BEHAVIOR AND PERFORMANCE OF UTW ON THIN ASPHALT PAVEMENT

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ABSTRACT

During the 1999 spring, two sections of ultra-thin whitetopping (UTW) were built in an urban street in São Paulo City over a thin 45 mm asphalt layer. The UTW was constructed at a bus stop within the University campus, defining two panels of squared 0.6 and 1.0 m and 95 mm slabs. A high strength concrete was applied and the test sections were fully instrumented with top and bottom thermal resistors and strain gages.

The instrumentation allowed the analysis of temperatures and thermal gradients on slabs as well as the curling deformation induced by these gradients. Temperature measurements showed daytime thermal gradients up to 11.7°C and nighttime gradients up to –4.2°C. Concrete stresses due uniquely to curling reached very low values in all seasons and were deemed negligible.

Open to traffic since November 1999, 19 months later no cracks or faulting were observed on UTW sections submitted to a daily traffic of 120 buses and trucks making it possible, on the basis of measured flexural stresses on slabs, to predict a good performance of the UTW even over a thin 45 mm asphalt layer once no distresses have been verified as yet.

INTRODUCTION

Ultra-thin whitetopping (UTW) can be a very interesting restoration technique for existing asphalt pavements when rutting is the major distress on pavement surface and no structural distresses are present, namely fatigue cracking of asphalt mixture and base deterioration. Among several particularities well known for the employment of UTW, the thickness of the remaining asphalt layer after milling is a key for the good performance of the concrete slabs as related mainly by Hawbaker (1996), Mack (1993) and ACPA (1996), and a minimum thickness of 75 mm is required for good practice according to Mack *et al.*(1998), Cole *et al.* (1998) and ACPA (1998). Indeed, besides the thickness of the asphalt layer over 70 mm, the bond at the interface UTW- asphalt layer is required to allow the pavement structure to take advantage of the composite pavement behavior in order to modify the position of the neutral axis on the concrete slab, allowing low tensile stresses due to loads.

Those basic requirements are attained by milling, once it can ensure good bond at interface, but anyway under a predefined thickness, in order to maintain a minimum of asphalt layer that allows the composite behavior for the pavement structure by taking advantage of the non-distressed asphalt layer. Since the beginning of the dissemination of UTW solution throughout the world, engineers recognized that the application of such a technique could be very constrained due to the fact that, in several countries, the existing flexible urban pavements were built with thin asphalt layers, rarely thicker than 70 mm. To avoid excessive grade changes, a reasonable milling depth should be mandatory; also, the condition of a minimum remaining thickness of asphalt layer is not verified in several situations.

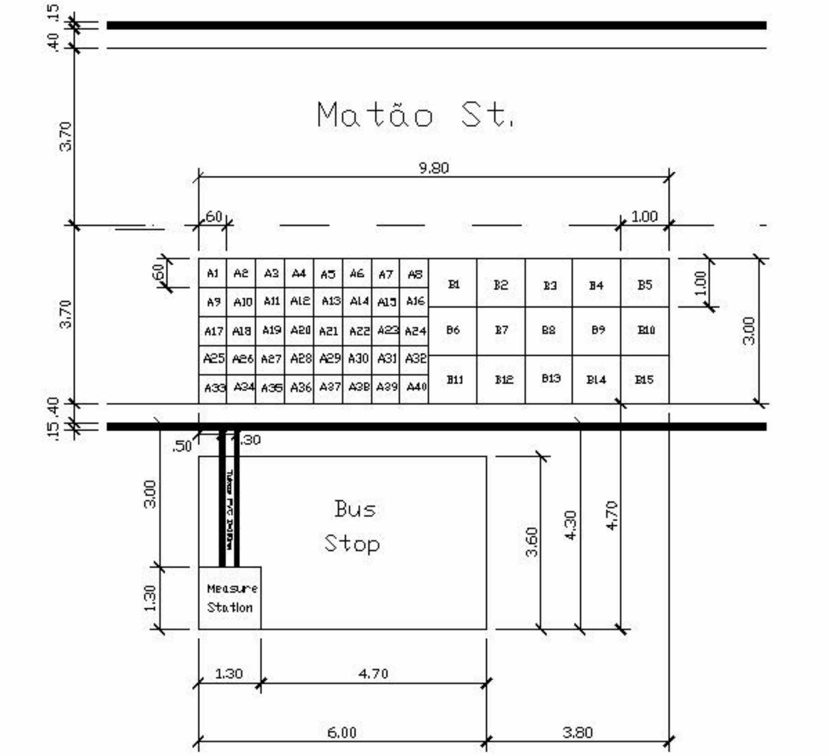
That is specifically the case of urban buses corridors like those in São Paulo City: using concrete surfaces would be of paramount importance to avoid constant and premature rutting due to canalized traffic with very short lateral wander, but constrained by the restriction of leveling the concrete with asphalt pavements of the lateral lanes, and moreover, by existing asphalt layers normally up to 100 mm. The only condition for future employment of UTW for such a situation was to analyze an experimental UTW pavement taking into account the typical technical conditions to be found on urban bus lanes; the research project should consider all constraints to be fruitful and to support the decision-maker alternatives for restoration.

