Generalized convection and power-law models to represent the residence time distribution for non-ideal laminar flow in a double-pipe heat exchanger

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ABSTRACT

The residence time distribution (RTD) is essential for the design and analysis of continuous thermal processing equipment. For the processing of liquid foods, flow is usually laminar and the classic RTD models often can not characterize non-ideal laminar flow. Such deviations are associated with coils, curves or tube roughness/corrugation. Generalized forms of the theoretical convective and power-law models were proposed in order to characterize non-ideal laminar flow in tubes. Introduced parameters were the breakthrough time and the flow index, respectively. The combined PFR+CSTR model was also considered. To test the models, experimental data was obtained from a sanitary double-pipe heat exchanger using an ionic tracer and a conductivity flow cell. Tube diameter was 4.5 mm with a total length of 19.5 m and internal volume of 311 mL. Tested flow rates were between 12 and 20 L/h of distilled water (960 < Reynolds number < 1620). The model E-curve was fitted to experimental data after numerical convolution with the E-curve of detection unit to correct the signal distortion. Adjusted parameters were correlated with flow-rate. The best fit was obtained with the combined model, followed by the generalized convection and generalized power-law models. The adjusted flow index was under 1.0, suggesting that the velocity profile was flatter than the theoretical parabola. This can be explained by the high roughness to diameter ratio of the tube. The higher breakthrough time obtained from the convection model also indicates a flatter velocity profile. The combined model was useful to determine the contribution of dead space and stagnation zones in the equipment, which are due to the curves and thermocouple connections. The proposed models proved to be useful to represent RTD of non-ideal laminar flow in a tubular system.

Keywords: residence time distribution; thermal processing; heat exchanger; laminar flow

INTRODUCTION

Many liquid food products, such dairy, fruit or egg products are subjected to continuous thermal processing for inactivation of undesired microorganisms and enzymes that compromise the food safety and shelf-life. Heat exchangers are used for rapidly heating and cooling the liquid stream, while a holding tube ensures the desired holding time at the processing temperature. For the proper design and analysis of continuous thermal processing equipment, the residence time and temperature distributions of the product inside the holding tube and heat exchangers must be known. The association of these distributions provides the lethality for given safety or quality attributes [1–5].

Characterization of the hydrodynamics of a vessel can be conducted by interpretation of its experimental residence time distribution (RTD) data. The RTD of a vessel (single fluid, constant density, steady state) can be determined by a stimulus response technique by which a tracer is introduced into the inlet stream and its concentration $C(t)$ is recorded at the outlet stream. This technique is widely used for the evaluation of chemical reactors and packed beds. For an instantaneous pulse input signal, the age distribution function (E-curve) is obtained from Eq. (1), where C_0 is the tracer background concentration, if existent [6, 7].

$$E(t) = \frac{C(t) - C_0}{\int_0^\infty C(t) - C_0 \; dt} \tag{1}$$

For tube flow, the RTD is associated with the velocity profile and the turbulence. The ideal cases of parabolic velocity profile (laminar flow) and flat velocity profile (plug flow) rarely describe the flow through real systems. Since liquid foods often have moderate viscosities, flow is usually laminar with a considerable radial velocity profile, which scatters the RTD. The departure from the idealized flow patterns can be accounted for using RTD models to represent the experimental data. The well known RTD models of “axial dispersion” or “tanks-in-series” can be used to describe non-ideal turbulent flow. Small deviations from

laminar flow are more difficult to represent since the RTD curve is asymmetric. Deviations of ideal laminar flow are associated with coils or curves and also with tube roughness or corrugation.
The objective of this work is to propose generalized forms of the theoretical convective and power-law RTD models in order to characterize non-ideal laminar flow in tubes and to test these models on a sanitary double-pipe heat exchanger.

