IMPROVEMENTS TO A QUEUE AND DELAY ESTIMATION ALGORITHM UTILIZED IN VIDEO IMAGING VEHICLE DETECTION SYSTEMS

Marshall T. Cheek (Corresponding Author)
Engineer II
Lockwood, Andrews & Newnam, Inc.
2925 Briarpark Drive, Suite 400
Houston, Texas 77042
Phone: (713) 266-6900
E-mail: mtcheek@lan-inc.com

H. Gene Hawkins Jr., Ph.D., P.E.
Associate Professor, Zachry Department of Civil Engineering
Research Engineer, Texas Transportation Institute
3136 TAMU
Texas A&M University
College Station, TX 77843-3136
Phone: (979) 845-9294
Fax: (979) 845-6481
E-mail: gene-h@tamu.edu

and

James A. Bonneson, Ph.D., P.E.
Research Engineer
Texas Transportation Institute
3135 TAMU
The Texas A&M University System
College Station, TX 77843-3135
Phone: (979) 845-9906
Fax: (979) 845-6254
E-mail: j-bonneson@tamu.edu

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ABSTRACT
Video Imaging Vehicle Detection Systems (VIVDS) are steadily becoming the dominant method for the detection of vehicles at a signalized traffic approach. This research investigated the use of VIVDS for quantitatively estimating queue length at signalized intersections. The technique proposed in this research uses a series of strategically placed virtual detectors to produce queue length measurements. Queue length measurements are then processed and corrected using numerous statistical techniques and ultimately provide accurate estimates of queue length. The results of this research show that a linear regression method using previous queue measurements to establish a queue growth rate, plus the application of a Kalman Filter for minimizing error and controlling queue growth produced accurate estimates of queue length. During validation tests, the linear regression technique was capable of describing 86 percent of the variance in observed baseline queue length data. The researcher would recommend the implementation of the linear regression technique with a Kalman Filter, because this method requires little calibration, while producing an adaptive queue estimation method that has proven to be accurate. This system provides a cost effective method for producing quantitative estimates of queue length which may be used in adaptive or traffic responsive control systems or so that the traffic engineer can more easily determine MOEs at a signalized intersection.

INTRODUCTION
There is a need for real-time queue and delay estimation of vehicles at signalized intersections, as often times, modern traffic signal controllers are able to use real-time queue and delay data to optimize signalized intersection performance. Queue length estimates can provide a valuable indication to the traffic engineer as to roadway conditions, and can allow the engineer to assess the performance of a roadway. Historically, inductive loop detectors have been used to collect this information. More recently, VIVDS are steadily becoming the preferred method for detecting vehicles at signalized intersections. VIVDS are progressively replacing inductive loop detectors at signalized intersections due to the high cost of maintenance and frequency of repair involved with non-VIVDS detection (1). However, VIVDS have some limitations when used to detect traffic (particularly queues) located further from the intersection. After implementing initial versions of the QDA, it was realized that using the furthest active detector from the stop line without a correction mechanism, did not produce accurate queue estimates. Therefore, mathematical correction mechanisms are essential for eliminating noise associated with measured queue length values collected by VIVDS hardware. In this research, a queue length estimation algorithm was developed for VIVDS to produce reasonable estimates of queue length, while minimizing noise associated with measured queue length estimates collected by VIVDS hardware.

Problem Statement
This research focused upon evaluating the potential accuracy that could be realized in a VIVDS queue detection and delay estimation algorithm (QDA) during the red phase of a signal cycle. The original algorithm was developed as part of National Cooperative Highway Research
Program (NCHRP) Project 3-79 (1). This QDA was based on a weighted average of previous and current estimates of queue length in order to produce output queue length estimates. However, the original algorithm contained a mathematical bias, leading to estimates output from the QDA that were inherently low. It also required some effort to calibrate. The researchers sought to produce an improved QDA that minimized the bias, required less calibration, and produced accurate estimates of queue length. They achieved this by using a different queue estimation technique and applying a Kalman Filter to minimize error and control queue growth. The research also uses functions common to most VIVDS hardware, such that the proposed mathematical technique can be easily implemented using existing VIVDS functions and technology.

**LITERATURE REVIEW**

This section is intended to give an overview of the fundamental concepts and principles involved in the determination of a queue estimation model. The literature review introduces VIVDS concepts, applications of VIVDS and the origins and theoretical explanation of the Kalman Filter.

