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Thermo Mechanical Model for Orthogonal Metal Cutting

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ABSTRACT: A finite element model of the orthogonal metal cutting process with a coupled thermal-elastic-plastic constitutive equation under large deformations based on update Lagrangian formulation is presented. Plane strain conditions with a flow stress function of strain, strain rate, work hardening and temperature is assumed. Principle of virtual work is applied in order to construct an equilibrium rate solution. The problem is formulated, discretized and the resulting equations solved with the finite element method. As an application the temperature field is computed for the case of machining AISI 1020 steel. The stress field is also shown in order to present the coupled problem. The interface problem is modeled with a simple Coulomb friction surface. The model produced good agreement with results presented in the literature.

1 INTRODUCTION

Metal cutting is a quite complex phenomenon, of multi physics type, where elasticity and plasticity, fracture, contacts, heat transfer, among others takes place simultaneously. Much work has been devoted to its understanding. Initially having an experimental character and afterwards theoretical as well. Since the forties several attempts were made pursuing a linking between empirical knowledge and theoretical modeling. Such attempts begun at forties (Piispanen, 1948) and (Merchant 1945), concentrating in explain the mechanisms of the chip formation. A recent review can be seen in (Mackerle 1999).

In metal cutting the material is subjected to extremely high strains in two principal regions: the shear zone or primary deformation zone, and another zone, known as secondary deformation zone. Determination of stresses and temperatures in this region is a central problem in understanding machining processes, as the problem is of a coupled one.

Many models were developed to address this solution. Of particular success were the ones considering the finite element tool, using update Lagrange scheme. Here it is presented some details of one such a solution, regarding some previous work in the area (Lin, 1990, 1992, 1995), (Komvopoulos, 1991), (Tay 1976 and Stevenson 1983), (Rebelo & Kobayashi, 1980) and (Muraka & all, 1979). A quasi-static solution with typical machining parameters will be formulated and implemented in this work.

2 FORMULATION

2.1 Orthogonal Cutting Model

The chip formation model to analyze has the geometry shown in Figure 1. In it is possible to distinguish two regions separated by a division line AB, the cutting line, marking the positions separation is going to take place. Dimensions for this part of the workpiece are shown in mm, in the same figure. It corresponds to a region of the workpiece where deformation is localized. For the mechanical and thermal problems, boundary conditions shown in Figure 2 apply. In the left lower side, it was prescribed null velocities, for the horizontal, 1, and vertical, 2, directions. Temperatures have a fixed value T_o.

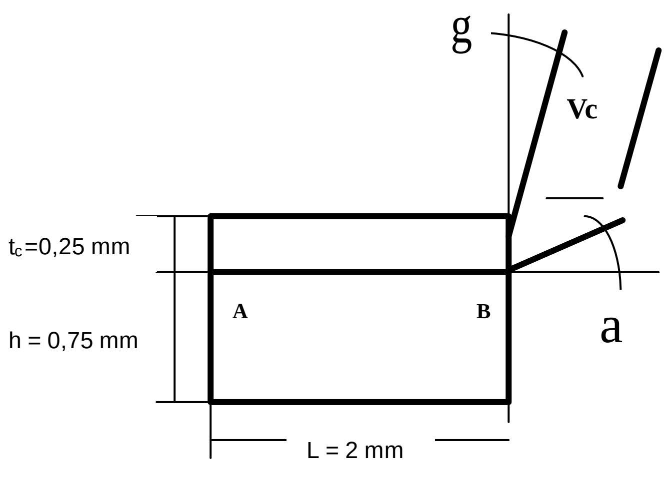

Figure 1. Chip formation model

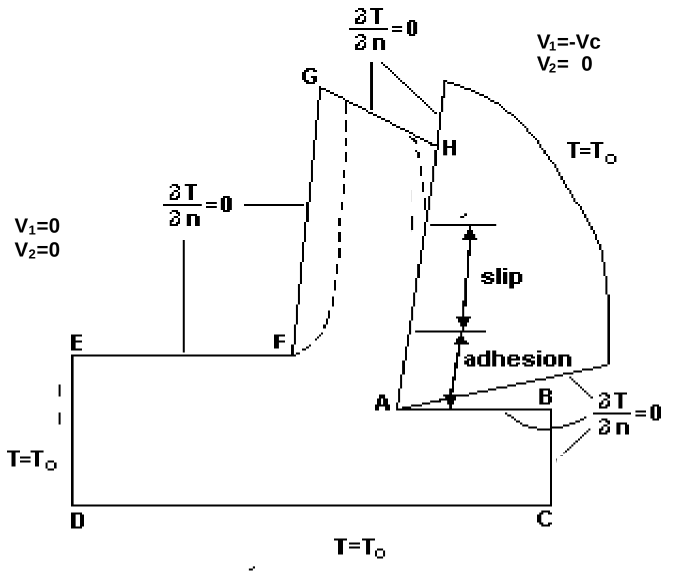

Figure 2. Set of boundary conditions.

