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SHEET BENDING THEORY APPLIED TO A THREE ROLL PROCESS

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Abstract. *This paper presents an analytical solution developed to compute, in an approximated form, bending moments from curvatures at different positions occupied by a specific section of a sheet being bent while in a three-roll-bending machine. The distribution of bending stresses is obtained, as well as the residual stresses resulting from springback of the component at the end of the process. Elasto-plastic conditions under the viewpoint of the deformation theory of plasticity are considered for a power law material. For a particular application considered residual stresses as well as springback is computed. Results obtained compare well with published results.*

Keywords: *Sheet-bending, Bending moment expression, Springback, Residual Stresses*

1. INTRODUCTION

Several researchers along history have treated the problem of bending of metallic sheets. An overview of the contributions, though incomplete, could start with initial solutions devised for the elasto-plastic bending under plane strain conditions (Hill, 1950). From there on, stress and strain distributions were determined (Lubahn and Sachs, 1950), with springback and work hardening added later on (Denton, 1966). Experimental and theoretical analyses for determination of residual stresses were developed during the seventies and eighties with acceptable results. Improved solutions were obtained recently (Tan et al., 1994) as applied to pure bending. No direct application of these solutions, however, appears for the case of roll bending.

In the present work basic equations are developed for this problem, under the framework of the theory of deformation plasticity, alleviating some restrictions, while still getting a closed form solution.

2. BASIC EQUATIONS

2.1 Model

In continuous sheet bending of thin plates, normal planes may be considered to remain plane on bending and to converge to a center of curvature, as illustrated in Figure 1. Assuming that the principal directions of stress and strain coincide with the tangential, radial and normal directions, respectively, allows us to deal with only three variables in each case. For roll bending, internal efforts include some traction and shear components, but their effect may be considered small compared to the effect of the bending moment. The problem translates therefore to the determination of the values of the bending moment at different

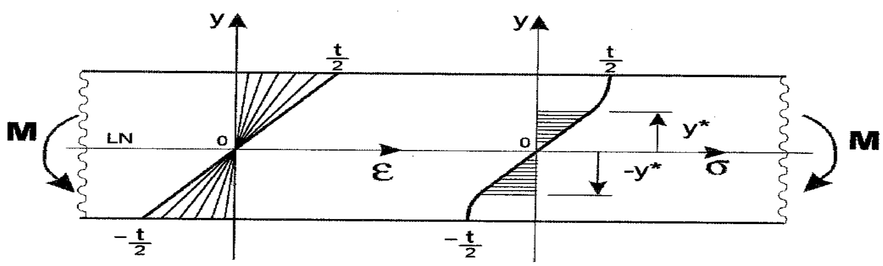

Figure 1. Strain and stress distribution in a typical section of the sheet element.

stations where curvatures are supposed known.

2.2 Constitutive relations

In a principal coordinate system, as described above, the components of the stress tensor σ are restricted to the tangential t, radial r and normal z component. For all points in an elastic state we may write (Sidebottom & all, 1979)

$$\begin{aligned}\varepsilon_t^e &= \frac{1}{E}[\sigma_t - \nu(\sigma_r + \sigma_z)]\\ \varepsilon_r^e &= \frac{1}{E}[\sigma_r - \nu(\sigma_z + \sigma_t)]\\ \varepsilon_z^e &= \frac{1}{E}[\sigma_z - \nu(\sigma_t + \sigma_r)]\end{aligned} \tag{1}$$

Where E and ν are the elastic modulus and Poisson's ratio, respectively. Under plane strain conditions, the z-component may be considered null, so that equations above simplify to render

$$\varepsilon_t^e = \frac{\sigma_t}{E'} \qquad E' = \frac{E}{1-\nu^2}; \qquad \varepsilon_r^e = \frac{-\nu}{E}(1+\nu)\sigma_t; \qquad \varepsilon_z^e = 0 \tag{2}$$