**Video Imaging Vehicle Detection Systems**

Early development of VIVDS began in the 1970s in the United States and throughout the world (2). Today, VIVDS are becoming an increasingly popular method for detecting vehicles at signalized intersections. VIVDS are primarily used for presence detection near the stop line of a signalized approach. VIVDS cameras are typically placed on mast arms or on mast arm poles. VIVDS technology utilizes a series of virtual video detection zones placed on the roadway through the use of specialized hardware typically consisting of cameras and controller cards.

The primary benefits of these systems reside in their cost efficiency and adaptability compared to alternative detection methods such as inductive loop detectors (3).

**VIVDS Application Research and Development**

VIVDS utilize technology that has existed since the 1950s. While limited in scope with respect to the applicability of these systems, most early VIVDS systems were developed to provide presence detection on signalized intersection approaches. In the 1990s, research was conducted that investigated the feasibility of using VIVDS for purposes other than presence detection. Research conducted by Michalopoulos et al. investigated the possibility of using VIVDS for more advanced traffic data measurements (2, 4). This research measured speed, and travel time associated with vehicles traveling along a corridor. The results of this research showed that given the advances in VIVDS technology at the time, VIVDS measurements could be relied upon to make accurate measurements of speed and travel time. Results of this study showed that advanced VIVDS technology used in the study proved to be 95-97 percent accurate for measuring the speed of vehicles through a corridor. Furthermore, the results of this study showed that for simple presence detection, VIVDS performed just as well as loops during experimentation. Research performed by Michalopoulos et al. also mentions the early realization and possible development of VIVDS technology for the purposes of producing quantitative queue estimates, as well as estimating measures of effectiveness such as delay, number of stops, and energy consumption (2). However, no documents could be found that present results as to the findings of this type of research.
Most research involving VIVDS and queue length detection, involves the simple process of identifying when queues are present on a subject approach (5). These detection systems offer only a mechanism by which to qualitatively indicate whether a queue has formed. Research conducted by Rourke and Bell investigated the use of fast Fourier transforms (FFT) in order to detect the formation of queues. This method was able to detect queue presence by defining an analysis window, then utilizing the frequency and power of the spectrum associated with images produced within this analysis window (5, 6). Furthermore, methods developed by Hoose utilized a full frame approach for queue detection (6). The full frame method is able to obtain an image no matter the position of the object on the screen. Hence, the full frame is utilized in the analysis, as opposed to the previous method that only analyzes objects within a specified analysis window. The full frame method is then able to track the obtained image, in this case a vehicle, and is able to track the object through a succession of frames. Both of these methods have been used to establish queue presence detection algorithms. The queue presence information can then be passed to a traffic signal controller, and a controller response can be initiated. Additionally, this information can provide a monitoring system for alerting traffic management personnel of roadway conditions (5, 6).

Limited research pertaining to the quantitative measurement of queue length using VIVDS could be found. The researcher was able to identify only one application of VIVDS technology where researchers claim to have successfully implemented VIVDS to estimate the length of a traffic queue. In 1995, the Institution of Electrical Engineering in Great Britain published a paper entitled Real-time Image Processing Approach to Measure Traffic Queue Parameters (7). The objectives of this research were intended to quantitatively establish measurements in real-time pertaining to traffic queue length.

The algorithm utilized by the authors of this paper consisted of two components, motion detection and vehicle detection. The motion detection algorithm described in this paper is essentially the same process by which standard VIVDS detectors operate. This process involves the comparison of consecutive frames. While applying noise and background filters, the algorithm is capable of distinguishing differences in vehicle location between the successive frames. Thus, if imaging properties associated with vehicles surpasses a specified threshold, a detection event is recorded. The second algorithm, vehicle detection, incorporates edge detection. Edge detection utilizes a technique that analyzes the boundaries of objects that appear in each frame of an image. These areas represent areas of substantial structural properties when viewing the full frame image produced by VIVDS. Edges are also known to be less sensitive to variations in ambient lighting. Thus edge detectors were believed by these researchers to be an optimal method for detecting precisely where vehicles are located on a roadway by placing edge detectors where vehicle outlines are likely to exist (7).

The combination of motion and vehicle detection algorithms ultimately produces the estimate of queue length. The motion detection algorithm is used to distinguish areas of relatively little motion, to areas where substantial motion is present. Then, the vehicle detection algorithm serves as a refinement tool, whereby the areas of relatively little motion are analyzed by edge detectors to determine if vehicles are present within this region. If a queue is detected, a queue length is reported based on the calibration input by the engineers (7). Limited documentation of the actual experimental procedure could be located, nor could other documents that reference this technique. This method implements advanced imaging hardware that is not typical of a standard VIVDS setup. This distinguishes this research from that proposed in this
research, whereby a queue and delay estimation algorithm is to be implemented in a generic way so that a variety VIVDS hardware can use the technique.

