Modeling of temperature and lethality distributions in continuous thermal processing using a tubular system

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ABSTRACT

The thermal processing of liquid foods using double-pipe heat exchangers for heating and cooling can produce safe products but, over-processing is usual because of the assumptions used to simplify the process design (negligible changes during heating and cooling and isothermal holding at minimum residence time). The growing demand for food products with minimal deterioration of quality attributes is making producers rethink traditional equipment design and operational conditions. In order to provide an important tool for evaluating the real impact of thermal processing in a food product, a mathematical model was developed for the processing of a non-Newtonian fluid in laminar flow through a double-pipe heat exchanger taking into account heat and mass dispersions, velocity profile and heat losses. The model comprises differential equations for heat and mass balances applied to the heating and cooling sections and to the holding tube and was solved using software gPROMS (PSE). The model was used to simulate the thermal processing of soursop juice (a pseudoplastic fluid) and the results showed a significant effect of the model assumptions. For instance, the calculated S_{value} (number of decimal reductions) regarding yeast and molds was 1.5 considering an isothermal holding tube at the maximum velocity. The S_{value} increased to 1.7 when the velocity profile was introduced in the model and further to 2.3 by introducing the heating and cooling sections. The introduction of mass and thermal dispersions yielded a S_{value} of 3.1. Alternatively, considering the heat losses (natural convection) increased the temperature at the entrance of the holding tube and, consequently, the S_{value} was 3.2. The estimated S_{value} using the combination of dispersion and heat losses was 5.4. It is expected that this study can contribute with the design of continuous thermal process of liquid foods in order to optimize equipment design and operational conditions.

Keywords: Mathematical modeling; thermal processing; non-Newtonian flow; heat transfer

INTRODUCTION

Most liquid foods have a complex flow behavior due to the high apparent viscosity and are commonly processed in tubular heat exchangers [1]. The flow of non-Newtonian foods, like concentrated fruit juices and emulsions in tubular system is usually laminar with a considerable velocity profile. The most common assumptions used to simplify the process design and ensure the food safety are: plug-flow at the maximum velocity (minimum residence time), isothermal process, no mass or thermal dispersion and no heat losses. Moreover, it is considered that the lethality occurs exclusively in the holding tube [2]. However, besides the additional unnecessary energy costs, these simplifications can also impact negatively on nutritional and sensory attributes of foods and, consequently, influence product acceptance by the consumers, which are increasingly demanding regarding these aspects [3].

The study of heat transfer, mass and thermal dispersions, the type of flow and residence time distribution are important factors that have been studied in order to better understand the food processing in heat exchangers and therefore meet the market demand [4,5]. The objective of this work was to develop and test a mathematical model for the continuous thermal processing of a non-Newtonian fluid in laminar flow through a tubular system taking into account the radial mass and thermal dispersions and the heat losses to the ambient.

MATERIALS & METHODS

The development of the mathematical model was based on continuous, steady-state and non-isothermal laminar flow of a non-Newtonian fluid in a double-pipe heat exchanger (concentric tubes), comprising heating section, holding tube and cooling section. In the heating and cooling sections, the fluid product flows through the inner tube and the heating or cooling fluid flows in the annulus countercurrently. The outer surface of the outer tube, in the heating and cooling sections, was considered in direct contact with ambient air. On the other hand, for holding tube, a layer of thermal insulation was considered (see Figure 1).

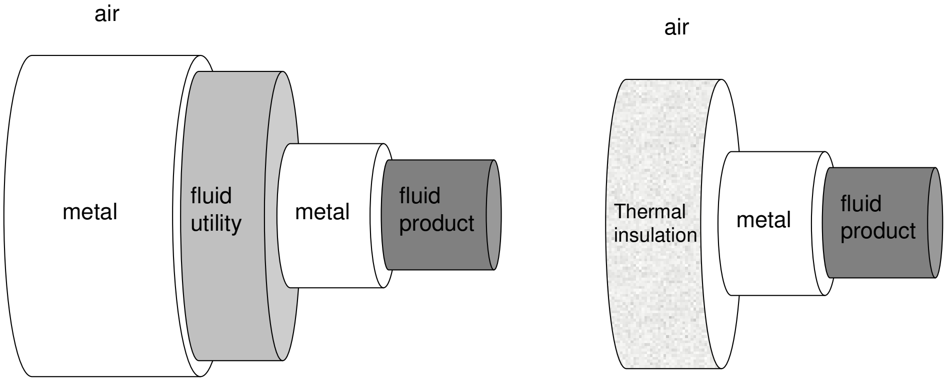

Figure 1. Illustrations of the control volumes considered in the mathematical model. At left: heating or cooling sections. At right: holding tube.