Plastic strain components, on the other hand, may be determined from the deformation theory of plasticity, for isotropic materials, under continuous loading on the hypothesis of proportionality between principal shearing strains and the corresponding principal shear stresses (Marin, 1952)

$$\begin{aligned}\varepsilon_t^p &= \frac{2k}{3}[\sigma_t - \frac{1}{2}(\sigma_r + \sigma_z)]\\ \varepsilon_r^p &= \frac{2k}{3}[\sigma_r - \frac{1}{2}(\sigma_z + \sigma_t)]\\ \varepsilon_z^p &= \frac{2k}{3}[\sigma_z - \frac{1}{2}(\sigma_t + \sigma_r)]\end{aligned} \tag{3}$$

Where the quantity 2k/3 is sometimes known as the plasticity modulus, being k a constant. With only two strain components to worry about, and assuming volume constancy in the plastic range, Equation (3) simplify to render

$$\varepsilon_t^p = \frac{k}{2}(\sigma_t - \sigma_r); \qquad \varepsilon_r^p = \frac{k}{2}(\sigma_t - \sigma_r); \qquad \varepsilon_z^p = 0 \tag{4}$$

Combined values of strain and stress may be compared to one-dimensional results if equivalent quantities are used. For many metallic materials equivalent values of strain and stress are related through the Hollomon formula, which is a power law model with a work-hardening index n,

$$\bar{\sigma} = k\bar{\varepsilon}^n \qquad \bar{\varepsilon} = \sqrt{\frac{2}{3}(\varepsilon_t{}^2 + \varepsilon_r{}^2 + \varepsilon_z{}^2)} \qquad \bar{\sigma} = \sqrt{\frac{3}{2}(\sigma_t'^2 + \sigma_r'^2 + \sigma_z'^2)} \tag{5}$$

Relating equivalent stresses to equivalent strains. This expression is dependent on the deviatoric components of stress tensor σ'

$$\sigma_t' = \sigma_t - \sigma_h; \qquad \sigma_r' = \sigma_r - \sigma_h; \qquad \sigma_z' = \sigma_z - \sigma_h$$
$$\sigma_h = \frac{\sigma_t + \sigma_r + \sigma_z}{3} \tag{6}$$

And on the total strains, that may be decomposed into elastic and plastic components (Lee and Liu, 1967)

$$\varepsilon_t = \varepsilon_t^p(1 + \frac{\varepsilon_t^e}{\varepsilon_t^p}); \qquad \varepsilon_r = \varepsilon_r^p(1 + \frac{\varepsilon_r^e}{\varepsilon_r^p}) \tag{7}$$

A simplification of these expressions may devise if we consider Equations (4) and (7) plus the fact that the radial components of stress are small compared to the tangential ones, so they may disregarded. The result is:

$$\varepsilon_t \cong \varepsilon_t^p(1 + \frac{2}{kE'}); \qquad \varepsilon_r \cong \varepsilon_r^p[1 + \frac{2\nu(1+\nu)}{Ek}] \tag{8}$$

Which means that the equivalent strain and stress may be written in terms of the tangential components as

$$\bar{\varepsilon} \cong \varepsilon_t^p\sqrt{\frac{4}{3} + [\frac{8}{3kE'} + \frac{8\nu(1+\nu)}{3Ek}]} \cong \frac{2}{\sqrt{3}}\varepsilon_t^p; \qquad \bar{\sigma} = \frac{\sqrt{3}}{2}\sigma_t \tag{9}$$

The above expression (9) is useful to generate an approximation to the tangential stresses. In order to do this we may get back to Equation (3), substitute the above result, expand the power term into a Taylor series, approximated to the first term, and find

$$\sigma_t \cong k\left(\frac{2}{\sqrt{3}}\right)^{n+1}\varphi\,\varepsilon_t^n; \qquad \varphi = \frac{1}{1 + k\left(\frac{2}{\sqrt{3}}\right)^{n+1}\varepsilon_t^{n-1}\frac{n}{E'}} \tag{10}$$

So that formally

$$\begin{aligned} &\varepsilon_t^e = \frac{\sigma_t}{E'}; \qquad \varepsilon_r^e = -\frac{\nu}{E}(1+\nu)\sigma_t; \qquad \varepsilon_z^e = 0 \\ &\varepsilon_t^p = \frac{k}{2}\sigma_t; \qquad \varepsilon_r^p = -\frac{k}{2}\sigma_t; \qquad \varepsilon_z^p = 0 \end{aligned} \tag{11}$$

Will be the set of bending strains required to define the components of the strain tensor. Components of the stress tensor may be obtained directly from the tangential component, Equation (10) and the results above applied to Equation (1) and (3).

