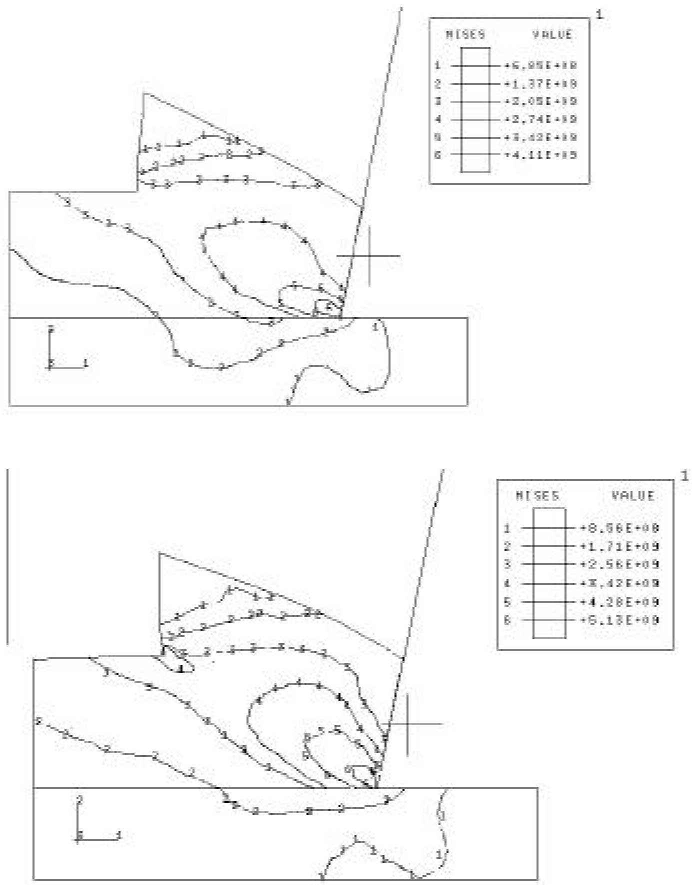
Table 1. Physical properties used in the model

E=200. Gpa	**ν=0.30**	**S_{yt}=700 MPa**	**S_{ut}=3700 MPa**	**ρ=7833 kg/m^3**
α=1.2x10^{-5} ^{0}C^{-1}	**η=0.90**	**k = 52. W/m ^{0}C**	**c = 586 J/kg ^{0}C**	**μ=0.10**

4. Conclusions and comparisons

Fig. 3 shows a cross-sectional view of the orthogonal cutting process for the instants corresponding to step 1 and step 4. Equivalent stress distribution is the parameter of comparison. The high-stress zone corresponding to high deformation is found to be located near the primary deformation zone. This is because elements in this zone experience drastic distortion-type deformation. In addition, in the secondary deformation zone, due to the plastic deformation produced by the chip–tool contact interface, the stress in this zone is also relatively high. The shown results compare well with reported results, eg (Komvopoulos, 1991).

Figure 3 :Equivalent Stresses : step. 1. for tool advancement s=0.0375 mm. and step 4 for s=0.15 mm.The temperature field computed with the procedure shown is represented in two



different steps, while working with strain rates of 10^{+4} s^{-1}. Maximum temperatures occur inside the shear zone and at the secondary shear zone as before. Effect of cracking in the field was not investigated at this time, as well as the strain rate effect, or use of lubricants. Heat transfer to the tool was not considered at this time, as well as the conductive or radiation effects. All these effects should be addressed later on this research.

Overall comparison of the obtained temperatures, in magnitude as well as in form agree well with presented results in the literature (Stevenson & all, 1983).

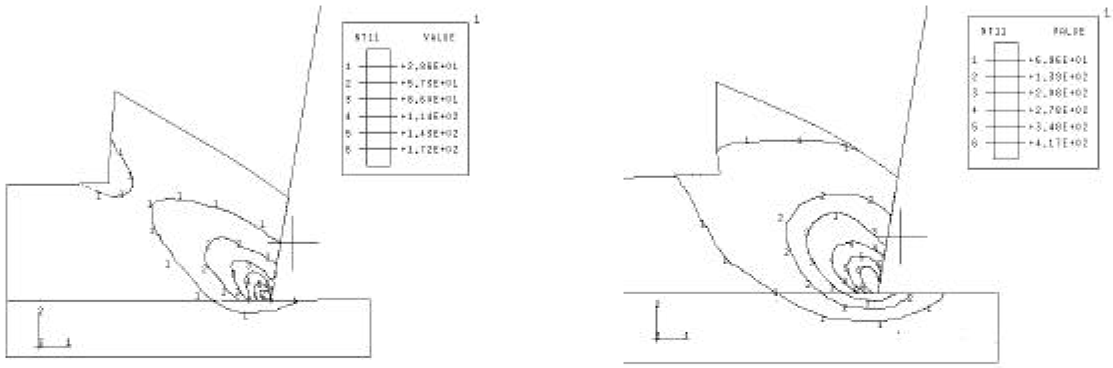


Figure 4 : Temperatures Field for step. 1 and step 4.

REFERENCES

Muraka, P. D, Barrow, G. & Hinduja, S., 1979, Influence of the Process Variables on the Temperature Distribution in Orthogonal Machining Using The Finite Element Methods, Int. J. Mech. Science, vol. 21, pp. 445-456.

Tay, A..O. & Stevenson, M.G., 1976, “A Numerical Method for Calculating Temperature Distribution in Machining from Force and Shear Angle Measurements”, Int. Journal. Mech. Tool Design. And Research, vol. 16, pp. 335-349

Tay, A..O., 1983, A Review of Methods of Calculating Machining Temperatures, Journal of Materials Processing Technology, vol. 36,.pp 225-257.

Rebelo, N & Kobayashi, S., 1980a, A coupled Analysis of Viscoplastic Deformation and Heat Transfer-I, Int. J. Mech. Sci., vol 22, pp. 699-705

Rebelo, N. & Kobayashi, S., 1980b, A coupled Analysis of Viscoplastic Deformation and Heat Transfer-II, Int. J. Mech. Sci., vol 22, pp 707-718

Smith, A. J. R. & Armarego, E. J. A., Temperature Prediction in Orthogonal Cutting with a Finite Diference Approach, Annals of the CIRP, vol. 30.,pp. 9-13.

Stevenson, M. G., Wright, P. K. & Chow, J. G**,** 1983. Further Developments in Applying the Finite Element Method to the Calculation of the Temperature Distribution in Machining and Comparisons with the Experiments,. Journal of Eng. for Ind.,vol. 105; Aug, pp149-154.

Ramesh, M. V.; Seetharamu, K. N.; Ganesan N.; Kuppuswamy, G.,1999 Finite Element Modeling of Heat Transfer Analysis in Machining of isotropic Materials, International Journal of Heat and Mass Transfer., vol. 42., pp. 1569-1583

Madrigal, M. R.; Batalha, G. F.; Aguiar, J. B, 2000,A Finite Element Simulation of the Chip Formation in a Orthogonal Metal Cutting., 8th Inter. Conf. Metal Forming. 2000, Krakow, Poland.

Komvopoulos, K., 1991, Finite Element Modeling of Orthogonal Metal Cutting**,** Journal of Eng. For Ind, vol.113,.pp. 253-26