MATERIALS & METHODS

RTD Models

The theoretical E-curve associated with the parabolic radial velocity profile of laminar tube flow is given by Eq. (2) that is known as the "convection model". Variable τ is the mean residence time, which is calculated as V/Q (volume/flow-rate). Likewise, Eq.(3) presents the theoretical RTD curve for the tube flow of a power-law fluid, where n is the flow index [6, 7].

$$E(t)=\left(\frac{1}{2}\right)\frac{\tau^2}{t^3} \qquad t\geq\left(\frac{1}{2}\right)\tau \tag{2}$$

$$E(t)=\left(\frac{2n}{3n+1}\right)\cdot\frac{\tau^2}{t^3}\cdot\left[1-\frac{n+1}{3n+1}\cdot\frac{\tau}{t}\right]^{\frac{n-1}{n+1}} \qquad t\geq\left(\frac{n+1}{3n+1}\right)\tau \tag{3}$$

Non-ideal laminar flow in tubes occurs when the relative roughness of the wall interferes with the velocity profile or when the tube has curves or it is coiled. In this case, Eqs. (2) and (3) would fail to predict the RTD and an experimental study is necessary. Large deviations from ideal flow with considerable dispersion could be represented by the classic axial dispersion or the tanks-in-series models; however, the long tail of tracer that is characteristic of laminar flow can not be captured correctly.
Since Eqs. (2) and (3) are theoretical, both can be modified by the introduction of parameters. The first parameter would be the active volume of the vessel, V_{active}, that is associated with the measured mean residence time as $V_{active} = Q\tau$. Consequently, the dead volume of the vessel is given by $V_{dead} = V - V_{active}$. The dead volume is related to undesirable stagnation and recirculation areas.
Adaptations of the convection model were previously used by other authors to describe laminar flow in helical coils and chemical reactors [5]. In this work, the approach is the parameterize the breakthrough time in Eq. (1) that is the minimum residence time given by $\alpha\tau$ (α = ½ for the ideal case). Taking into account that the area under the E-curve must be unitary, Eq. (1) can be rewritten as Eq. (4), the "generalized convection model".

$$E(t)=\frac{1}{(1-\alpha)\,t}\left(\frac{\alpha\,\tau}{t}\right)^{\frac{1}{1-\alpha}} \qquad t\geq\alpha\,\tau \tag{4}$$

The proposed parameterization of Eq. (3) is simpler: the power-law index n is disassociated from the rheological behaviour of the liquid and turned into a free parameter. The flow index is related to the shape of the radial velocity profile in tube flow through Eq. (5), where v_b is the mean (bulk) velocity and R is the internal radius of the tube. Note that for n = 1.0 the Newtonian parabolic profile is obtained. Equation (3) with free parameter n constitutes the "generalized power-law model".

$$v(r)=v_b\left(\frac{3n+1}{n+1}\right)\left[1-\left(\frac{r}{R}\right)^{\frac{n+1}{n}}\right] \xrightarrow{n=1.0} 2v_b\left[1-\left(\frac{r}{R}\right)^2\right] \tag{5}$$

Besides the generalized convection and power-law models, a third RTD model is also considered in this work, which is the combined PFR+CSTR model. Its E-curve can easily model a long tail of tracer. Equation (6) provides the E-curve of the combined model, where the parameters are τ_{plug} and τ_{mix} (mean residence times of the plug flow and mixed flow regions). The mean residence time is $\tau = \tau_{plug} + \tau_{mix}$ and the active volume is obtained from $V_{active} = Q\tau$ [6, 7].

$$E(t)=\frac{1}{\tau_{mix}}\,exp\left(-\frac{t-\tau_{plug}}{\tau_{mix}}\right) \qquad t\geq\tau_{plug} \tag{6}$$

Figure 1 presents the dimensionless E-curves ($E_\theta = \tau E$) of the three models using a dimensionless time variable that is $\theta = t/\tau$.

RTD Experiments

The RTD experiments were conducted on the heating section of a tubular continuous pasteurizer with nominal capacity of 20 L/h (see Figure 2). The equipment was previously used for the pasteurization of banana puree and mango puree. Tested fluid in this work was distilled water in laminar flow (flow-rate between 12 and 20 L/h with Reynolds number between 960 and 1620). The heating section was a double-pipe heat exchanger with five hairpins connected in series. The inner diameter was 4.5 mm and the total length was 19.5 m (including the return bends). The internal volume was 311 mL.