2.3 Bending moment

Focusing attention into the specific case of the three roll bending process, Figure 2 shows that, each section of the sheet, at some moment, is submitted to different conditions of straining. If we follow a specific section, while rolling from position A till position E, we will notice that the stresses are small, in the elastic range. Bending is predominant, with values of bending moment ranging from zero, position A, to the maximum elastic bending moment m_y, position E, whose value may be computed from :

$$m = \int_{-\frac{t}{2}}^{\frac{t}{2}} \sigma_t\, y\, dy; \qquad m_y = E'\frac{t^3}{12} r_y^{-1}; \qquad r_y = r_E \tag{12}$$

Where the radius of curvature at position E is such that the maximum stresses in the section will reach the yield value S_y, being t the thickness of the metallic sheet and r the radius of curvature. In order to expand the above expressions, logarithmic strains were put as

$$\varepsilon = \varepsilon_0 + \varepsilon_b; \qquad \varepsilon = \ln\left(1 + \frac{y}{r}\right) \cong \frac{y}{r} \tag{13}$$

For the case where we have very neither small radius of curvature nor stretching of the sheet.

The condition where the first yield stresses appear does not correspond, however to the point where curves for the elastic part of the one-dimensional stress-strain diagram intercept the elasto-plastic region (Queener and Angelis, 1968).

$$\varepsilon_t^{e0} = \frac{1-\nu^2}{\sqrt{1-\nu+\nu 2}}\left(\frac{k}{E}\right)^{\frac{1}{1-n}} \tag{14}$$

So that, in general, elastic plus elasto-plastic conditions will be present in any section after point E is surpassed. For any position y up to the transition point y_0, elastic stresses prevail contributing to the total bending moment in the section with:

$$m_e = 2\int_0^{y_0} E' r^{-1} y^2 dy; \qquad y_0 = r\varepsilon_t^{e0} \tag{15}$$

$$m_e = \frac{2}{3} E' r^2 [\frac{(1-\nu^2)}{\sqrt{1-\nu+\nu^2}}](\frac{k}{E})^{\frac{3}{1-n}} \tag{16}$$

While the remaining stresses, which obey the elastoplastic conditions, contribute with:

$$m_p = 2\int_{y_0}^{\frac{t}{2}} k(\frac{2}{\sqrt{3}})^{n+1} \varphi (\frac{y}{r})^n dy = 2k(\frac{2}{\sqrt{3}})^{n+1} \int_{y_o}^{\frac{t}{2}} \frac{(\frac{y}{r})^n}{1 + k(\frac{2}{\sqrt{3}})\frac{n}{E'}(\frac{y}{r})^{n-1}} dy \tag{17}$$

The above expression may be evaluated numerically. Fortunately its denominator differs very little from one, so that a close-form approximation can be written:

$$m_p \cong \frac{k}{\sqrt{3}^{n+1}} \frac{2^{n+2}}{(n+2)r^n} \{(\frac{t}{2})^{n+2} - [\frac{(1-\nu^2)}{\sqrt{1-\nu+\nu^2}}]^{n+2} r^{n+2} (\frac{k}{E})^{\frac{n+2}{1-n}}\} \tag{18}$$

And the total bending moment will be the sum of m_e plus m_p. (Tan & all, 1994)

2.4 Springback

After passing the position of maximum flexure, station B, Figure 2, the bending moment is gradually decreased to zero, station C. When this happens, stresses try to relieve themselves by straightening the bent plate. This causes springback and leaves the sheet with residual stresses. In order to describe the first effect, let r_b and r_c denote the radii of curvature before and after unloading. Unloading is elastic.