Kalman Filters
In 1960, the creation of a mathematical filtering procedure for the optimization of discrete-data linear filtering problems was published by Rudolph Kalman. The filter was designed to provide recursive solutions to multiple-input, multiple-output systems intended to find optimal solutions based on noisy outputs (8). The Kalman Filter minimizes the mean-squared error. In other words, it minimizes the squared difference between an estimator and the value in which the estimator is approximating. The appeal of the Kalman Filter involves this technique’s ability to minimize error in real-time associated with a system’s theoretical performance based on measured performance of the system collected at regular intervals. Furthermore, drastic improvements in computer technology around 1960 aided the widespread acceptance of the Kalman Filter for a multitude of applications and made this technique ideally suited for real-time estimation procedures (9).

The Kalman filter is designed to minimize the variance of the estimation error experienced during the output of a linear system. Accordingly, in order for a Kalman Filter to be implemented, the process must be described in linear terms (10). A linear system is simply the process that can be described by the following two equations involving the state equation (Equation 1), and the observed measurement equation (Equation 2) (9, 11):

\[ x_k = Ax_{k-1} + Bu_{k-1} + w_{k-1} \quad (Equation 1) \]
\[ z_k = Hx_k + v_k \quad (Equation 2) \]

where
- \( x_k \) = process state vector at time \( t_k \),
- \( A \) = matrix relating \( x_{k-1} \) to \( x_k \),
- \( B \) = matrix relating optional control input, \( u_{k-1} \), to the state, \( x_k \),
- \( u_k \) = optional control input,
- \( w_k \) = assumed to be a white noise sequence with known covariance, \( Q_k \),
- \( z_k \) = vector measurement at time \( t_k \),
- \( H \) = matrix giving the ideal noiseless connection between the measurement and the state vector at time \( t_k \), and
- \( v_k \) = measurement error, assumed to be a white noise sequence with known covariance, \( R_k \).

To start the iterative process, there must a set of initial conditions from which to begin. The terms \( Q_k \) and \( R_k \), representing process noise covariance and measurement noise covariance respectively, are usually measured during offline calibration before the implementation of the Kalman Filter. The process and measurement covariance error terms can be determined by knowing the error terms \( w_k \) and \( v_k \) (9).

\[
E[w_i w_j^T] = \begin{cases} 
  Q_i & i = k \\
  0 & i \neq k
\end{cases} \quad (Equation 3)
\]
\[
E[v_i v_j^T] = \begin{cases} 
  R_i & i = k \\
  0 & i \neq k
\end{cases} \quad (Equation 4)
\]
where
\[ Q_k = \text{Covariance matrix associated with } w_k, \text{ and} \]
\[ R_k = \text{Covariance matrix associated with } v_k. \]

While the measurement noise covariance, \( R_k \), is generally easy to determine, the process noise covariance term, \( Q_k \), can often prove difficult to obtain. This is due to the fact that it is often impossible to directly observe the process we are estimating. Therefore, \( Q_k \) must often times be estimated at the discretion of the researcher. The proper calibration of \( Q_k \) and \( R_k \) can lead to superior Kalman Filter performance. As such, care should be applied in determining these values (9).

The beginning sequences of the Kalman Filter requires that the process state equation, \( \hat{x}_k \), be structured based on knowledge of an \textit{a priori} state estimate, \( \hat{x}_k^- \), where the “hat” denotes an estimate, and the super-minus represents the fact that a term is an \textit{a priori} estimate. Additionally, the \textit{a priori} error covariance associated with the \textit{a priori} estimate is given by the term, \( P_k^- \). These terms are determined by evaluating the following equations (8, 9, 11):

\[
\begin{align*}
\hat{x}_k^- &= A\hat{x}_{k-1}^- + Bu_{k-1} \quad (\text{Equation 5}) \\
I_k^- &= AP_{k-1}A^T + Q_k \quad (\text{Equation 6})
\end{align*}
\]

where
\[
\begin{align*}
\hat{x}_k^- &= \text{A priori estimate of the process state vector}, \\
P_k^- &= \text{A priori error covariance matrix associated with } \hat{x}_k^- , \text{ and} \\
Q_k &= \text{Process noise covariance}.
\end{align*}
\]

