Characterization of non-ideal laminar velocity profile in a tubular thermal process through residence time distribution

P.R. Pegoraro, R.N. Silva, J.A.W. Gut*

University of Sao Paulo, Escola Politecnica, Dept of Chemical Engineering, POBox 61548, Sao Paulo, SP, 05424-970, Brazil
jorgewgut@usp.br

Abstract

The velocity profile in tube flow is useful for the calculation of the lethality of a thermal process. For the laminar flow of viscous liquid foods, the power-law velocity profile is commonly assumed. However, wall roughness, tube corrugation, curves or coiled tubes can distort the ideal velocity profile. Such deviation can be characterized through the experimental determination of the residence time distribution (RTD) of the process. In this work, generalized forms of the power-law laminar velocity profile and turbulent velocity profile in tubes are proposed to characterize non-ideal laminar flow in tubes. The RTD models were derived from the velocity profile equations and they were tested to represent the flow of a 1,0% carboxymethyl cellulose solution (pseudoplastic) in a sanitary double-pipe heat exchanger. Experimental RDT data was obtained using an ionic tracer and a conductivity flow cell. The model curve was fitted to experimental data after numerical convolution with the RTD curve of detection unit in order to correct the signal distortion associated with the flow in the cell. Adjusted model parameters were correlated with the flow rate in order to model the effect of the flow rate on the shape of the velocity profile. The proposed models proved to be useful to represent RTD of laminar flow of non-Newtonian fluids in tubular systems, which are usually found in the continuous thermal processing of liquid foods.

Keywords: residence time distribution, thermal processing, laminar flow

Introduction

Continuous thermal processing with plate or tubular heat exchangers is much used for inactivation of undesired microorganisms and enzymes that compromise the safety and shelf-life of liquid foods. The equipment contains a heating section, a cooling section and a holding section that ensures the desired time at the processing temperature. The degree of thermal treatment can be determined by the time spent in the holding section; however, not all the fluid particles spend the same amount of time. The distribution of the times of the particles leaving the system is called the E-curve or age distribution function, $E(t)$, which characterizes the residence time distribution (RTD) of the process (Fellows, 2000; Jung, and Fryer, 1999; Rao and Loncin, 1974).

The RTD can be determined by the pulse experiment, where a very small amount of a non-reactive tracer is instantaneously injected into the fluid inlet stream and its concentration is continuously recorded at the outlet: $C(t)$. The E-curve is then obtained from Equation (1), where C_0 is the tracer background concentration and the area under the curve is unity, as in Equation (2) (Levenspiel, 1999; Torres and Oliveira, 1998).

$$E(t) = \frac{C(t) - C_0}{\int_0^{\infty} C(t) - C_0 dt} \tag{1}$$

$$\int_0^{\infty} E(t)dt = 1 \tag{2}$$

Residence time distribution studies provide important information about the process and it has been commonly used in determining the performance of industrial heat exchangers; however, RTD of real processes arise from a complex interaction between the velocity profile, diffusion, turbulence and heat transfer (Fillaudeau et al., 2009).

The mean residence time is obtained from Equation (3) and the dimensionless E-curve is obtained from Equation (4), where $\theta = t / t_m$ is the dimensionless time.

$$t_m = \int_0^\infty tE(t)dt \cong \sum_i t_i E(t_i)\Delta t_i \tag{3}$$

$$E_\theta(\theta) = t_m E(t) \tag{4}$$

The spatial time of ideal mean residence time , τ, is defined as the ratio between the volume of the vessel and the flow rate, Q, according to Equation (5).

$$\tau = \frac{V}{Q} \tag{5}$$

In a tubular system, the velocity profile and dispersion are associated with the RTD. The ideal cases of parabolic velocity profile (laminar flow) and flat velocity profile (plug flow) rarely describe the flow in real systems (Gutierrez et al., 2010). Viscous liquid foods usually exhibit laminar flow in continuous reactors, which promotes a considerable dispersion on the residence time (Jung and Fryer, 1999). Using the minimum residence time for the sizing of the holding tube is a conservative way that leads to overprocessing of the food product. Studies of the behavior of fluid particles in a vessel allow a correct sizing of a holding tube and reduce the loss of nutritional value.