UTW PROJECT IMPLEMENTATION

The bus lane chosen for the UTW construction is located in Matão St., a double waystreet within the campus of Universidade de São Paulo. Several options for location were taken into account, but the bus stop location (Fig. 1) was chosen for presenting the most convenient conditions. A bus stop is actually a critical point of the bus lane, due to acceleration and stopping stresses. The existing asphalt pavement at that location, at the time, presented good conditions (no cracks and 10 mm maximum rut depth); the asphalt layer was 50 mm thick over a macadam base of 100 mm, representing typical pavements of urban streets; the subgrade is a lateritic clay soil. Resilient modulus of the thin pavement layer backcalculated by ELSYM-5 program resulted 3,500 MPa for the hot mixed asphalt and 150 MPa for both macadam base and subgrade.

The UTW was designed in two sections: (A) 0.6 and (B) 1.0 m squared slabs, each section 5 m long, in a lane width of 3.0 m (Fig.1). For both sections the thickness was 95 mm. Before concrete laying, the asphalt surface was milled 5 mm deep, resulting in a remaining 45 mm asphalt layer ; after milling, the surface was cleaned by brush washing. Forms were fixed on the asphalt surface by using steel nails defining an area of 30 m^2 to be filled by the high strength concrete.

Figure 1 UTW location and dimensions



Instrumentation

The instrumentation was a set of strain gages (Kyowa) and thermal resistors (PT-100) buried under the concrete (Fig.2), and it was positioned at the top and at the bottom of the concrete, the strain gages with the aid of steel stands (Fig.3) and the thermal resistors attached at the extremities of a PVC cut out pipe (Fig.4). In section A, top and bottom thermoresistors were buried on four slabs and top and bottom strain gages on twelve slabs; using the same scheme, section B has five slabs with thermal-resistors and 11 slabs with strain gages.

These devices were checked after the conclusion of works, and two top thermoresistors were out-of-order; also four strain gages were not operational. Positions of instruments have been chosen in order to allow the measurements of temperatures and deformations in several positions of slabs: centers, corners and edges.

All the instruments were previously calibrated and the cables connected to a measurement station located by /near the bus stop. The signal conditioner works at 2.5 V and can be connected to 32 channels, allowing measurements every 30 seconds, operating continuously for days and weeks; this device is connected to a lap top Pentium III PC and controlled by a software especially developed for the experiment.