Now that the time update equations have been established in Equations 10 and 11, the measurement update equations must be established. The first step of this process requires the calculation of the Kalman gain, \( K_k \), also known as the “Blending Factor” (see Equation 7). The next step is to actually measure the process so that \( z_k \) can be obtained, and a \textit{posteriori} state estimate can be calculated (see Equation 13). The final step in the measurement update process is to make a \textit{posteriori} error covariance estimate by evaluating Equation 9 (9, 11).

\[
\begin{align*}
K_k &= P_k^-H^T\left(HP_k^-H^T + R_k\right)^{-1} \quad (\text{Equation 7}) \\
\hat{x}_k = \hat{x}_k^- + K_k\left(z_k - H\hat{x}_k^- \right) \quad (\text{Equation 8}) \\
P_k = (I - K_kH)P_k^- \quad (\text{Equation 9})
\end{align*}
\]

where
\[
\begin{align*}
K_k &= \text{Kalman gain, “Blending Factor”}, \\
\hat{x}_k &= \text{Posteriori of the process state vector}, \text{ and} \\
P_k &= \text{Posteriori estimate of the error covariance associate with the process state vector}.
\end{align*}
\]
Once each phase has been completed (time update and measurement update), the *posteriori* state estimate is recycled to create a new *a priori* estimate of the process state vector. A graphical illustration of the Kalman Filter process can be seen in Figure 1.

**Figure 1 - Kalman Filter Illustration (9)**

### DATA COLLECTION

The intersection of George Bush Drive and Wellborn Road in College Station, Texas served as the test site for this analysis. This site offered ample space for setting up video cameras adjacent to the roadway and had existing VIVDS hardware in place. During this study, three types of data were recorded. First video data were recorded from the VIVDS camera. Second, the phase status of traffic signals was recorded using an industrial computer. Lastly, video data were recorded for the purposes of establishing baseline measurements involving queue length and vehicle counts on the subject approach.

**VIVDS Data Collection Procedure**

The City of College Station allowed a research team to use the VIVDS video feed from the intersection of George Bush Drive and Wellborn Road to record video data (see Figure 2). These data would then be reduced and used in the laboratory for the design, calibration and validation of the queue estimation technique. The VIVDS camera was mounted on a 5 ft riser arm and is located at an approximate height of 24 ft above the roadway. Video data were
recorded for one approach at this intersection. Video data from the VIVDS camera was transformed from an analog signal output from the VIVDS camera and converted to a digital signal where it was then stored to an industrial computer. Later, this digital video data were transferred to DVD, where the data were replayed, extracted and archived for future analysis.

Figure 2 - George Bush Drive and Wellborn Road in College Station, Texas

Phase Status Data
The phase status of the indication displayed by the traffic signal was recorded from the traffic signal controller during the same time period that video data were being recorded from VIVDS cameras. The phase status data was eventually used under laboratory conditions and replayed at the same time video data were being played. Essentially video data from the VIVDS camera and phase status data were synchronized on an industrial computer as if recorded events were happening in real-time.

Baseline Data Collection Procedure
Video cameras placed adjacent to the roadway were able to capture queue formation as far as 400 ft upstream from the stop line on the subject approach. Video cameras were placed adjacent to the roadway at an approximate distance of 280 ft from the roadway. Video cameras recorded video data concurrently with video footage obtained from the VIVDS camera as well the traffic signal phase status data. This was necessary so that researchers could compare VIVDS queue estimates to baseline queue estimates determined by cameras adjacent to the roadway. For the purposes of this study, the length of a traffic queue was defined as the rear of the furthest stopped
vehicle behind the stop line, or a vehicle that is slowing that is within 20 ft of the furthest stopped vehicle during the red phase of a traffic signal cycle.

Once data collection concluded, data from the video cameras were then extracted manually. Data pertaining to queue length and vehicle counts were recorded every 10 seconds during video playback. These data then allowed the researcher to obtain baseline queue length data.

**Laboratory Procedure**

Once data were collected using the data collection procedure described, data were analyzed under laboratory conditions. VIVDS camera data were output utilizing the recorded DVD video footage of the subject approach and were fed to an *Autoscope “Rackvision”* VIVDS processing unit. It is believed that this procedure involving the use of recorded DVD video footage offers many advantages over conducting these experiments under field conditions. For instance, using recorded footage allows the researcher to notice the affects of small refinements in queue logic, detector design, or other experimental modifications. In total, 24 hours of video data were analyzed, four were used for algorithm development and 20 hours were used for algorithm validation.