The objective of this work is to propose generalized forms of the velocity profile of power-law laminar tube flow and turbulent tube flow in order to characterize the RTD of non-ideal laminar flow in tubular systems and to adjust the models to the experimental data to represent the flow of a 1,0% carboxy-methyl-cellulose solution (pseudoplastic) in a sanitary double-pipe heat exchanger used for the pasteurization of liquid foods.

Residence time distribution models. RTD models are usually derived from ideal models accounting for the deviations implied in real systems. The flow model shall describe the real flow pattern, as much as possible, with few adjustable parameters. If the predicted and experimental response curves match closely, the model is assumed to reflect the actual flow pattern (Rao and Loncin, 1974). Models with one parameter, such as the well known axial dispersion model or the tanks-in-series model, are usually adequate to represent tubular systems (Levenspiel and Bischoff, 1963).

In this study, four RTD models were used to characterize non-ideal laminar flow in a tubular system. Distortions of the ideal laminar velocity profile in tube flow are associated to wall roughness, tube corrugation, curves or coils. The models of axial dispersion and PFR+CSTR association are derived from ideal flow models. The generalized power-law profile and the generalized turbulent profile models are derived from the equations that describe the velocity profile in a circular tube.

Axial dispersion. The axial dispersion model is widely used to represent small deviations from the plug flow and other non-ideal flow patterns in tubular systems (Rao and Loncin, 1974). To characterize the tracer spreading, it is assumed that it diffuses axially in plug-flow. The model parameter is the Peclet number (*Pe*). Ideal plug-flow is obtained with *Pe* approaches infinity. On the other hand, the perfectly mixed tank behavior is obtained when *Pe* is close to zero (Levenspiel, 1999). An adaptation of the Nauman (1985) simplified model provides Equation (6) that result in a good approximation for $Pe > 16$.

$$E_\theta(\theta)=\left(\frac{Pe+1}{4\pi\theta^3}\right)^{1/2}\exp\left(\frac{-(Pe+1)(1-\theta)^2}{4\theta}\right) \qquad (6)$$

PFR+CSTR association. Combined or compartment models consist of ideals reactors associated in series or parallel, which are the plug-flow reactor PFR and the continuously stirred tank reactor CSTR. In this work, a series association of one PFR and one CSTR with the presence of a dead volume was considered, as in Equation (7). The only model parameter is the dimensionless breakthrough time: $\theta^P = \tau^P/(\tau^P+\tau^M)$, where τ^P and τ^M are the spatial times of the PFR and CSTR reactors, respectively.

$$E_\theta(\theta)=\frac{1}{1-\theta^P}\exp\left(\frac{\theta^P-\theta}{1-\theta^P}\right) \qquad \theta\geq\theta^P \qquad (7)$$

Generalized power-law velocity profile. Equation (8) with parameter m = 2 represents the theoretical laminar velocity profile in tube flow. For a non-Newtonian fluid characterized by the rheological model of power law, $m = (n+1)/n$, where n is the flow behavior index. In this work, the flow behavior index is dissociated from the rheological model of the fluid and is turned into a free parameter to adjust the velocity profile (Figure 1a) to the RTD data, according to Equation (9) (Levenspiel, 1984; Garcia-Serna et al., 2007).

$$v(r)=v_{max}\left(1-\left(\frac{r}{R}\right)^m\right) \qquad (8)$$

$$E_\theta(\theta)=\frac{1}{\theta^3}\frac{2}{(m+2)}\left(1-\frac{m}{(m+2)\theta}\right)^{\frac{2-m}{m}} \qquad \theta\geq\frac{m}{m+2} \qquad (9)$$