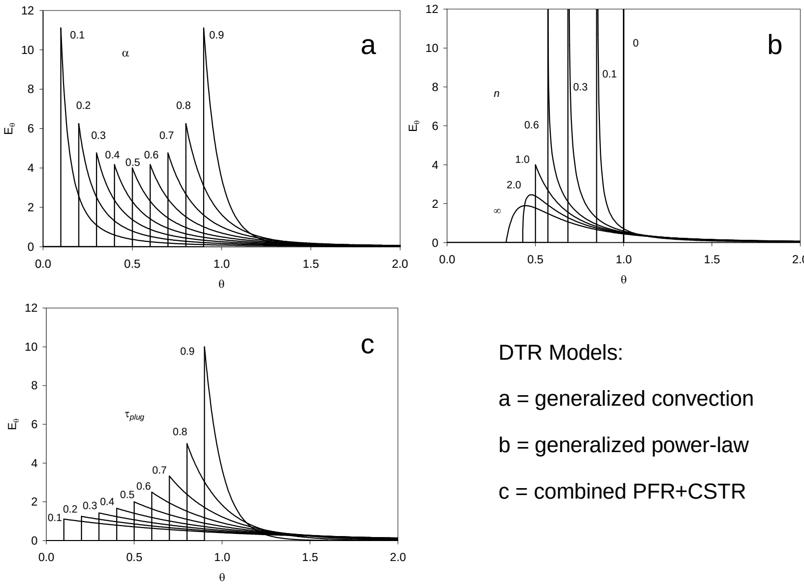

Figure 1. Dimensionless E-curves for the RTD models considered in this work. Curves are from Eqs. (4), (3) and (6).

Figure 2. Photograph of the pasteurizer. Cooling heat exchanger at left and heating heat exchanger at right, with the holding tube in between.

Sodium chloride was used as tracer, which was detected with a conductivity meter with flow-through cell (YSI, Ohio, USA). Data acquisition frequency was 1 Hz. A low background concentration of NaCl was kept to improve the strength of the conductivity signal. A small volume of saturated NaCl solution was injected through a Tee at the entrance of the exchanger using a syringe. All experimental measurements were run in quadruplicate. The tracer concentration was calculated from the electrical conductivity and temperature data using a calibration equation [5]. The experimental values of $E(t)$ were obtained through Eq. (1), using a trapezoidal method to evaluate the integral.

Model adjustment

The RTD data from a pulse response experiment is affected by the way the tracer is injected and by how it is detected. When measuring short residence times, the efficiency of the injection and detection units is of considerable importance [8]. In order to take into account the signal distortion promoted by the detection unit, its E-curve was determined for the same flow-rate and temperature conditions of the RTD experiments and the model curve was convoluted with this E-curve before the model adjustment. The technique is described in details elsewhere [5].

The two model parameters were adjusted in order to minimize the sum of squared errors of the convoluted curve. The convolution was accomplished numerically in the time domain using a time step of 0.05, 0.10 or 0.20 s, depending on the time horizon. The number of discrete time points was 500. The Solver feature of Excel (Microsoft, Redmond, USA), which uses a GRG2 algorithm, was applied to minimize the error function after a good first estimate of the parameters was obtained by trial and error.

RESULTS & DISCUSSION

An example of E-curve fitting is presented in Figure 3 for a given experimental run. Each $E \times t$ plot provides the model curve, the convoluted curve and the experimental points. The clear difference between the model curve and the convoluted curve shows that the signal distortion promoted by the detection unit is significant. The experimental RTD data presented a long tail of tracer that is characteristic of laminar flow and/or stagnation areas.