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The tool, on the other hand, is supposed constructed with high speed steel capable of heating and approaching the workpiece with a velocity V_c. It is assumed rigid. Geometry of it depends on the rake angle γ and the clearance angle α.

No restrictions are imposed to the velocities in the other surfaces. Contact with heat transfer associated occurs in the chip tool interface, where normal velocity components equal the prescribed velocity in the same direction, whenever there is no open condition in the contact interface.

2.2 Constitutive Law

During contact between tool and workpiece, a part from the large strains of the process, friction occurs. Energy consumed in the process is largely transformed into heat, raising the temperature of the interacting elements. Thermal strains are naturally induced by the change in temperature.

Assuming the total deformation rate as being the sum of three parts, an elastic, a plastic and a thermal one:

$$\mathbf{D} = \mathbf{D^e} + \mathbf{D^p} + \mathbf{D^t} \quad (1)$$

Allows us to describe the elastic portion of the constitutive relation by

$$\mathbf{D^e} = \mathbf{C^{e^{-1}}} : \overset{\nabla}{\mathbf{s}} \quad (2)$$

Being $\mathbf{C}_e$ the elastic tensor relating the elastic part of the deformation gradient with the Jaumann rate of the Cauchy stress. Plastic deformations are assumed to comply with normality, so that

$$\mathbf{D^p} = \dot{\lambda}\frac{\partial F}{\partial \mathbf{s}} \quad (3)$$

Which is an expression that depends on the plastic potential F, assumed associated to a Mises type of loading function f,

$$f = f(\mathbf{s}, R) \quad R = R(\overline{D}^p, \dot{\bar{\varepsilon}}, q)$$
$$f = \bar{s} - R \quad \bar{s} = \sqrt{\frac{3}{2}\mathbf{s}' : \mathbf{s}'} \quad (4)$$
$$\mathbf{s}' = \mathbf{s} - \frac{1}{3} tr\,\mathbf{s}\,\mathbf{I}; \quad \overline{D}^p = \sqrt{\frac{2}{3}\mathbf{D}^p : \mathbf{D}^p}$$

The determination of the parameter λ, which may vary throughout the straining, comes from the consistency condition:

$$f = \overset{\nabla}{f} = 0 \quad (5)$$

And leads to the following

$$\overset{\nabla}{s} = \mathbf{C^{ep}} : (\mathbf{D} - \mathbf{D^t}) + \left(\frac{\partial R}{\partial \dot{\bar{\varepsilon}}}\ddot{\bar{\varepsilon}} + \frac{\partial R}{\partial q}\dot{q}\right)\mathbf{S}$$

$$\mathbf{S} = \frac{\mathbf{C}^e : \mathbf{N}}{H' + \mathbf{N} : \mathbf{C}^e : \mathbf{N}}\mathbf{N}; \quad H' = \frac{\partial R}{\partial \overline{D}^p}; \quad \mathbf{N} = \frac{\partial f}{\partial \mathbf{s}} \quad (6)$$

Where H' is the strain hardening parameter, for conditions of isotropic hardening, and **N** the normal to the yield surface, and $\overline{D}^p$ the equivalent plastic strain rate, respectively. On performing the indicated operations, the final form of the elastic-plastic matrix will be

$$\mathbf{C^{ep}} = \mathbf{C^e} - \frac{\mathbf{C^e} : \mathbf{N}\,\mathbf{N} : \mathbf{C^e}}{\mathbf{H'} + \mathbf{N} : \mathbf{C^e} : \mathbf{N}} \quad (7)$$

Where

$$\mathbf{C^e} = 2G\mathbf{J}' + K\,\mathbf{II} \quad (8)$$

With G and K the elastic shear and bulk moduli, respectively, and the **J**'and **I** the unit tensors, the first one in the deviatoric plane (Aravas, 1995).