The radial dimensionless domain varied from 0 (center of the tube) to 1 (inner tube wall) and the axial domain varied from 0 to 1 (heating), from 1 to 2 (holding) and from 2 to 3 (cooling). It was admitted that the fluid product presented a generic component "A", which showed changes with temperature and followed first order thermal destruction kinetics according to equation 1 [6]:

$$r_A = \frac{2.303}{D_A} C_A \tag{1}$$

where: C_A = mean concentration, D_A = decimal reduction time (s) and r_a = reaction rate.
Equation 2 was used to calculate the number of decimal reductions of the thermal process for component "A", called S_{value} [6]:

$$S_{value} = \log\left(\frac{C_{Ao}}{C_A}\right) \tag{2}$$

where: C_{Ao} = process inlet concentration and S_{value} = number of decimal reductions (dimensionless).
The velocity profile for a non-Newtonian fluid (power-law theological model), flowing in a tube with circular section in laminar regime is described by equation 3, which was incorporated in the model [6]:

$$v_p = v_{a-p}\left(\frac{3n+1}{n+1}\right)\left(1 - x^{\frac{n+1}{n}}\right) \tag{3}$$

where: n = flow behavior index (dimensionless), v_p = fluid product velocity (m s $^{-1}$), v_{a-p} = average velocity (m s $^{-1}$) and x = radial domain (dimensionless).
Mathematical modeling was done through the use of differential equations of mass and energy conservation with appropriate boundary conditions [7]. The assumptions for the fluid product modeling were as follows: incompressible fluid, developed laminar steady-state flow, radial dispersion (mass and thermal), uniform physical properties in each section and negligible viscous dissipation. The mass and energy balances for the generic component "A" in the fluid product are described by equation 4 and 5, respectively:

$$x \cdot \frac{v_p}{L} \cdot \frac{\partial C_A}{\partial \eta} = \frac{D_{ef-p}}{(r_i)^2} \cdot \frac{\partial}{\partial x}\left(x \cdot \frac{\partial C_A}{\partial x}\right) - x \cdot r_A \tag{4}$$

$$\frac{v_p}{L} \cdot \frac{\partial T_p}{\partial \eta} = \frac{k_{ef-p}}{\rho_p \cdot cp_p \cdot (r_i)^2} \cdot \frac{\partial}{\partial x}\left(x \cdot \frac{\partial T_p}{\partial x}\right) \tag{5}$$

where: cp_p = fluid product specific heat (J kg^{-1} K^{-1}), D_{ef-p} = fluid product effective radial mass diffusivity (m^2 s^{-1}), k_{ef-p} = fluid product effective thermal conductivity (W m^{-1} K^{-1}), L = section length (m), η = dimensionless axial domain, ρ_p = fluid product density (kg m^{-3}), r_i = inside diameter of the inner tube (m) and T_p = fluid product temperature (K).

The assumptions for the utility (service) fluid were: incompressible fluid, developed turbulent steady-state flow, uniform physical properties in each section, thermal axial dispersion and negligible viscous dissipation. The equation 6 is the energy balance regarding the utility fluid:

$$v_{a-s} \cdot \frac{\partial T_s}{\partial \eta} = \frac{k_{ef-s}}{\rho_s \cdot cp_s \cdot L} \cdot \frac{\partial^2 T_s}{\partial \eta^2} - \frac{\dot{Q}_s \cdot L}{\rho_s \cdot cp_s} \tag{6}$$

where: cp_s = utility fluid specific heat (J kg^{-1} K^{-1}), k_{ef-s} = utility fluid effective thermal conductivity (W m^{-1} K^{-1}), Q_s = heat transferred from the utility fluid to the neighborhood (W m^{-3}), ρ_s = utility fluid density (kg m^{-3}), T_s = utility fluid temperature (K) and v_{a-s} = fluid product velocity (m s $^{-1}$).