Generalized turbulent velocity profile. The dimensionless residence time distribution of this model was derived from the equation of velocity profile for turbulent flow in pipes in Equation (10), where the only parameter model is y (Figure 1b). For turbulent flow y = 1/7 is a usual value but, in this work, parameter y is adjusted through the RTD data of the vessel. The equation representing this RTD model is shown in Equation (11), where $\beta=(y^2+3y+2)/2$.

$$v(r)=v_{max}\left(1-\frac{r}{R}\right)^y \qquad (10)$$

$$E_\theta(\theta)=\frac{1}{\theta^2}\frac{2}{y}\left(\frac{1}{\beta\theta}\right)^{1/y}\left[1-\left(\frac{1}{\beta\theta}\right)^{1/y}\right] \qquad \theta\geq\frac{1}{\beta} \qquad (11)$$

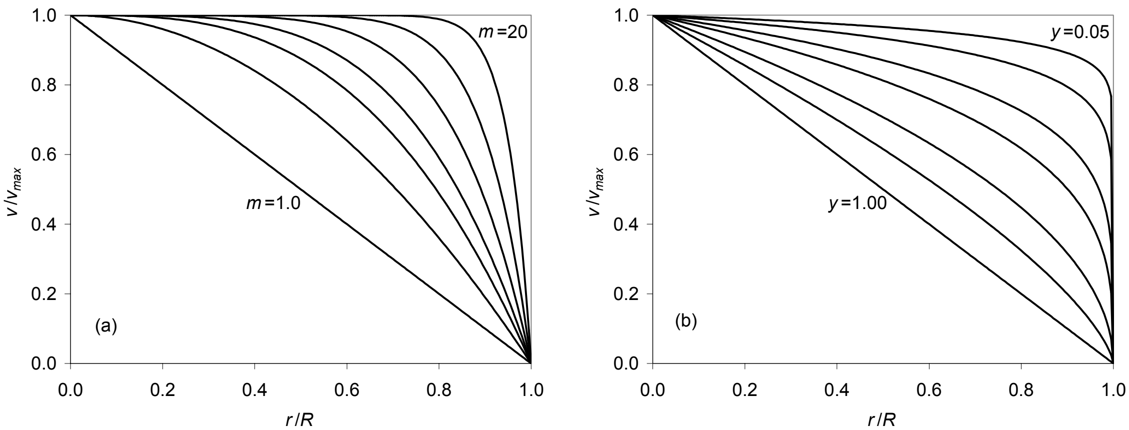

Figure 1. Velocity profile for tube flow: (a) power-law laminar flow and (b) turbulent flow.

Materials and Methods

The experimental setup (Figure 2) consisted of a custom made double-pipe heat exchanger with heating and cooling sections and a holding tube. Each section contained five hairpins and the overall length was 19.3 m per section. The length of the holding tube was 4.7 m. At the end of each hair pin, a thermocouple was inserted for measuring the product temperature. The inner tube internal diameter was 4.5 mm. A positive displacement pump (Netzsch) with a frequency converter (Danfoss) was used for pumping. The tested fluid was a 1.0% CMC (carboxymethyl cellulose) solution, which has a pseudoplastic behavior (Carezzato et al., 2007). The RTD experiments were conducted at ambient temperature. Due to the high pressure at the pump outlet, it was not possible to inject the tracer in the entrance of the heat exchanger, therefore, only one hairpin of the exchanger was considered.

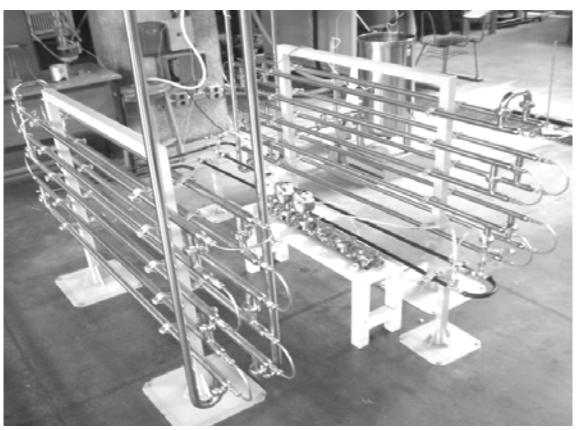

Figure 2. Continuous-flow thermal processing equipment with cooling section (right-side), heating section (right-side) and holding tube (center)