The VIVDS processing unit contains an imaging file that was merged with the output VIVDS camera footage. The imaging file containing sensors designed by the researcher, and created virtual detection zones on the VIVDS camera footage. Using these sensors, the queue estimation algorithm was able to produce estimates based on specified assumptions, design guidelines, and traffic engineering principles specified by the researcher.

As can be seen in Figure 3, video imaging data and phase status data are merged when the algorithm estimates queue length in 10 second intervals. The phase status data alerts the queue estimation algorithm as to the current phase status, and allows it to initiate or terminate subroutines and algorithms for the estimation of MOEs during a particular phase during a cycle.

A typical VIVDS sensor layout for queue detection in the right-hand lane can be seen in Figure 4. Each horizontal bar in Figure 4 represents a detector placed at a pre-determined distance from the stop line. This setup consists of eight distinct detection zones associated with distances such that queue lengths of 50, 100, 150, 200, 250, 300, 350 and 400 ft from the stop line can be reported (1)

Notice in Figure 4 that the two nearest detectors to the stop line (those that report 50 and 100 ft) incorporate two detectors placed in close proximity to one another. The reasoning behind this detector design is that it is believed that this design adds increased reliability due to detector redundancy. A Boolean logic function “OR” joins the two detectors and if either is switched “on,” the associated queue length is reported.

When vehicles begin to accumulate at a signalized intersection, the algorithm is allowed to report queue length once a detector has been switched “on” for a certain period of time. Hence, detectors function on a delay and vehicles must be present on a detector for a specified duration of time in order to place a call. Once a detection zone reports that the queue length has reached a certain distance from the stop line, these data are sent to a laboratory computer, and its value is analyzed by the algorithm.
Monitor Phase Status (Green, Not Green)

Phase Status

Determine Queue Length per Lane and Record to File Every 10 Seconds

Sync Time

Synchronized Phase Status with Video Time-Stamp

Queue Detector Output (On, Off)

Queue Estimation Algorithm (Kalman Filter)

Recorded Video

Video Image

DVD PLAYER

Video Image

Process Video Image

Configure Queue Detectors

AUTOSCOPE RACKVISION

INDUSTRIAL COMPUTER

Queue Length

Figure 3 - Hardware Setup for QDA Experimentation (I)
DATA ANALYSIS
The research analysis used four different types of data:

- Baseline data,
- VIVDS measurement data,
- Estimates from measurement data, and
- Kalman Filter adjusted estimates.

Baseline data (or ground-truth data) were obtained from video cameras placed adjacent to the roadway. These data represent true queue length during any one time interval. The next component is VIVDS measurement data. These data are obtained directly from VIVDS system detectors placed at 50 ft intervals from the stop-line. The third component involves estimates developed from measurement data. Estimates are calculated from various modeling techniques using queue growth trends associated with previous measurements that are aimed at obtaining results that are reasonably close to baseline queue lengths. The last component involves the utilization of the Kalman Filter to adjust estimates and minimize error associated with estimated queue lengths.

During algorithm development, a number of techniques were evaluated for their potential for adequately estimating queue length using queue measurements from previous intervals. A technique was developed whereby the slope of the two previous intervals was used to predict the queue length in the current interval (incremental slope technique). Furthermore, a technique was
developed that used the measured queue length in the initial interval of the red phase and the previous queue measurement to estimate queue length in the current interval (moving slope technique). Both of these techniques however, were found to cause more than desired fluctuations in queue length estimation. The use of a technique that performed a linear regression analysis during every time interval was found to outperform all techniques analyzed. A description of the linear regression technique is given in the following subsection. The linear regression technique was found to be able to better describe baseline queue data, and was able to describe approximately 83 percent of baseline queue measurement. This is opposed to 73 percent and 80 percent produced by the incremental slope and moving slope techniques, respectively.

**Linear Regression Technique**

A technique for modeling linear, deterministic queuing was achieved by performing a linear regression analysis using previous queue measurements. This method essentially took measured queue lengths recorded from previous intervals, and used these measurements to establish the rate of growth of the traffic queue. Queue lengths were recorded every 10 seconds. If during the red phase, 60 seconds had elapsed since the beginning of the red phase, this would mean that approximately six polling intervals were recorded by the QDA. The QDA would then take these six stored measurements and perform a linear regression analysis and would create a “best-fit” trendline corresponding to these points. The slope of this trendline would then serve as the growth rate to be implemented in the Kalman Filter. The growth rate is recomputed every interval, hence, another linear regression would be carried out at 70 seconds, using 7 measurements of queue length. Figure 5 illustrates the linear regression method.