Equations 7, 8 and 9 represent the energy balance for the inner and the outer tubes (heating and cooling sections) and for the thermal insulation (holding section), respectively:

$$\frac{k_{tb-i}}{\rho_{tb-i} \cdot cp_{tub-i} \cdot L^2} \cdot \frac{\partial^2 T_{tb-i}}{\partial \eta^2} = \frac{Q_{tb-1}}{\rho_{ttb-i} \cdot cp_{tb-i}} \tag{7}$$

$$\frac{k_{tb-o}}{\rho_{tb-o} \cdot cp_{tb-o} \cdot L^2} \cdot \frac{\partial^2 T_{tb-o}}{\partial \eta^2} = \frac{Q_{tb-o}}{\rho_{tb-o} \cdot cp_{tb-o}} \tag{8}$$

$$\frac{k_{insu}}{\rho_{insu} \cdot cp_{insu} \cdot L^2} \cdot \frac{\partial^2 T_{insu}}{\partial \eta^2} = \frac{Q_{insu}}{\rho_{insu} \cdot cp_{insu}} \tag{9}$$

where: cp_{tb-i} = inner tube specific heat (J kg^{-1} K^{-1}), cp_{tb-o} = outer tube specific heat (J kg^{-1} K^{-1}), cp_{insu} = thermal insulation specific heat (J kg^{-1} K^{-1}), k_{tb-i} = inner tube conductivity (W m^{-1} K^{-1}), k_{tb-o} = outer tube conductivity (W m^{-1} K^{-1}), k_{insu} = thermal insulation conductivity (W m^{-1} K^{-1}), Q_{tb-i} = heat transferred by the inner tube with the neighborhood (W m^{-3}), Q_{tb-o} = heat transferred by the outer tube with the neighborhood (W m^{-3}), Q_{insu} = heat transferred by the thermal insulation with the neighborhood (W m^{-3}), ρ_{tb-i} = inner tube density (kg m^{-3}), ρ_{tb-o} = outer tube density (kg m^{-3}), ρ_{insu} = thermal insulation density (kg m^{-3}), T_{tb-i} = inner tube temperature (K), T_{tb-o} = outer tube temperature (K) and T_{insu} = thermal insulation temperature (K).

It was considered that mass and thermal Peclet numbers for radial diffusion (Pe_M and Pe_T) were equal. Equations 9 and 10 were used for the product and utility, respectively:

$$Pe_{M-p} = \frac{v_{a-p} \cdot r_i}{D_{ef-p}} = Pe_{T-p} \frac{v_{a-p} \cdot r_i \cdot \rho_p \cdot cp_p}{k_{ef-p}} \tag{10}$$

$$Pe_{T-s} = \frac{v_{a-s} \cdot L \cdot \rho_s \cdot cp_s}{k_{ef-s}} \tag{11}$$

where: Pe_{M-p} = fluid product radial mass Peclet number (dimensionless), Pe_{T-p} = fluid product radial thermal Peclet number (dimensionless), Pe_{T-s} = utility fluid axial thermal Peclet number (dimensionless).

The software g-PROMS (Process System Enterprise, version 3.2) was used for simulation and 400 axial points and 30 radial points were used for the discretization of the variables.

RESULTS & DISCUSSION

The model was applied for the simulation of the thermal processing of 18 ºBrix soursop juice (a pseudoplastic fluid) in laminar flow considering the lethality of yeast and molds ($D_{82.2ºC}$ = 0.57 s, z = temperature gradient to reduce D by a factor of 10 = 7 ºC) in a small-scale equipment (internal diameter of 4.5 mm and each section of the double-pipe heat exchanger with 5.0 m length). Mean thermo-physical and rheological properties were defined regarding the estimated mean temperatures for each section. Inlet temperature of hot water was set to obtain a mean temperature of 74 ºC for the fluid product at the end of the holding tube. Thermal and mass Peclet numbers were stated as 1000 for product and fluid services. Table 1 shows the assumptions made in each simulation case and the resulting S_{value}.

Table 1. Simulation study cases and resulting S_{value}.