Sodium chloride was used as tracer, which was detected using a YSI3200 conductivity meter with YSI3445 flow-through cell (YSI) and a personal computer. Data acquisition frequency was between 1 and 5 s. The conductivity cell was a 15 mL annular glass tube with two small platinum-iridium electrodes. A small background concentration of NaCl was kept to improve the strength of the conductivity signal. A volume of 0.35 mL of CMC solution saturated with NaCl was injected through the silicon tubing at the desired process point using a 10 mL syringe. Five flow rates were investigated: 10, 20, 30, 40, and 50 L/h. Experiments were conducted in triplicates for the determination of the RTD of the heat-exchanger hairpin and of the detection system.

In order to fit the theoretical models with the experimental data, it was necessary to take into account the distortion of the RTD curve caused by the detection system. This is necessary because the volume of the conductivity cell was significant when compared to the volume of the studied vessel. The convolution of signals procedure proposed by Gutierrez et al. (2010) was followed. The model curve was first convoluted with the E-curve of the detection system prior to parameter fitting. The sum of squared errors on the E-curve was minimized by varying both the model dimensionless parameter (Pe, θ^P, n and y) and the mean residence time (t_m). The numerical convolution of signals, the evaluation of integrals and the minimization of the errors were performed using software Excel (Microsoft).

Results and Discussion

RTD of the detection system. The best fit for the RTD of the detection system was obtained with the axial dispersion model in Equation (6). The flow rate dependence of the mean residence time is presented by Equation (12). Since it was not possible to detect a clear flow rate dependence on Pe, the mean value of 0.87 ± 0.74 was considered. Since the simplification of Equation (6) is valid for $Pe > 16$, the RTD curves with $Pe < 16$ using this Equation fall between the axial dispersion model and the perfectly mixed tank model.

$$t_m = 88.8Q^{-0.846} \tag{12}$$

RTD of the section studied. The Solver feature of Excel was used to adjust the parameters of each model to minimize the sum of squared errors between the experimental data of the heat exchanger and the convolutional curve. The results from the sum of squared errors for each of the four models studied for the section of the heat exchanger are shown in Figure 3. It is possible to check that the model that best fitted the experimental data was the generalized turbulent velocity profile, followed by the axial dispersion, the combined PFR+CSTR and then by the generalized power-law velocity profile model. In fact, the generalized Power-Law velocity profile model did not provide reliable results because the parameter n presented a large variation and often tended to infinity during the minimization of the error.

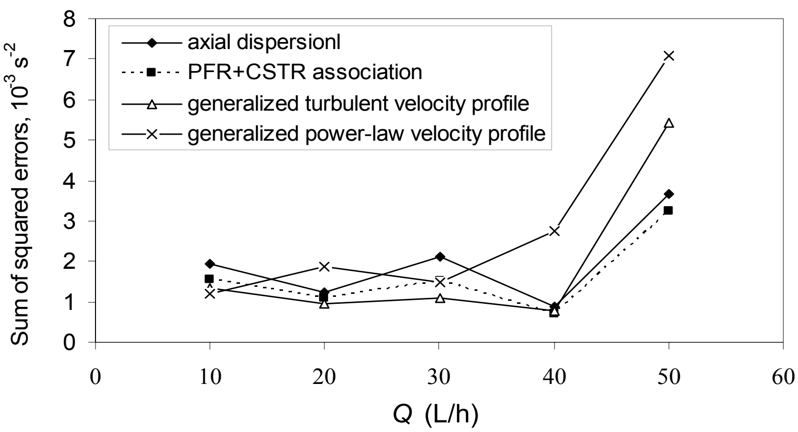

Figure 3. Sum of squared errors on model fitting of the E-curves.