Figure 2 Location of instruments on UTW

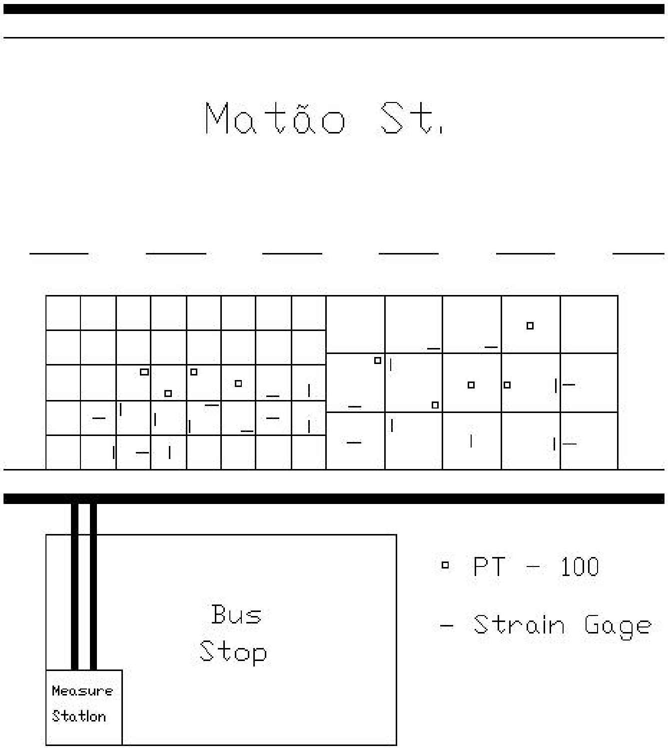


Figure 3 Position of the strain gages

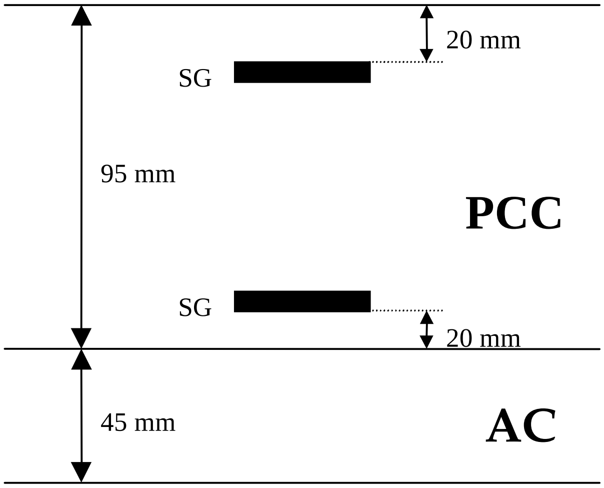
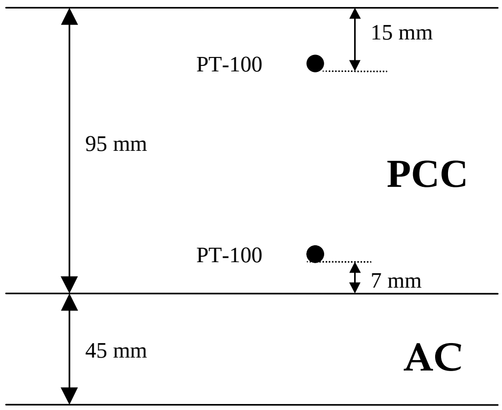


Figure 4 Position of thermal-resistors (PT-100)



Concrete Mixture and Strengths

Previous numerical simulations of the designed sections indicated that the thin asphalt layer should not provide a significant contribution for lessening stresses on concrete slabs; based on that information, the decision was to employ high strength concrete in order to prepare the slabs to resist reasonable stresses, here included the possibility of loss of bond for some plates, which was not verified later. The chosen concrete mixture was composed of 476 kg/m^3 of early strength Portland cement, 28.5 kg/m^3 of silica fume, 642 kg/m^3 of sand, 1,029 kg/m^3 of granite crushed stone (19 mm max. diameter), w/c ratio of 0.4, and plasticizer and superplasticizer on the ratios of 1.43 and 2.38 L/m^3 of concrete respectively. All the materials were mixed in a central plant and the superplasticizer added on the truck at the site; the reached concrete slump was 140 mm. Silica fume was tentatively employed just for practical understanding of its proper difficulties for concrete mixing and curing once the test section was small and thus easy to control shrinkage cracks by using moist curing during three days.

Nearly three hours after laying and finishing the concrete, the green saw cut of the joints was carried out, defining the final dimensions of the UTW slabs. The curing process employed both chemical curing (PVA) during the first hours before the cutting and a wet blanket for the following three days. The controlling concrete samples reached flexural strengths of 5.2 MPa at 48 hours and 7.0 MPa and 7.2 MPa after 7 and 28 days respectively. Modulus of elasticity of concrete reached 38,000 MPa. Hot mixed asphalt was employed to built the frontal and lateral slopes between existing pavement and new UTW before opening to traffic. Construction occurred on November 1st, 1999 and opening to traffic one week later.

Traffic Loads

The lane close to bus stop location is served by 2,574 urban buses and around 135 medium trucks (back dual wheel single axle) monthly. The experimental sections are daily loaded by 120 commercial vehicles (on average) and by 600 cars.