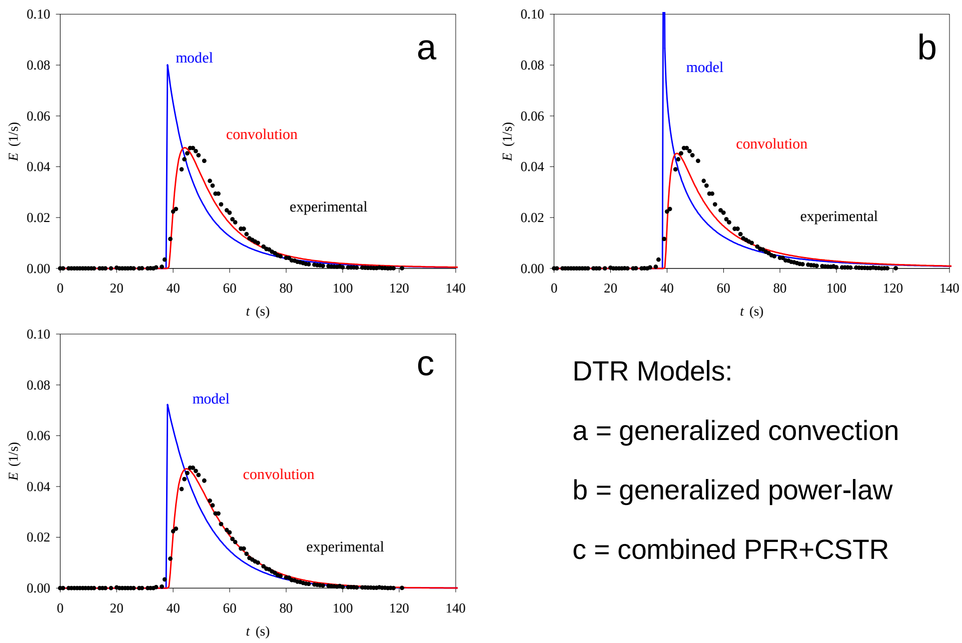

Figure 3. Example of RTD model adjustment for a given experimental run (Q = 16.5 L/h). The sums of squared errors on E were: a) 6.6×10^{-4}, b) 14.1×10^{-4} and c) 3.3×10^{-4}.

The model that provided the best fit for the considered flow-rate range was the combined PFR+CSTR model, followed by the generalized convection model and by the generalized power-law model. The means of the sum of squared errors on E were 4.4×10^{-4}, 8.3×10^{-4} and 14.5×10^{-4} s^{-2}, respectively. The adjusted parameters are presented in Figure 4.

According to Eq. (1), the theoretical breakthrough time for laminar flow is half of the mean residence time, which corresponds to $\alpha = 0.5$. In Figure 4a it can be seen that $\alpha > 0.5$ was observed with a positive effect of the flow rate. This deviation indicates the existence of secondary flow circulation that delayed the breakthrough time for laminar flow in helical coils [9, 10]. Since the exchanger have ten return bends and some tees for thermocouple connection, some internal recirculation may exist.

Figure 4b shows that the flow index for water was $n < 1.0$ that is characteristic of a pseudo-plastic fluid. The velocity profile of a pseudoplastic fluid in laminar flow is flatter than the theoretical parabola for Newtonian tube flow. The results indicate that the relative tube roughness altered the velocity profile by inducing some turbulence near the wall, which consequently made the velocity profile flatter. The return bends and tees may also have this effect locally. The negative effect of the flow-rate on n indicates a tendency towards plug-flow with increasing flow rate.