2.3 Equilibrium Condition

Equilibrium of the part P, workpiece, on a rate form derives from equating internal and external virtual work rates (Hill, 1959):

$$\int_{V_o} \frac{\partial d\dot{\mathbf{u}}}{\partial \mathbf{X}} : \dot{\mathbf{s}}\, dV = \int_{S_o} \dot{\mathbf{f}}_o \cdot d\dot{\mathbf{u}}\, dS + \int_{V_o} \dot{\mathbf{b}}_o \cdot d\dot{\mathbf{u}}\, dV$$

$$\mathbf{s} = J\mathbf{F}^{-T}\mathbf{s} \qquad \mathbf{F} = \frac{\partial \mathbf{x}}{\partial \mathbf{X}}; \quad J = \det \mathbf{F} \quad (9)$$

Where V_o and S_o are the volume and surface of the body in the reference configuration, respectively.

The virtual velocity field is assumed time independent and obeying the boundary conditions. The nominal, or Lagrange stress is denoted by **s**, the nominal traction on S_o by $\mathbf{f_o}$ and body force by on V_o by $\mathbf{b}_o$ (Yamada, 1973). If we consider the expressions above and perform the indicated operations we may write that

$$\dot{\mathbf{s}} = J\mathbf{F}^{-1}[\overset{\nabla}{\mathbf{s}} + \mathbf{W}\cdot\mathbf{s} - \mathbf{s}\cdot\mathbf{W} - \mathbf{L}_m.\mathbf{s} + tr\,\mathbf{L}_m\mathbf{s}] \quad (10)$$

$$\mathbf{W} = \frac{1}{2}(\mathbf{L} + \mathbf{L}^T) \quad \mathbf{D} = \frac{1}{2}(\mathbf{L} + \mathbf{L}^T) \quad \mathbf{L} = \frac{\partial \mathbf{v}}{\partial \mathbf{x}}; \mathbf{v} = \dot{\mathbf{u}}_s$$

Where its geometric part may have the velocity gradient **L** factored out through a matrix $\mathbf{D}^g$ (Obikawa, 1966). Discretized the part with finite elements, being the nodal velocities **V** the independent variables, results in the matrix form:

$$(\mathbf{K}^{ep} + \mathbf{K}^g)\mathbf{V} = \dot{\mathbf{F}};$$
$$\mathbf{K}^{ep} = \int_{V_t} \mathbf{B}^T{}_L\mathbf{C}^{ep}\mathbf{B}_D\,dV; \quad \mathbf{K}^g = \int_{V_t} \mathbf{B}_L{}^T\mathbf{D}^g\mathbf{B}_L\,dV \quad (11)$$

With the loading rate term composed of the parts:

$$\dot{\mathbf{f}}_0 = \int_{S_o} \mathbf{N}^T\dot{\mathbf{f}}_o\,dS + \int_{V_o} \mathbf{N}^T\dot{\mathbf{b}}_o\,dV; \quad \mathbf{v} = \mathbf{NV}$$

$$\dot{\mathbf{f}} = \int_{V_t} \mathbf{B}_l{}^T(\mathbf{C}^{ep}\mathbf{D}^t - \frac{\partial R}{\partial q}\dot{q}\mathbf{S})\,dV; \quad \mathbf{D}^t = (a\,q)^{\cdot}\mathbf{I} \quad (12)$$

$$\dot{\mathbf{f}} = -\int_{V_t} \mathbf{B}_L{}^T\frac{\partial R}{\partial \dot{\bar{e}}}\dot{\bar{e}}\mathbf{S}\,dV; \quad \dot{\bar{e}} = \sqrt{\frac{2}{3}\mathbf{D}:\mathbf{D}}; \quad \mathbf{B}_L = \frac{\partial \mathbf{N}}{\partial \mathbf{x}}$$

Where α, thermal expansion and θ, temperature, couple the problem.