TEMPERATURES AND THERMAL GRADIENTS ON SLABS

The temperature measurements began in May 2000, after the acquisition of the signal conditioner and the calibration of all the operational instruments. The measurements were continuously performed during sequential days with data collection every ten minutes. During the year (2000), the measurements were held on 31 autumn days, 81 winter, 57 spring and 15 summer days .

In order to verify the occurrence of different thermal gradients for different slabs and positions of the instruments, during the data collection, several commutation of data acquisition were performed once the equipment allows only 32 instruments connected; for that reason, during collection of data from section A, section B was disconnected and vice-versa. The comparison of data, as shown as an example in Table 1, has permitted to verify that the top temperature is always the same, independent of the position (center, corner or edge) or slab dimension; the same conclusion is valid for bottom temperatures. Thermal gradients were considered as the difference between top and bottom temperature for pairs of thermal resistors.

Table 1 Temperatures on sections at the same time, in ^{o}C

Section	Average Top Temperature	Average Bottom Temperature	Average Thermal Gradient
A	42.9	32.6	10.5
B	44.9	33.9	11.0

For the determination of typical received conditions during the year seasons, from the data a sequential set of maximum top temperature and the correspondent bottom temperature were collected, as well as the minimum top temperature and its correspondent bottom temperature; these values allowed to define the present thermal gradients (not rectified) on slabs. During the typical seasonal weeks, maximum, minimum and average temperatures could be found for both top and bottom and their correspondent non-rectified thermal gradients.

In Table 2, the received data mentioned are presented, from which it can be inferred that there are sensible changes in surface temperatures during the seasons, the maximum varying from 25.4^{o}C during the winter to 48.3^{o}C during the summer; the averages still changing season by season from 22.4^{o}C (winter) to 43.1^{o}C (summer).

On the other hand, with regard to the minimum temperatures, no significant changes occurred from autumn to winter, when maximal temperatures of 14.3^{o}C and 15.2^{o}C were found; the averages were 12.4^{o}C for autumn and 11.1^{o}C for winter. From spring to summer more significant changes were received, when averages jumped from 17.9^{o}C to 21.6^{o}C.

The average positive (non-rectified) thermal gradients during daytime shown in Table 2 did not present important variations in autumn, winter or spring, with values of 7.4^{o}C, 5.6^{o}C and 6.1^{o}C respectively. The thermal gradients presented high values mostly for the summer, reaching the average of 9.6^{o}C and a maximum isolate value of 11.7^{o}C. It must be realized that typical values of gradients during the summer can be reached during the spring, as shown in Table 2 (9.4°C isolate received during spring). A typical spring day is also found in Table 2, when the minimum value of daytime gradient was 0.4°C; that was a rainy day of spring and the temperature remained the same practically all day long.

Table 2 Top temperatures and thermal gradients during typical days of each season

	Maximum Temperatures (°C)				Minimum Temperatures (°C)			
	autumn	winter	spring	summer	autumn	winter	spring	summer
Maximum	29.7	25.4	40.0	48.3	14.3	15.2	20.1	22.8
Minimum	26.5	17.3	25.2	38.9	10.2	5.4	16.5	20.3
Average	28.0	22.4	33.3	43.1	12.4	11.1	17.9	21.6
Sd	0.9	2.7	6.4	3.6	1.2	3.4	1.5	0.9
	Maximum Gradients (°C)				Minimum Gradients (°C)			
	autumn	winter	spring	summer	autumn	winter	spring	summer
Maximum	8.4	7.8	9.4	11.7	-1.3	-1.1	-1.7	-1.9
Minimum	5.4	2.2	0.4	7.6	-3.1	-2.4	-4.2	-3.6
Average	7.4	5.6	6.1	9.6	-2.1	-1.8	-2.8	-2.4
Sd	0.8	2.2	4.2	1.6	0.8	0.5	0.8	0.6

Where, Sd is the standard deviation.

The nighttime gradients (negative) were practically unchanged during the seasons, as can be inferred from the averages presented in Table 2. The minimum negative gradient was recorded during the spring, reaching –4.2°C. All the measurements were connected to solar time and not official time. São Paulo is located at latitude 23° 27’ S (Tropic of Capricorn) and the daytime maximum gradients occurred between 12 and 1:30 p.m. The maximum negative gradient generally occurred between 6 and 7 a.m.