Cases	**Product temperature distribution**	**Product velocity**	**Mass and thermal dispersion**	**Heat losses**	S_{value}
A	isothermal *	plug flow at v_{max}	no	no	1.5
B	isothermal*	velocity profile	no	no	1.7
C	temperature profile	velocity profile	no	no	2.3
D	temperature profile	velocity profile	yes	no	3.1
E	temperature profile	velocity profile	no	yes	3.2
F	temperature profile	velocity profile	yes	yes	5.4

* Considering only the holding tube (without heating and cooling sections).

It is clear that the model assumptions can impact meaningfully in the lethality of micro-organisms. For instance, using the assumption that there are no temperature changes during the process and considering the minimum residence time (case A) provides an S_{value} that is 3.6 times lower than the one in case F. Using the assumptions of case A, it would be necessary to use a holding tube with approximately 18 m length (instead of 5 m length considered in the simulations) to reach the S_{value} of 5.4.

The temperature at the center of the tube ($x = 0$) at the heating section is shown in Figure 2 regarding cases C, D, E and F. As the dispersions and heat losses were negligible in case C, the estimated temperature of the product is the lowest, that has influence in the process lethality estimation. The opposite is observed in case F in which all assumptions were considered. In cases C and E, heat is dispersed radially through the product taking into account only its thermal conductivity, while in cases D and F the turbulence effect is included through the *Peclet* number.

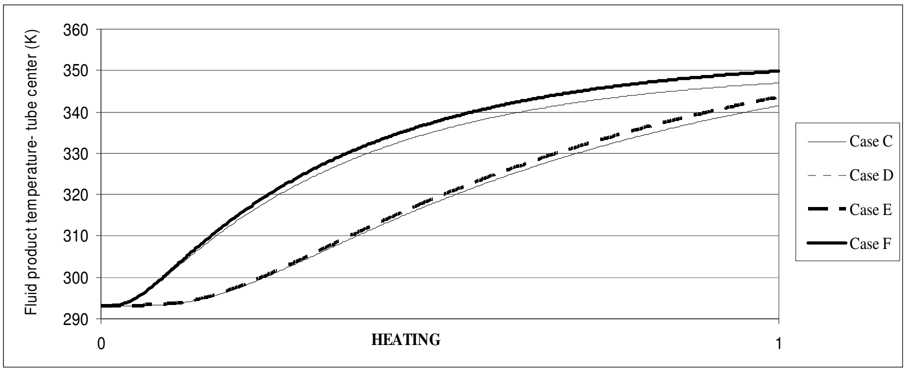

Figure 2. The temperature of the fluid product at the tube center depending on the case simulated.

The results presented on Table 2 can emphasize the importance of considering the lethality that occurs in the heating section to analyze the thermal process. If this lethality is neglected, i.e. if it is considered that the lethality occurs exclusively in the holding tube, the product is kept under high temperature longer than necessary yielding quality reduction and unnecessary energy consumption.

Table 2. S_{value} of the heating section, regarding yeast and molds, according to the studied case.

Cases			
C	D	E	F
0.3	0.6	0.4	1.4

Figure 3 shows the influence of the product Peclet number in the S_{value} (keeping Pe = 1000 for the utility fluid) for case F. It can be said that the more intense the dispersion (lower Pe) the more effective is the thermal processing. The determination of experimental Pe that represents how is the dispersion of the product during the process is essential for the determination of the effective dispersion (mass and thermal) and consequently, for the correct modeling of the process. The determination can be achieved through residence time distribution experiments.

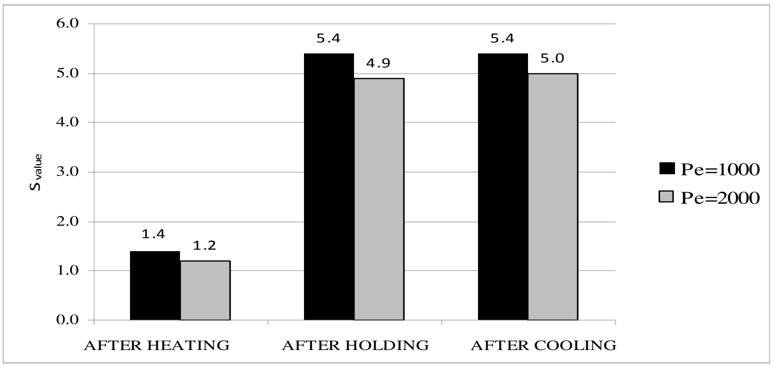

Figure 3. The impact of Peclet number in the S_{value}.