The construction of the linear regression equation is important for the understanding of the trends associated with queue data. However, it is the slope of the trendline that is of particular interest. The slope provides a rate of growth that can be incorporated in the Kalman Filter to project the growth of estimates computed in the iterative calculations.
The Kalman Filter Applied to Queue Estimates

A linear queuing model is used as the primary model for describing queue growth using VIVDS. This simplistic model results in the use of a single linear equation for describing growth of a traffic queue. As such, the Kalman Filter described in the Literature Review of this paper reduces from a system of linear equations, best solved through linear algebra processes, to a system described by scalar equations (i.e., a system incorporating $1 \times 1$ matrices).

The scalar Kalman Filter begins by obtaining values of $Q_k$ and $R_k$, which represent the covariance of the estimation error and measurement error respectively. These values are obtained by taking offline measurements and estimates and comparing these values to baseline queue measurements. The measurement and estimation error that are produced are then statistically analyzed, and the standard deviation of each error term obtained. The error covariance with respect to the measurement error and estimation error are determined as follows (9):

\[
Q_k = (\sigma_{\text{Estimation Error}})^2 \quad \text{(Equation 10)}
\]
\[
R_k = (\sigma_{\text{Measurement Error}})^2 \quad \text{(Equation 11)}
\]

where

$\sigma_{\text{Estimation Error}} = \text{Standard deviation of the estimation error, ft, and}$

$\sigma_{\text{Measurement Error}} = \text{Standard deviation of the measurement error, ft.}$
The measurement error is obtained through offline measurements. Offline standard deviation calculations were made using measurement error readings for an approximate 15 minute period. Equation 12 illustrates how the error of measurements was calculated compared to baseline values and Equation 13 demonstrates how the standard deviation of the error terms was determined.

\[ e_k = q_k - q'_k \] (Equation 12)

\[ \sigma_{\text{Measurement Error}} = \sqrt{\frac{\sum_{k=1}^{N} (e_k - \bar{e})^2}{N}} \] (Equation 13)

where

- \( e_k \) = Measurement error at time “\( k \)”, ft,
- \( q_k \) = Baseline queue length at time “\( k \)”, ft,
- \( q'_k \) = Measured queue length at time “\( k \)”, ft,
- \( \bar{e} \) = Average measurement error, ft, and
- \( N \) = Number of observations.

During offline measurements, the standard deviation of the measurement error was determined to be 45.67 feet. This value was inserted into Equation 10, and used in the iterative processes involved in the Kalman Filter.

There are many methods for obtaining the estimation or process error covariance, denoted “\( Q_k \)”. Of these techniques, there exists an approximation method proposed by Welch and Bishop. According to Welch and Bishop, acceptable results can be obtained if one “injects” enough uncertainty into the process via the selection of “\( Q_k \)” (11). Essentially, Welch and Bishop are making assumptions based on previous knowledge of a process.

The estimation error term was assumed to be 50 ft for the initial interval. As Welch and Bishop state in their description of the Kalman Filter, it is often common to begin the calibration of the error term \( Q_k \) by assuming a reasonable value for this input (11). The 50 ft value is an assumed value and is believed to be reasonable as it reflects the distance between detectors and closely resembles the quantities obtained for the standard deviation of the measurement error. This assumption for obtaining the estimation error, is common, and is supported by Welch and Bishop (11). It is important to note that the assumption of 50 ft estimation error is only true for the initial iteration of offline runs of the Kalman Filter. Once the 15 minute offline runs were complete, the Kalman Filter has modified the estimation error. The 50 ft assumption merely provides a place to begin the iterative process.

The assumed value of 50 ft for estimation error was utilized in offline testing for the approximate 15 minute duration. The resulting estimates produced during the offline procedure were then compared to baseline queue values corresponding to the same time period of offline analysis. The estimation error was calculated using Equation 14, and the corresponding standard deviation of the error was calculated as shown in Equation 15. The standard deviation of the estimation error in Equation 15 was used in Equation 10 to ultimately produce the estimation error (\( Q_k \)).

\[ e_k = q_k - \hat{q}_k \] (Equation 14)
\[ \sigma_{\text{Estimation Error}} = \sqrt{\frac{\sum_{k=1}^{N} (e_k - \bar{e})^2}{N}} \quad (\text{Equation 15}) \]

where
- \( e_k \) = Estimation error at time “\( k \)”, ft,
- \( q_k \) = Baseline queue length at time “\( k \)”, ft,
- \( \hat{q}_k \) = Estimated queue length at time “\( k \)”, ft,
- \( \bar{e} \) = Average estimation error, ft, and
- \( N \) = Number of observations.