Another interesting aspect to be discussed is the UTW slab bottom temperature (at interface UTW/AC). In Table 3, the bottom average temperatures during the seasons are presented, which allows to conclude that asphalt layer temperature during cold or hot days in the Tropics could strongly fall once UTW slabs are built. Acquired data showed typical asphalt surface layer temperatures of 40°C (for autumn and winter) and 65°C (for spring and summer), quite different (near the double) from the measured temperatures on UTW. Consequently, asphalt layers on UTW pavements are submitted to lower temperatures, which is strictly related to its plastic deformation and fatigue behavior, probably improved by that condition. Another remarkable implication to this temperature context is that it was possible to predict that bond strength at UTW/AC interface should be reduced from autumn-winter period (20°C of temperature) to spring-summer (30°C) due to increase in temperature.

Table 4 presents the ratio thermal gradient by UTW thickness found during the seasons as well as the correction of the mentioned gradients by considering its real location on slabs and the full thickness of the UTW (Note that, as shown in Fig. 4, the thermal resistors were placed at a certain distance from the top and from the bottom of the slabs, and therefore the rectification or correction of thermal gradients for the full thickness of slabs was done by simple linear extrapolation) . From those calculations, the more significant changes for daytime gradients during the seasons is confirmed (0.160°C/mm for summer and 0.107°C/mm for winter). Moreover, the rectification of the thermal gradients shows that daytime gradients reached up to 15.2°C; this adjusted gradient must be very significant for curling stresses on larger slabs (dimensions over 1.80 m are found in international experience with UTW).

Table 3 Bottom slab average temperature

Season	**Daytime**	**Nighttime**
Autumn	20.6	14.5
Winter	16.8	12.9
Spring	27.2	20.7
Summer	33.5	24.0

Table 4 Ratios between thermal gradients and thickness

Season	**ΔT (^{o}C)**	**ΔT/t (^{o}C/mm)**	**Rectified ΔT (^{o}C)**
Daytime Gradients			
Autumn	7.4	0.115	10.9
Winter	5.6	0.107	10.1
Spring	6.1	0.129	12.2
Summer	9.6	0.160	15.2
Nighttime Gradients			
Autumn	-3.1	-0.042	-4.0
Winter	-2.4	-0.033	-3.1
Spring	-4.2	-0.056	-5.3
Summer	-3.6	-0.049	-4.7

It must be taken into account that in regions very close to the tropics, several times the weather is not exactly like that of inter-tropical zones or of temperate climate; it can thus be said that regions like São Paulo have transition climate particularity due to that. As a matter of fact, the climate in São Paulo (hot and moist during the summer and moderate and dry during the winter) is not absolutely constant along the seasons; therefore, the end of winter may be very similar to spring in terms of temperatures (high values), but still dry. On the other hand, in the summer, due to the cold masses originating in Patagonia, two or three cloudy, cold and moist days are very likely to occur.

In order to exemplify the assertive above, during the autumn a top temperature of 32.6°C occurred on May 28 around 1 p.m., with the bottom correspondent temperature of 25.1°C, resulting in a 7.5°C gradient; on September 21 (last winter day) the top temperature of 43.8°C and positive gradient of 9.1°C at 2 p.m. was recorded.

Table 5 was developed on the basis of 184 days of measurements up to December 31, 2000; the statistics are considered as a first tentative to classify the distribution of gradients (nighttime and daytime) during the seasons by means of bands. From the results received, it can be concluded that the occurrence of different bands of thermal gradients should be taken, for simplicity, in two blocks: autumn-winter and spring-summer. The reasonable origin of those results is to have in mind that, during part of spring, the weather is summer-like; the same occurs for autumn and winter, and the consequence is very close solar radiation for both seasons.