If the length of an element located a distance y from the neutral layer, between two sections Δl_0 apart, before unloading, station B, is

$$\Delta l_y^b = (r_b + y)\Delta\alpha_b \tag{19}$$

Where $\Delta\alpha_b$ is the angle involved before unloading, and

$$\Delta l_{y'}^c = (r_c + y')\Delta\alpha_c; \qquad r_b\Delta\alpha_b = r_c\Delta\alpha_c; \quad y \cong y' \tag{20}$$

Then the corresponding elastic strains obtained upon unloading will be

$$\varepsilon_{ut}^e = \frac{\Delta l_{y'}^c - \Delta l_y^b}{\Delta l_y^b}; \qquad \varepsilon_{ut}^e = \frac{yr_b}{y + r_b}(\frac{1}{r_c} - \frac{1}{r_b}) \tag{21}$$

Whereas the unloading bending moment to be applied to the maximum bending moment results to be

$$m_u = 2\int_o^{\frac{t}{2}} E'\varepsilon_{ut}^e y dy; \qquad m_u = Max(m_e + m_p) = [m_e + m_p](r_b); \tag{22}$$

Therefore, solving Equation (21) for r_c we get

$$r_c = \frac{r_b}{\dots}; \quad b = [2r_b^2 - 2r_b^2 \ln(\frac{r_b + \frac{t}{2}}{\dots}) \dots t^2] \tag{23}$$

Defined as the final expression for the determination of the curvatures after springback.

2.5 Residual stresses

In order to determine the stresses left in the sheet after removal of the loading, we may subtract the elastic stresses, caused by unloading at every position y of the sheet, from the stresses already there. The resulting stresses will be the residual ones. Starting at Equation (23), and performing the subtraction, we get the final result:

$$s_{rt} = s_t - s_{ut}; \qquad s_{ut} = -\frac{m_u y}{(r_b + y)h} \tag{24}$$

3. APPLICATIONS

Expressions derived above may be applied directly to the problem of bending a sheet on a three-roll device. The arrangement illustrated in Figure 2 may be used in order to show this. It is regarded as known the final desirable curvature of the plate at C, and desired to know what the maximum curvature, which happens to occur at B, should be so that after springback the required final form is obtained. If it is considered Equation (23) on setting the curvature at B as the free variable, curvature r_b results to be the root of equation:

$$r_b + \frac{(g - b)r_b^{\,2} + a\, r_b^{\,-n}}{E'(ar_b^{\,2} + br_b + c)} r_c = r_c$$

$$a = x\left(\frac{t}{2}\right)^{n+2}; \quad b = x\left(\frac{1-u^2}{\sqrt{1-u+u^2}}\right)^{n+2}\left(\frac{K}{E}\right)^{\frac{n+2}{1-n}}; \quad x = \frac{K}{\sqrt{3}^{\,n+1}}\frac{2^{n+2}}{(n+2)} \tag{25}$$

$$a = 3 - 2\ln\left(\frac{r_b + \frac{t}{2}}{r_b}\right); \quad b = t; \qquad c = -\frac{t^2}{4}$$

And therefore all we have to do is to solve the above for r_b. Newton’s method may be used among others, (Hughes, 1987).

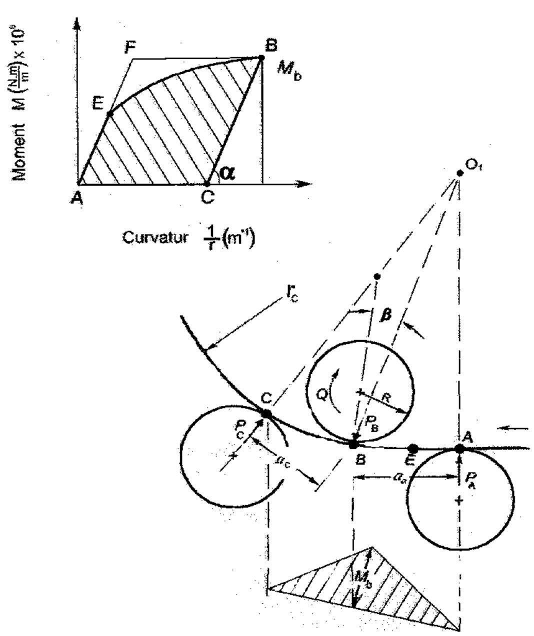

Figure 2. A roll bending apparatus (Marciniak, 1991)