2.4 Unsteady State Thermal Conduction Analysis.

The integral form of the energy equation to be solved is given by the following expression

$$\int_{V_t} r\,c\frac{\partial q}{\partial t}\,dV = \int_{S_t} \mathbf{k}\frac{\partial q}{\partial \mathbf{x}}.\mathbf{n}\,dS + \dot{W}_f + \dot{W}_p \quad (13)$$

Where r is the density, c is the specific heat, **k** is the thermal conductivity, and W is the rate of dissipation due to friction and plasticity effects:

$$\dot{W}^f = \int_{S_t^{csl}} m\mathbf{t}\cdot\dot{\mathbf{u}}\,dS; \quad \dot{W}_p = \int_{V_t} h\,\bar{s}\,\bar{D}^p\,dV \quad (14)$$

Where η measures the part of the plastic work converted into heat while μ represents the friction coefficient associated with tangential forces **t**. On applying the principle of virtual temperatures and upon discretizing the same volume with the above interpolation function (Rebelo and Kobayashi, 1980) as:

$$\mathbf{C}\dot{\mathbf{T}} + \mathbf{K}_k\mathbf{T} = \dot{\mathbf{Q}}_p + \dot{\mathbf{Q}}_f + \dot{\mathbf{Q}}_k$$
$$\mathbf{C} = \int_{V_t} \mathbf{N}^T c\mathbf{N}\mathbf{dV}; \quad \mathbf{K}_k = \int_{V_t} \mathbf{B}^T\mathbf{k}\mathbf{B}\,dV_t \quad (15)$$
$$\dot{\mathbf{Q}}_\mathbf{p} = \int_{V_t} \mathbf{N}^T h\dot{W}_p\,dV; \quad \dot{\mathbf{Q}}_f = \int_{S_t^{csl}} m\dot{W}_f\,dS$$

Friction dissipation at the interface in slip condition.

2.5 Contact Problem

Solution of the contact problem, between tool and workpiece is based on Coulomb's law. In it, localized stick or slip conditions depend on the magnitude of the friction coefficient. For conditions of rigid stick, dissipation potential is added to the general virtual work statement used above. In the contact interface adhesion and slipping/sticking contact occur. Considered the discretization used above, the final result is the addition of stiffness term K_f to the left side of expression and a vector term to the right side (Cheng and Kikuchi, 1985).

*2.*6 Separation Criterion

As the tool advances, each node on the path line was assumed to separate at the cutting edge into a chip surface node and a machined surface node when the strain energy density reached a critical value (Lin, 1992).

3. MODEL IMPLEMENTATION

A quasi-static simulation, including the model parameters considered above was implemented and results are shows ahead. The model was run assuming a HSS tool having a rake angle of 8 degrees, moving with a velocity of 122 m/min. Coefficient of friction between 0.2 and 0.5 were considered. The workpiece was assumed made of AISI 1020, E= 207 GPa, ν=0,292, σ_o=210 MPa. The model was implemented using an ABAQUS FEM code (ABAQUS, 1994). Chip formation and separation during the process is shown in Figure 3. Figure 4 shows the equivalent plastic strains. It concentrates in the primary shear zone and close to the tool-chip interface, in agreement with the Merchant's theory. Figure 6 shows the isotherms of the temperature distribution in the tool, chip and workpiece. It shows the temperature rise due to the combined effects of two principal heat sources, the plastic strain in the shear plane and the frictional heat dissipation in the chip-tool interface. In generating this plot it was assumed room temperature as initial condition, T_o. Maximum temperatures occur inside the shear zone and at the secondary shear zone.