Table 5 Gradients frequencies distribution along seasons, in % of time

Gradient range	**Autumn**	**Winter**	**Spring**	**Summer**	**Total**
less than –6 °C	0.0	0.0	0.2	0.0	0.1
from –6 to –3 °C	1.1	0.8	4.1	1.0	1.9
from –3 to 0 °C	49.2	49.7	58.9	59.5	53.3
from 0 to 3 °C	40.0	37.7	18.9	18.7	30.5
from 3 to 6 °C	7.3	6.0	9.4	10.8	7.7
from 6 to 9 °C	2.4	4.6	5.9	7.8	5.0
more than 9 °C	0.0	1.2	2.6	2.1	1.6
Σ	100.0	100.0	100.0	100.0	100.0

For the block autumn-winter, big frequencies of gradients between –3°C an 0°C (by 50% of time) and between 0°C and 3°C (40% of time) are observed. A quick look at Table 5 shows less importance concerning nighttime gradients (negatives), proportionally. For the spring-summer period, the frequencies of gradients between -3°C and 0°C is about 60% and from 0°C and 3°C it is close to 20%. Although it could reveal an important difference in gradients distribution for different weather, gradients of the band mentioned have no influence on stresses on UTW, specially for shorter slabs.

The numbers show that in spring and summer there is a higher incidence of negative gradients than in the cold period. Contrariwise, in spring and summer there are high values of positive gradients. If positive gradients are important over 9°C, it is possible to say that, at least in the tropical environment focused in the study, there will be no need to compute thermal stresses for UTW with dimensions up to 1.2, as normally employed. Nevertheless, this conclusion could not be applied automatically to the inter-tropical regions where a great amount of developing and emergent countries is found, once daytime gradients essentially depend on air temperature and solar radiation, and thus, on the latitude.

Based on all measurements performed in 2000, it was possible to precisely conclude about the possibility of treating the data in two blocks. To close the discussion, the averages found during the seasons for top temperatures (non-rectified) and gradients were:

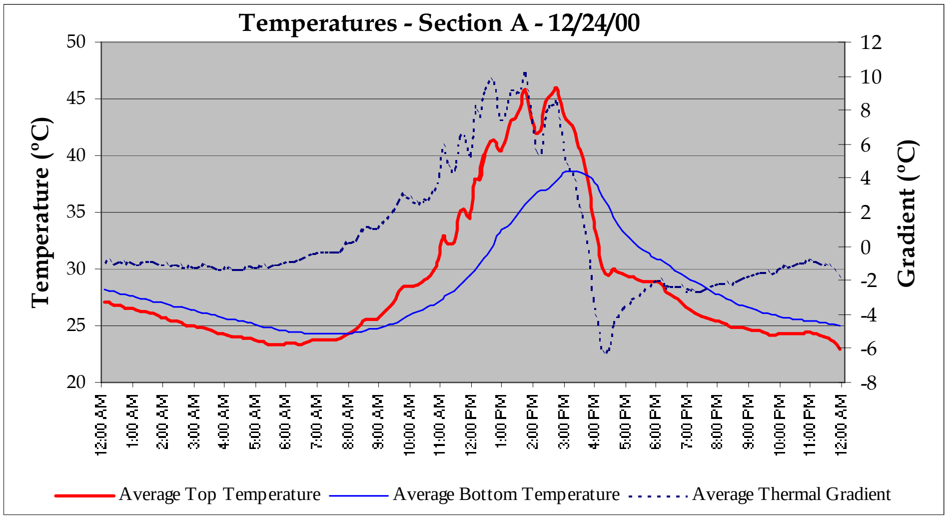
- Average of the maximal temperatures on top (by noon): 27,3°C (autumn); 28,3°C (winter); 39,3°C (spring); and 41,8°C (summer).
- Average of the minimal temperatures on top (morning): 12,4°C (autumn); 12,2°C (winter); 19,0°C (spring); and 21,3°C (summer).
- Average of daytime maximum thermal gradient: 6,8°C (autumn); 7,1°C (winter); 8,7°C (spring); and 9,1°C (summer).
- Average of minimum (negative) thermal gradient: -2,2°C (autumn); -2,3°C (winter); -3,8°C (spring); and –3,1°C (summer).

Concerning negative gradients, the study allowed to verify the effects of heavy tropical rain on the slabs. As presented in Fig. 5, on December 24, 2000, it rained from 3:16 p.m. to 4:26 p.m. The positive gradient at 2:46 p.m. was 8.1°C. With the rain, a decreasing strong slope and the inversion between top and bottom temperature are observed in such a way that, when the rain stopped at 4:26 p.m., the gradient, even during the day, was –8.7°C.