When the standard deviation of the estimation error was calculated, it was found to be 51.01. This results after an initial assumption of 50 feet for the standard deviation of the estimation error. The calculated value was obtained by using a combination of the linear regression growth technique as well as a Kalman Filter. The following sections will describe how to combine these two mathematical techniques to produce queue estimates.

Once these values are determined, attention must be turned to the term \( H_k \). The matrix \( H_k \) was described in the Literature Review as the connection between the measurement and state vector at a specific time, \( t_k \). For the estimation of queue lengths, the measurement of queue length has a direct relationship to that which is output from the current state equation. Accordingly, this direct one-to-one ratio results in the following reduction associated with Equation 2 describing the observed measurement (see Equations 16 and 17 for reduction).

\[ H_k = 1 \quad (\text{Equation 16}) \]
\[ z_k = (1) \times x_k + v_k \quad (\text{Equation 17}) \]

In Figure 1, the Kalman Filter begins with the \textit{a priori} estimate and the error term. The terms \( \hat{x}_{k-1} \) and \( P_{k-1} \) are both initially zero (see Equations 18 and 19). This simplifies the initial stages of the Kalman Filter process and results in the prediction portion of the filter to yield a perfect estimate, that is, the \textit{a priori} estimate is assumed perfect, with no error associated with the measurement. This seems logical, as during the beginning seconds of the red interval, it is common for no vehicles to be queued. While this assumption is highly dependant upon the definition of a queued vehicle, the researcher believes that for the purposes of establishing a reference point, the time at the beginning of the red phase where no vehicles are queued serves as a perfect estimate of queue length.

\[ \hat{x}_0 = 0 \quad (\text{Equation 18}) \]
\[ P_0 = 0 \quad (\text{Equation 19}) \]

Now that all of the parameters have been obtained, the recursive loop utilized by the Kalman Filter can begin. Initially, the Kalman Filter begins by inserting the values \( \hat{x}_0 \) and \( P_0 \). This allows for the evaluation of Equations 20 and 21. Recall that in the previous subsections that the slope of the linear regression model was discussed. This value is now used. The slope is
inserted for the variable \( u_{k-1} \), or for the first iteration this value is \( u_0 \). The 1×1 matrix, \( B \), is the time step, or in this case 10 seconds. The quantity \( Bu_{k-1} \) is an optional term provided in the Kalman Filter. It is an optional control input intended to provide for the adaptation of an estimate from one time step to the next. In this equation there will always be a direct relationship between the Kalman Filter estimate of queue length, and the time update (predicted estimate). Therefore, the \( A \) term will be one.

\[
\dot{x}_1 = Ax_0 + Bu_0 \quad \text{(Equation 20)}
\]

During time interval \( t_1 \), both the estimate from the previous time step, \( x_0 \), and the slope, \( u_0 \), are zero (assumed queue length is zero). This initial estimate of queue length, \( x_0 \), is assumed to be perfect. The term, \( P_0 \), the a priori error covariance, from the previous interval, is also zero. Thus, the error term, \( P_1 \), is then equal to the error covariance, \( Q \), associated with the estimate error (Equation 21).

\[
P_1^* = P_0 + Q = Q \quad \text{(Equation 21)}
\]

During this time step and subsequent time steps, the equation for determining the Kalman gain in the corrector portion of the Kalman Filter significantly reduces due to the fact that the matrix \( H_k \) is equal to unity.

\[
K_1 = P_1^* H^T \left( H P_1^* H^T + R \right)^{-1} \quad \text{(Equation 22)}
\]

\[
\Rightarrow K_1 = P_1^* (P_1^* + R)^{-1} \quad \text{(Equation 23)}
\]

Similarly, the updated estimated output from the filter, \( \hat{x}_k \), also reduces. In this research the variable \( z \) is representative of the measured queue length for the current time step. The equation for determining the Kalman Estimate of queue length reduces similar to that of the Kalman gain, as the \( H \) term is equal to one, and can be ignored.

\[
\dot{x}_1 = \hat{x}_1 + K_1 \left( z_1 - H \hat{x}_1 \right) \quad \text{(Equation 24)}
\]

\[
\Rightarrow \dot{x}_1 = \hat{x}_1 + K_1 \left( z_1 - \dot{x}_1 \right) \quad \text{(Equation 25)}
\]

The last step of this initial iteration of the Kalman Filter concludes with the calculation of a new error covariance term, \( P_1 \). Notice that the “\( F \)” term reduces to a 1×1 identity matrix. This makes this scalar value equal to one.