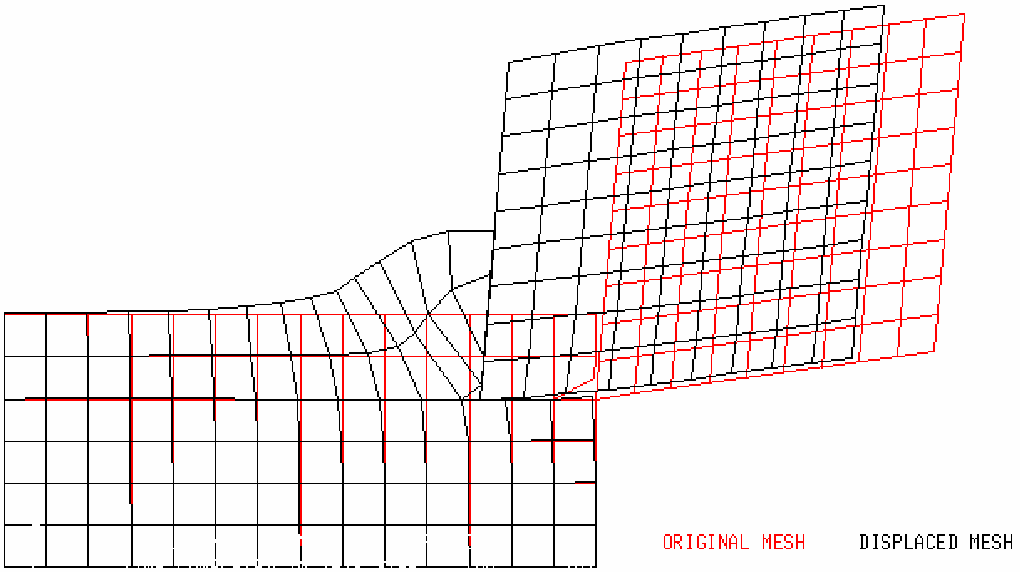

Figure 3. Chip formation: (red) undeformed, black, deformed

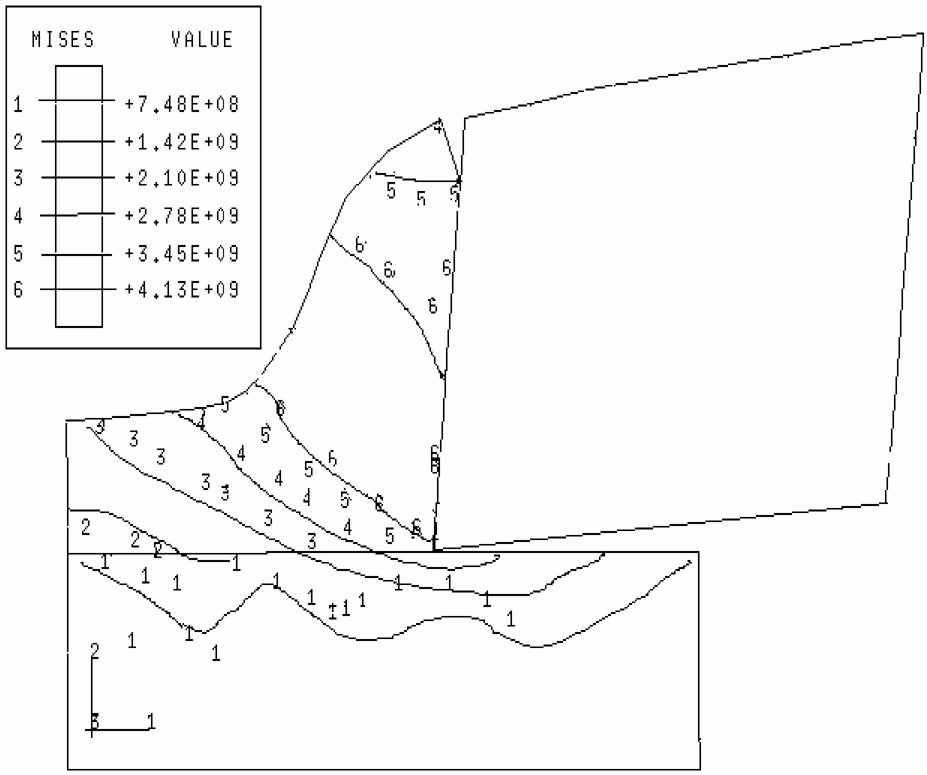

Figure 4. Equivalent Stress

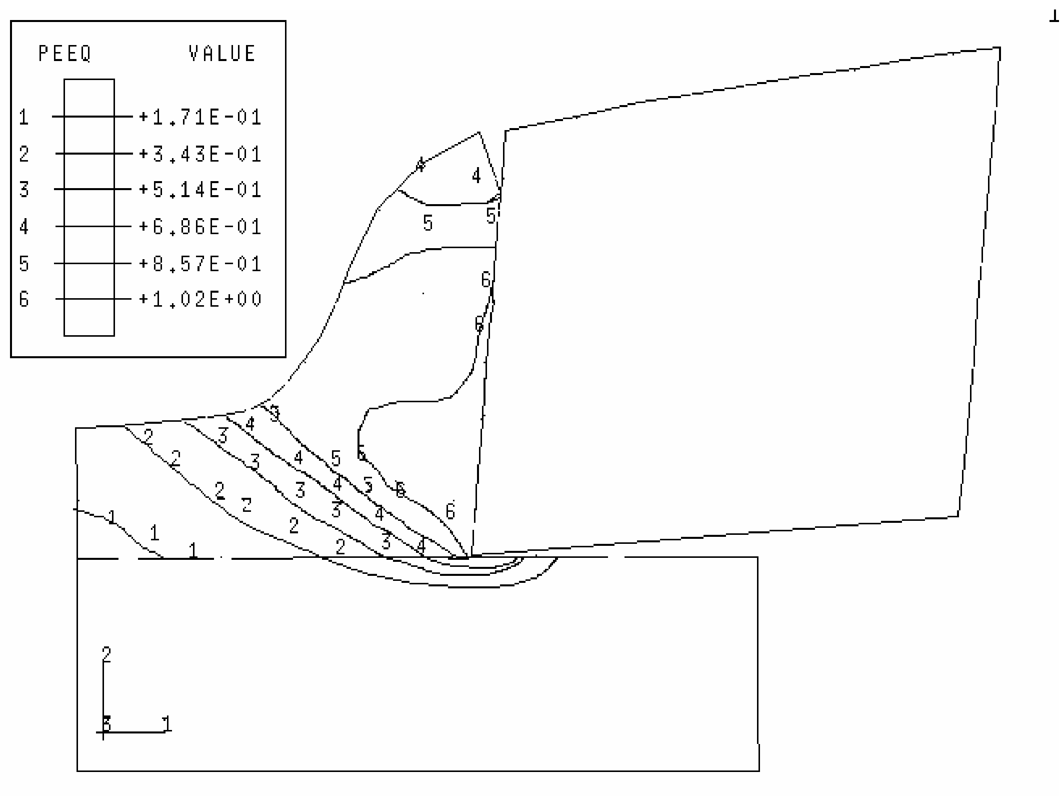