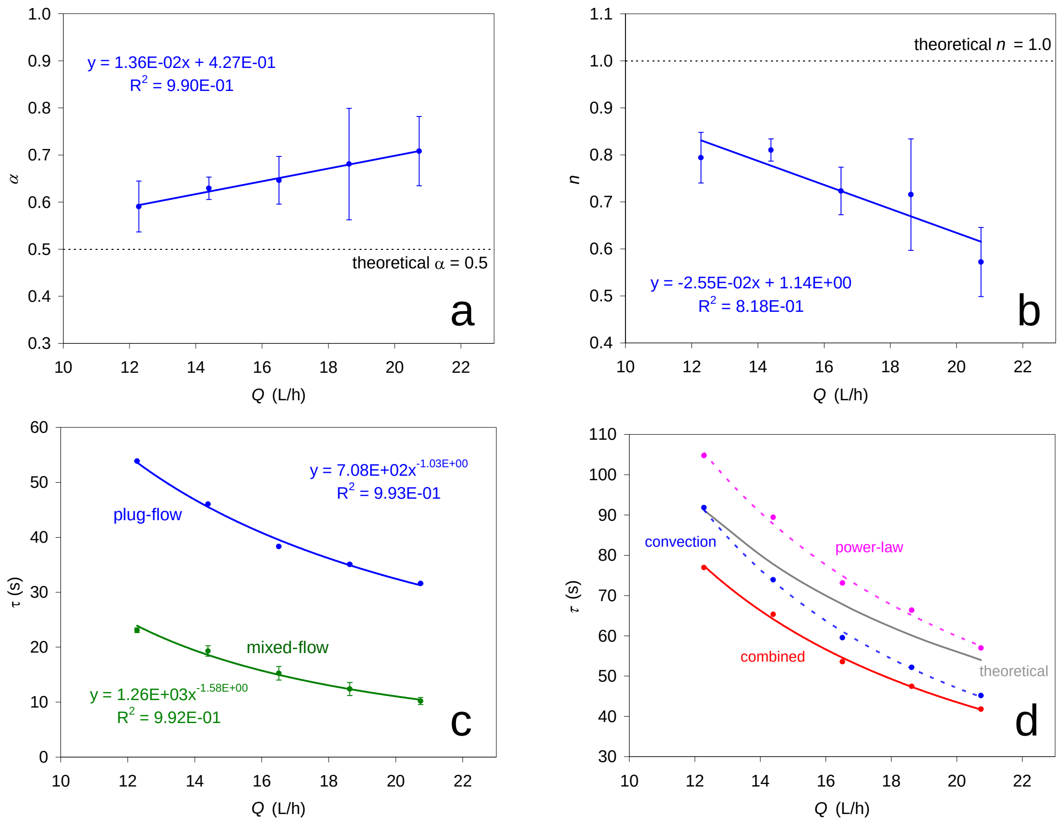

Figure 4. Summary of the results: a) parameter α of the generalized convection model; b) parameter n of the generalized power-law model; c) parameters τ_{plug} and τ_{mix} of the combined model (best fit in this work); d) mean residence time as predicted by the three models and theory (V/Q).

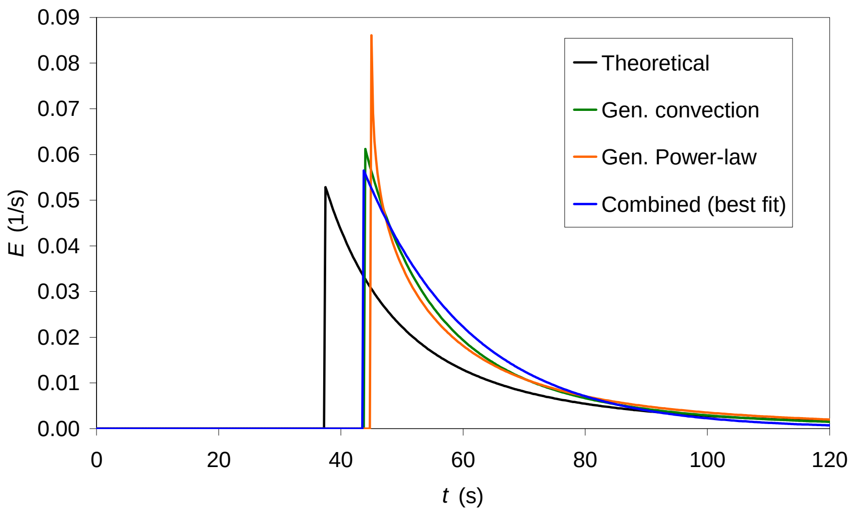

Figure 5. Example of RTD model adjustment for a given experimental run (Q = 16.5 L/h). The sums of squared errors on E were: a) 6.6×10^{-4}, b) 14.1×10^{-4} and c) 3.3×10^{-4}.

The best model fit was obtained with the combined PFR+CSTR model, whose parameters are presented in Figure 4c. For laminar flow, parameter τ_{plug} is actually the breakthrough time associated with the maximum velocity at the centre of the tube. The difference between the convection and combined models resides on the shape of the descending part or the curve. As can be noticed in Figure 2, the convection model better represented this part of the curve.
The difference between the theoretical residence time and the residence time predicted by the combined model can be seen in Figure 4d. This difference suggests that 19% of the internal volume of the exchanger is associated with stagnation and recirculation regions.
Figure 5 brings the theoretical E-curve and the predictions from the adjusted RTD models for laminar flow in the double-pipe heat exchanger with a flow rate of 15 L/h. It is clear that the proposed models could capture the nonideality of the flow. The combined model provided the best fit, but the difference among the model curves in Figure 5 is rather small. The curves in Figure 5 could be combined with the temperature distribution along the exchanger in order to evaluate the distributed and integrated lethality in the heating section of the exchanger [11].

CONCLUSIONS

The proposed models proved to be useful to represent RTD of non-laminar flow in tubular systems, which are usually found in the continuous processing of liquid foods. The E-curve of these models can represent the tail of tracer that is inherent in laminar flow. These models can be used in practice to represent the flow patterns of tubular vessels and to diagnose deviations from non-ideal flow. Future testes will be conducted using a glycerol/water mixture, which is a Newtonian viscous liquid, and a CMC solution, which is a pseudo-plastic viscous fluid.

ACKNOWLEDGMENTS

The authors wish to thank the São Paulo State Research Fund Agency (FAPESP 2009/07934-3) and the National Council for Scientific and Technological Development (CNPq PIBIC) for financial support.

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