An example of convolution and model adjustment with the generalized turbulent velocity profile model, which was convoluted with the detection system E-curve for the same flow rate, is shown in Figure 4. The error between the convoluted curve and the experimental points was minimized in order to adjust the model parameter, y, and the mean residence time, t_m. It can be seen that the distortion of the E-curve caused by the detection system is not negligible.

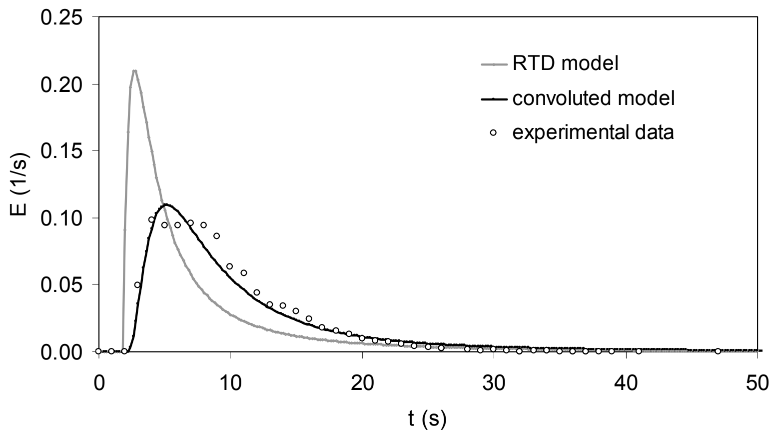

Figure 4. Example of adjustment of the turbulent profile model using numerical convolution.

Equation (13) brings the mean residence time correlation obtained by fitting the experimental data with the generalized turbulent velocity profile model.

$$t_m = 106Q(L/h)^{-0.603} \qquad (13)$$

The influence of the flow rate on model parameter y is shown in Figure 5. Adjusted linear correlation for this dependence is in Equation (14). Although this model had the best fit with experimental data, the result was unexpected because the values of the parameter y were above 1.0 (see Figure 1), except for the flow of 10 L / h. This means that the concavity of the velocity profile is inverted. This variation is probably due to the radial diffusion of the tracer, that is not accounted for in the Generalized Power-Law velocity profile and Generalized Turbulent velocity profile models. However, it can represent the RTD of the tubular system.

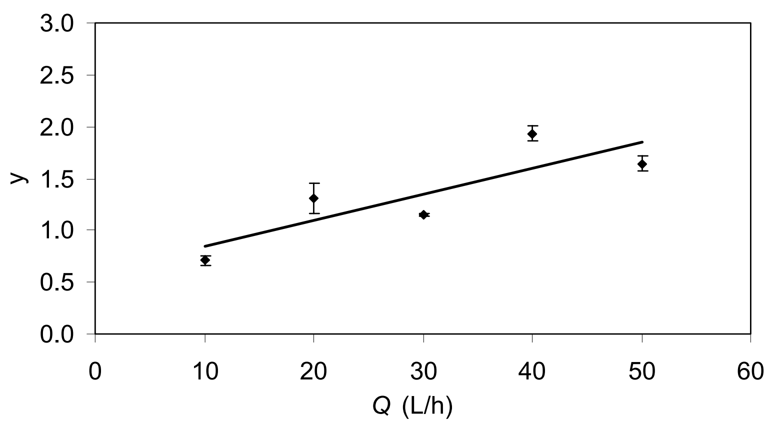

Figure 5. Breakthrough time for studied section as affected by flow rate

$$y = 0.0250Q(L/h) + 0.600 \qquad (14)$$

Conclusions

The RTD model that best fitted the experimental data of non-ideal laminar flow in the tubular system was the generalized turbulent velocity profile model, even though the parameter values were different form expected. The determination of the RTD for laminar flow is difficult because of the long residual tail of tracer that strongly affects the mean residence time. Further tests using different fluids (water and water/glycerin mixture) in the same system will be performed to better evaluate the application of the generalized power-law and generalized turbulent velocity profile models.

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