Figure 5. Equivalent plastic strain

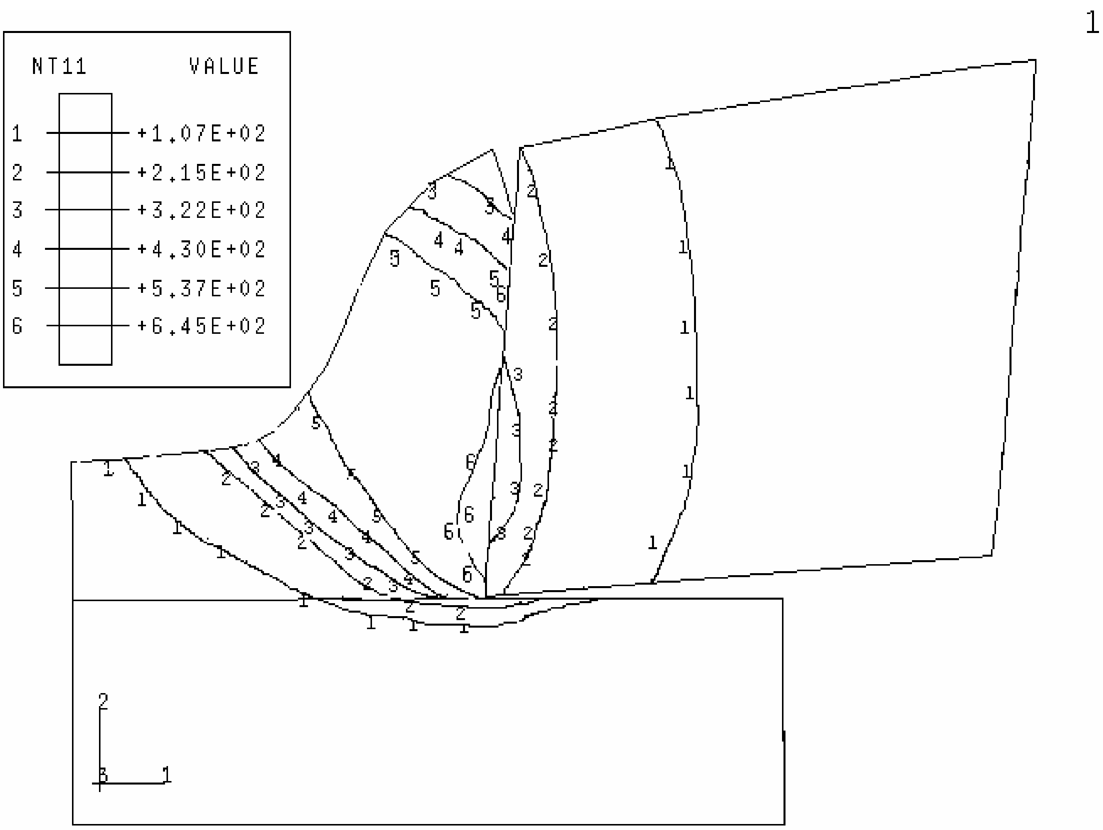

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Figure 6. Cutting temperature distribution