DEFORMATIONS DUE TO CURLING AND LOAD

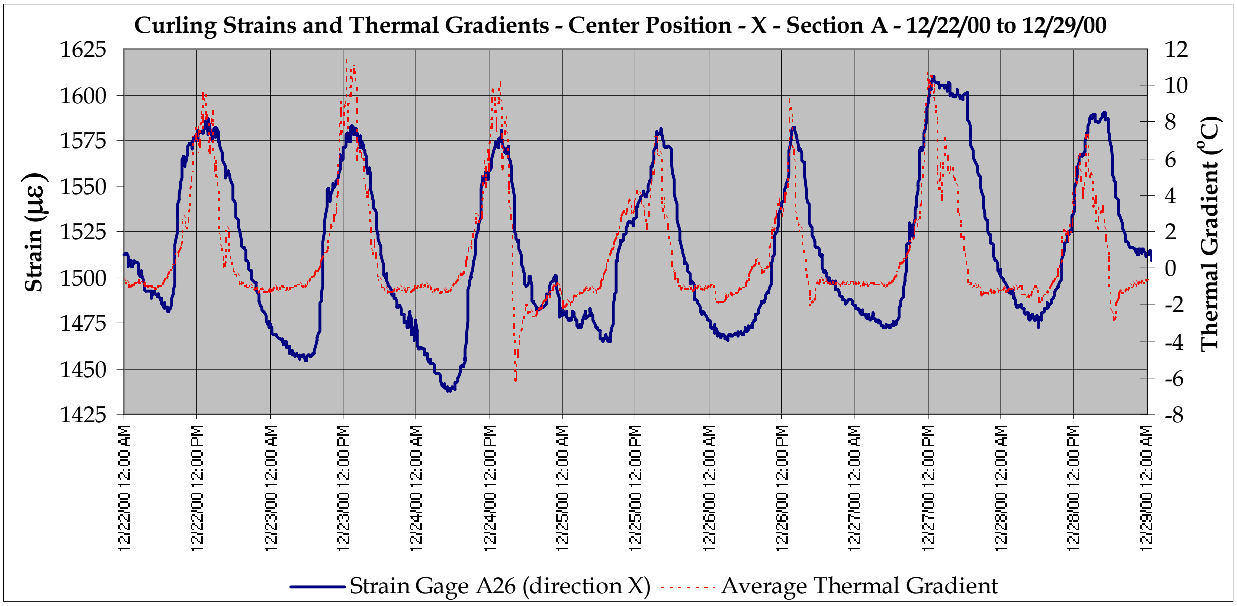
Strains exclusively due to induced thermal gradients on slabs were very difficult to analyze once it was necessary to divide them into two types: deformation of the strain gage induced by temperature and deformation induced by curling. For this purpose, it was necessary to perform a calibration of the strain gages simulating only strain measurements due to temperature on the range between 0°C to 50°C. In spite of the measurements, the quantitative results for strains are not reliable, providing but a rough idea of what was going on. Typical levels of stresses induced on concrete slabs ranged from 0.0015 to 0.0025 MPa, only allowing to judge the lack of importance of the thermal gradients for the UTW case, but not for a fair quantitative value.

Figure 5 Behavior of temperatures and gradients during a heavy summer rain



This kind of difficulty was early observed by Barenberg and Zollinger (1990) in former studies in the USA. Fig.6 presents a typical daily variation on gradients and their corresponding measured strains. The response of strain gages shows the extreme measures of strains occurring when the top temperature reaches its maximum or minimum values.

Figure 6 Variation of gradients and strain gage response during a day



On the other hand, measurements of load effects are well recorded by strain gages, even with static loads, on the condition that no temperature changes on concrete slabs occur before placing the load and the moment the slab is loaded. For instance, values of stress on concrete during the first load test (just one preliminary test was performed until December 2000) are more coherent with expected values got from numerical simulations by FEM as shown in Table 6. The test was carried out employing a dual single wheel of 22,5 kN.

Table 6 Results of preliminary load test

Slab	Max. Top stresses (MPa)	Bottom stresses (MPa)
A 26	-2.3	1.9
B 03	-0.4	0.4
B 06	-1.4	not responding
B 12	-0.8	0.9

Maximum stresses computed by simulations and equations previously adjusted by Pitta *et al.*(2000) for the slabs of the case study result no greater than 2.0 MPa. Although the field responses could not be directly correlated with the numerical results yet (the load tests are scheduled for 2001), the received stress levels by means of instrumentation were still compatible with former expectation. According to the instruments, received strains in the neutral axis is close to half the thickness of the slabs, showing a little contribution of the asphalt layer to decrease the stresses, except for the measurement on slab A 26, when its position is 43 mm from the bottom.