According to Figure 4, it can be noticed that the impact of just considering the thermal dispersion (neglecting mass dispersion) provides a lower S_{value} than when all the assumptions are taken into account (case F, Pe = 1000 for the product and utility). On the other hand, when the assumption of just mass dispersion was considered, the S_{value} estimated is the highest. It can be justified by the higher inlet hot water temperature necessary to guarantee a mean fluid product temperature of 74 ºC at the end of the holding section (process parameter) because of the lower heat transfer rate through the fluid. The inlet hot water for each situation was 80 ºC, 90 ºC and 80 ºC, regarding case F, mass dispersion and thermal dispersion, respectively. The results of process lethality needs to be analyzed together with the temperatures that the product is submitted so that it can be possible to evaluate the impact of the process on the sensorial characteristics of the product and energy costs.

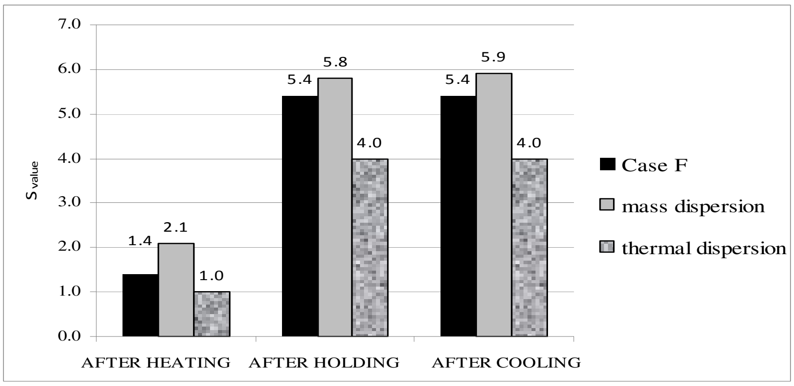

Figure 4. The impact of negligible mass or heat dispersion in the S_{value} (Pe = 1000 for fluid product and fluid service).

Figure 5 shows the lethality distribution (S_{value}) along the heat exchanger, regarding case F (for Pe = 1000 for fluid product and fluid service). The lethality of the process is 5.4 and most of it occurs in the holding tube (4.0). The lethality for the heating is 1.4 and represents 26 % of the total, reinforcing that it should not be neglected.

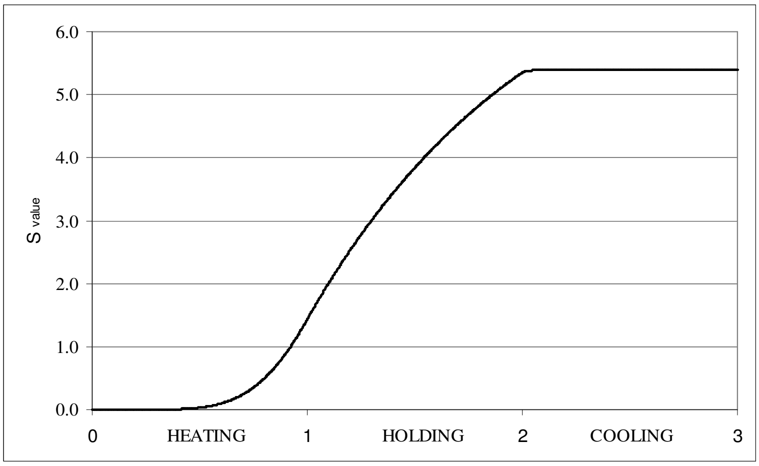

Figure 5. Lethality distribution (S_{value}) regarding case F.

CONCLUSION

The model of the thermal processing of a pseudoplastic fluid flowing in a double-pipe heat exchanger in laminar regime comprising mass and heat transfer, velocity profile and heat losses was presented. The simulations showed the considerable impact of the model assumptions in the estimated S_{value}. These results emphasize the importance of the correct modeling of food processing in order to obtain safe products avoiding the negative effect in product quality and energy consumption.

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