4. CONCLUSIONS

Overall observation of the results produced and comparison with some of the available results in the open literature show good agreement. Temperature fields, as well as stress fields behave like, and have values in the range of the ones obtained experimentally or numerically. In the primary zone, top temperatures in the vicinity of 600 degrees were obtained. Only conduction to the tool was allowed and that in a half-and-half proportion. Top temperatures a little bit lower should be expected that the heat transfer in the supposed isolated portions of the model do occur. Allowance for radiation or exchange with lubricants, when they are present, would include small modifications to the model. In what concerns the stresses, again good results are obtained, with discrepancies of the order of 30%. The model presented could be modified also to include anisotropic effects as well as incorporate cinematic hardening conditions. In it the Shirakashi expression for the description of behaviour of metallic materials in a equivalent setting was used (Usui & Shirakashi, 1982). Forces, can also be computed from this model, by integration of the stresses (Madrigal, Batalha & de Aguiar, 2000), and they present the trend and range of measured results.

5. AKNOWLEDGEMENTS

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6. REFERENCES

ABAQUS *1994. Theory Manual.* Version 5.4. Ed. Hibbitt. Karlsson and Sorensen. USA.

Aravas, N, 1987. On the Numerical Integration of a Class of Pressure-dependent Plasticity Models. *Int. J. for Num. Methods in Eng.* 24: 1395-1416.

Cheng, J.H. & N. Kikuchi, N. 1985. An incremental Constitutive Relation of Unilateral Contact Friction for Large Deformation Analysis. *J. Applied Mechanics*. 52: 639-648.

Hill, R., 1959. Some Basic Principles in the Mechanics of Solids without a Natural Time. *J. Mech. Phys. Solids.* 7: 209-225.

Komvopoulos, K. 1991. Finite Element Modeling of Orthogonal Metal Cutting. *Trans. ASME - J. Of Engineering for Industry.* 113:. 253-67.

Lin, Z. & S. Lin 1990. An Investigation on a Coupled Analysis of a Thermo - Elastic - Plastic Model during Warm Upsetting. *Int. J. Machine Tools and Manufacturing.* 30(4): 599-612.

Lin, Z. & S. Lin 1992. A Coupled Finite Element Model of Thermo-Elastic-Plastic Large Deformation for Orthogonal Metal Cutting. *Trans. ASME J. Engineering for Industry.* 114: 218-26.

Lin, Z. C. 1995. A Study of Orthogonal Cutting with Tool Flank Wear and Sticking Behavior on the Chip-Tool Interface. *Journal of Materials Processing Technology.* 52: 524-538.

Mackerle, J. 1999. Finite-element analysis and simulation of machining: a bibliography (1976 - 1996). *J. Mat. Processing Technology.* 86: 17-44.

Madrigal, M.R., Batalha, G.F., de Aguiar, J.B., 2000. A Finite Element Simulation of the Chip Formation in Orthogonal Metal Cutting, In: Pietrzyk, M. et al. (eds.), *Metal Forming 2000 - Proc. 8th Int. Conf. On Metal Forming, Krakow, 3-7 Sept. 2000:* 197-202. Rotterdam: Balkema.

Merchant, M. E. 1945. Mechanics of Metal Cutting Process - 1st part: Orthogonal Cutting. *Journal of Applied Physics.* 16: 267-275.

Muraka, P., G. Barrow & S. Hinduja 1979. Influence of the Process Variables on the Temperature Distribution in Orthogonal Machining Using the Finite Element Method. *International Journal Mechanical Science*. 21: 445-456.

Obikawa, T. & E. Usui 1996. Computational Machining of Titanium Alloy - Finite Element Modeling and a Few Results. *Trans. ASME - J. of Engineering for Industry*. 118: 208-215.

Piispanen, V. 1948. Theory of Formation of Metal Chips. *Journal of Applied Physics*. 19: 876-881

Rebelo, N. & S. Kobayashi 1980. A coupled analysis of viscoplastic deformation and heat transfer parts I and II, *Int. J. Mech. Science*. 22: 699-718.