\[
P_1 = (I - K_1 H) P_1^* \quad \text{(Equation 26)}
\]

\[
\Rightarrow P_1 = (I - K_1) P_1^* \quad \text{(Equation 27)}
\]

This ends the initial iteration of the Kalman Filter. Calculations can now begin for the second iteration, for time \( t_2 \). Once the initial iterations are complete, the recursive nature of the Kalman Filter becomes apparent. Those estimates from the previous time step, \( t_1 \), are used in the new prediction portion of the Kalman Filter in the current time-step, \( t_2 \) (see Equations 28 and 29).
Hence, a new estimate of queue length, given by $\hat{x}_2$, is produced based on the Kalman Filter output queue length from the previous time-step.

\begin{align*}
\hat{x}_2 &= A\hat{x}_1 + Bu_i \quad (\text{Equation 28}) \\
P_2 &= P_1 + Q \quad (\text{Equation 29})
\end{align*}

Note that during this iteration, the estimate is not perfect, queue length and the corresponding error will not be zero during this or subsequent time-steps.

**RESULTS**

The validation procedure was carried out using data from George Bush Drive and Wellborn Road. During this procedure, 20 hours of video data were analyzed, consisting of nearly 5500 measurement points.

Results indicate that the average error associated with each 10 second interval is 6.52 feet. The magnitude of the error, whether high or low, was shown to be 21.86 feet or approximately one car length. Furthermore, each 10 second interval was compared to baseline data for a corresponding interval. The results of this analysis is shown by the predicted vs. actual queue plot in Figure 6. The results show a coefficient of determination value ($R^2$) of 0.8574.

![Figure 6 - Predicted vs. Actual Queue Plot for the Linear Regression Technique (Validation)](image-url)
Discussion of Results

A noticeable trend was observed when comparing the predicted vs. actual queue plot in Figure 6. A coefficient of determination value of 0.8574 indicates that approximately 86 percent of queue length data can be explained by the queue algorithm. It was the researcher’s goal to create a queue estimation technique using VIVDS hardware and an algorithm that best describes the actual queue length at an approach, while capable of describing variances in actual queue length data. A method capable of describing 86 percent of the variance of a data set provides strong support for the implementation of this method for VIVDS queue estimation. Therefore, the researcher believes this technique using a Kalman Filter is a viable method for estimating the length of a queue formed at a signalized approach.

The linear regression method used to make queue estimates and queue growth rates is simplistic with respect to its mathematical procedure, and it can be argued that a more sophisticated model may provide a more accurate model for describing vehicle queuing. This analysis had to assume linear queue growth (due to the Kalman Filter), thus, selecting a model for queue growth was limited. A more complex queue estimation technique should incorporate more detectors with smaller graduations between detection zones. This should allow for the utilization of a more complex queue growth model, as smaller detector spacing will not cause as much initial error as detectors spaced at 50 ft introduces to the queue growth models. A simplistic hardware setup requires a simplistic modeling approach supported by correction procedures that ensure that the model is under control. The queue estimation technique described in this research, using a linear regression model and a Kalman Filter accomplishes the stated objective, and minimizes noise with respect to queue estimation. The queue estimation algorithm is shown to minimize error to a surprising degree given that only eight detectors over a 400 foot analysis area were used.

CONCLUSION

This research investigated a queue estimation algorithm using standard VIVDS hardware. This technique used measurements of queue length obtained from VIVDS detectors placed strategically along a signalized approach. Measurement queue length data was combined with queue length estimates using a Kalman Filter, which attempted to minimize error with respect to queue length estimates produced by the algorithm.

It was found that the magnitude of error produced by this technique for any 10 second interval is approximately 22 feet. Predicted vs. actual queue plots reveal that the algorithm is capable of explaining 86 percent of actual queue length data. The queue estimation algorithm described in this research and the use of a Kalman Filter requires relatively little calibration to implement. This technique is adaptive in the sense that it seems to automatically correct itself using the Kalman Filter and does not allow estimates to get out of control.

Applications of this research include the potential for real-time traffic signal control. The use of quantitative estimates of queue length provides an important MOE for making decisions with respect to traffic control. The use of quantitative queue estimates can be used for adaptive traffic signal control or a traffic responsive (TRSP) control system. The application of this research is simplistic in the sense that all VIVDS features used to interact with the queue estimation algorithms are common features to most VIVDS systems. The implementation of such a system would provide a flexible means of estimating queue length with little calibration, and upfront cost to install.
REFERENCES