4. RESULTS

The above development was used to compute springback as well as residuals stresses. This paper deals with the specific case of a sheet of SAE 1008 Steels, for which E = 200 GPa, ν = 0.33, K = 541 MPa and n = 0.252, with thickness of 3mm, bent to a final radius of 0.135 m. Results are shown in Fig. 3. Comparison with X-ray measurements as reported in the literature, is presented. The characteristic S-shape is obtained in both cases, with top stresses occurring at similar positions.

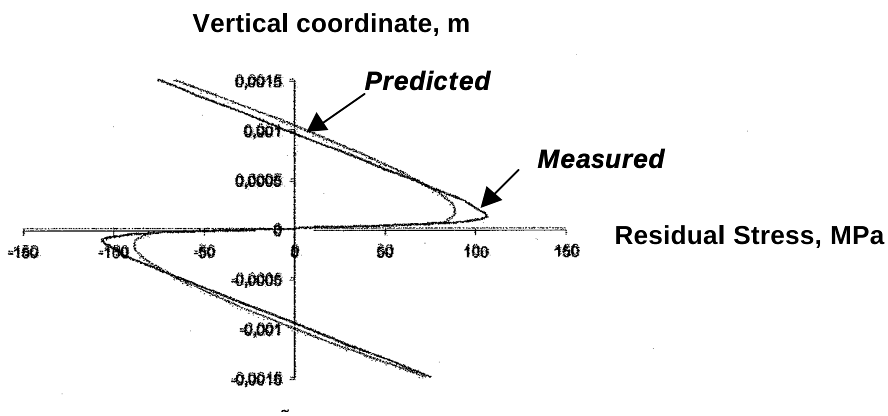

Figure 3. Predicted and measured curves for residual stresses after spring back for steel SAE 1008.

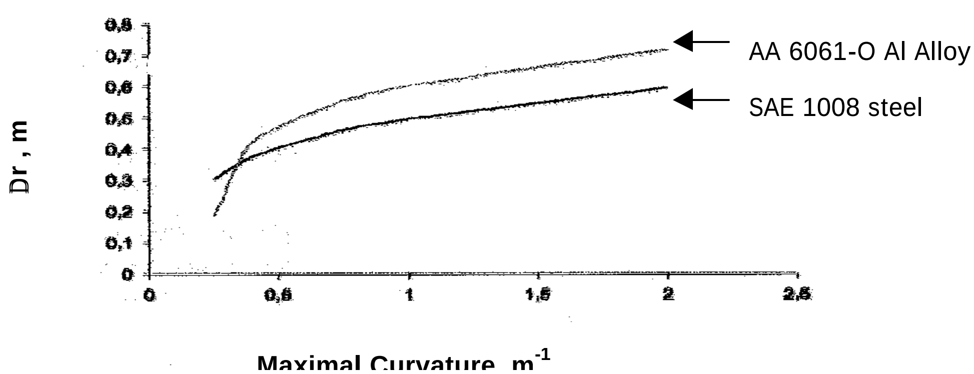

Figure 4. Predicted curvature variations (Springback) versus maximal curvature.

In much the same way, variations of curvature due to springback, as a function of the maximum curvature, may be considered. Figure 4 shows the behavior the variation of curvatures for SAE 1008 and an aluminum alloy AA 6061-O.

5. CONCLUSION

The development presented here, which covers some results included in the literature, though simple produces good results and might be very useful in industry. Further improvements in the model, including effects like damage and anisotropy, are predicted for the next steps of the work.

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