Stevenson, M., P. K. Wright & J. G. Chow 1983. Further Developments in Applying the Finite Element Method to the Calculation of the Temperature Distribution in Machining and Comparisons with the Experiment. *Trans. ASME-J. of Eng. for Industry*. 105: 149-154.

Tay, A. O. & M. G. Stevenson 1976. A Numerical Method for Calculating Temperature Distribution in Machining from Force and Shear Angle Measurements. *International Journal Machine Tool Design and Research.*16: 335-349.

Usui, E. & T. Shirakashi 1982. Mechanics of Machining - From Descriptive to Predictive Theory. In: ASME (ed.), PED 17- *On the Art of Cutting Metals-75 Years Later:* 13-35. Michigan: ASME.

Yamada, Y., K. Takatsu & K. Iwata 1973. Non-linear Analysis by the Finite Element Method and some Expository Examples. In: *Theory and Practice in Finite Structural Analysis.* Tokyo, University of Tokyo Press.

Merchant, M. E. 1945. Mechanic of Metal Cutting Process - 1st part: Orthogonal Cutting. *Journal of Applied Physics*: 16, 267-75.

Hinton, E. & D. R. J. Owen 1981. *An Introduction to Finite Element Computations*. Pineridge Press Limited.

Kleiber, M. 1989. *Incremental Finite Element Modelling in non-linear Solid Mechanics*. John Wiley & Sons. 61-72.

Komvopoulos, K. 1991. Finite Element Modelling of Orthogonal Metal cutting. *Trans. ASME - Journal of Engineering for Industry*: 113. 253-67.

Lin, Z. C. & S. Y. Lin 1990. An Investigation on a Coupled Analysis of a Thermo - Elastic - Plastic Model During Warm Upsetting. *Int. J. Machine Tools and Manufacturing:* 30. 4. 599-612.

Lin, Z. C. & S. Y. Lin 1992. A Coupled Finite Element Model of Thermo-Elastic-Plastic Large Deformation for Orthogonal Metal Cutting. *Trans. ASME J. Engineering for Industry:* 114. 218-26.

Lin, Z. C. 1995. A Study of Orthogonal Cutting with Tool Flank Wear and Sticking Behaviour on the Chip-Tool Interface. *Journal of Materials Processing Technology*: 52. 524-38.

Muraka, P. D., G. Barrow & S. Hinduja 1979. Influence of the Process Variables on the Temperature Distribution in Orthogonal Machining Using the Finite Element Method. *International Journal Mechanical Science*: 21, 445-56.

Obikawa, T. & E. Usui 1996. Computational Machining of Titanium Alloy - Finite Element Modelling and a Few Results. *Transactions ASME - Journal of Engineering for Industry:* 118. 208-15.

Piispanen, V. 1948. Theory of Formation of Metal Chips, *Journal of Applied Physics:* 19, 876-81.

Stevenson, M. G., P. K. Wright & J. G. Chow 1983. Further Developments in Applying the Finite Element Method to the Calculation of the Temperature Distribution in Machining and Comparisons with the Experiment. *Transactions ASME - Journal of Engineering for Industry:* 105. 149-54.

Strenkowski, J. S. & J. T. Carroll 1985. Finite Element Model of Orthogonal Metal Cutting. *Transactions ASME - Journal of Engineering for Industry*: 107. 349-54.

Strenkowski, J. S. & J. T. Carroll 1988. Finite Element Model of Orthogonal Cutting with Application to single Point Diamond Turning. *International J. Mechanical Science*: 30, 12, 899-920.

Strenkowski, J. S. & M. Kyoung-Jim 1990. Finite Element Prediction of Chip Geometry and Tool/Workpiece Temperature Distribution in Orthogonal Metal Cutting. *Transactions ASME - Journal of Engineering for Industry:* 112. 313-18.

Tay, A. O. & M. G. Stevenson 1976. A Numerical Method for Calculating Temperature Distribution in Machining from Force and Shear Angle Measurements. *International Journal for Mechanical Tool Design and Research:* 16, 335-49.

Usui, E. & T. Shirakashi 1982. Mechanics of machining -From Descriptive to Predictive Theory. *On the Art of Cutting Metals. 75 Years Later.* ASME. Publication PED. 17, 13-35.

Yamada, Y., K. Takatsu & K. Iwata 1973. Non-linear Analysis by the Finite Element Method and some Expository Examples. In: *theory and practice in finite element structural analysis.* University of Tokyo Press.