PERFORMANCE OBSERVED

From opening to traffic up to June 2001, a total amount of 110,314 axles have loaded the experimental sections, considering more than one month of closure of the campus during May-June 2000. After 14 months of operation, no cracks or faulting have been observed on UTW sections submitted to a daily traffic of 120 buses and trucks. Numerical simulations, as previously mentioned, suggested that the level of stress due to buses could be a long-lasting factor of the UTW without distresses; this can be confirmed through the stresses measured by instrumentation, and no early or short-term problems are likely to happen taking into account the concrete characteristics.

PCI values of 99 for section A and 100 for section B measured as soon as the UTW was opened to traffic still remained. The value of section A is due to two shrinkage cracks no longer than 100 mm at the edge of two slabs (A 25 e A 33) and spalling of the longitudinal external joint of two other slabs (A 39 and A 40). The shrinkage cracks occurred nearly one hour after laying the high strength concrete, and the spallings occurred when the lateral wood stand for the fresh concrete was removed; thus the distresses described should not be taken as functional nor structural distresses and, moreover, these distresses did not progress any further after their initial manifestation.

CONCLUSIONS

Concerning the conditions of the study, the temperature and deformation analysis on thin slabs has allowed to define that:

- Average values of daytime gradients during the summer are the maximum found during the seasons and reached 12.5^{o}C (rectified); maximum daytime gradients occur between 12 and 1:30 p.m.;
- Nighttime gradients are similar during all seasons and close to –2^{o}C; the maximum value got from measurements was –5.5^{o}C (rectified); the minimum gradients occur between 6 to 7 a.m.;
- There are no important changes on gradients in autumn, winter or spring.
- The ratio between thermal gradient and thickness is twice as great during the summer than in winter;
- Strains due to thermal gradients are very little on UTW so that they could be neglected for slab design purposes;
- Bottom temperatures at UTW/AC interface showed to be smaller than typical temperatures on AC surface (as little as half), a data to be taken into account for prediction on AC fatigue behavior;

- Preliminary results from load tests pointed out low values of stresses on UTW due to traffic, and, considering the achieved resistance of the concrete, it is possible to predict that the experimental pavement will not be subjected to fatigue damage (resulting on corner cracks) in short term;

Although previous reports suggested a minimum thickness of 70 or 80 mm for the remaining asphalt layer after milling, the present study showed that it is possible to achieve adequate behavior of UTW even over remaining asphalt layers with half of that thickness, respecting the basic condition of there being no structural distresses (cracks) on the asphalt layer, besides the pavement base being still in good condition. That is important mostly for offering the UTW alternative to several developing countries with tradition of laying slim asphalt concrete layers.

Dynamic load tests scheduled for 2001 on the UTW experiment will provide more information on stresses for the calibration of previous stresses computing models based on the finite element method developed in Brazil by Pitta et al. (2000).

REFERENCES

ACPA (1996) *Ultra-thin whitetopping - new choice for pavement overlay*. http://www.irmca.com/utw/.

ACPA (1998) *Whitetopping: state of the practice*. Engineering Bulletin, American Concrete Pavement Association.

Barenberg, E.J.; Zollinger, D.G. (1990) *Validation of concrete pavement responses using instrumented pavements*. Transportation Research Record 1286, pp.67-77, Washington D.C.

Cole, L. W., Mack, J.; Packard, R. (1998). *Whitetopping and ultrathin whitetopping - the U.S. experience*. Proceedings of the 8th International Symposium on Concrete Roads. AIPCR-CEMBUREAU, Theme IV, pp. 203-218, Lisbon.

Hawbaker, L. (1996). *Summary report of UTW in North America 1988-1995*. National Ready Mix Concrete Association & American Concrete Pavement Association.

Mack, J.W, Cole, L. W.; Mohsen, J. P. (1993). *Analytical considerations for thin concrete overlays on asphalt*. Transportation Research Record No. 1388, pp. 167-173, Washington, D.C.

Mack, J.W.; Hawbaker, L.D.;Cole, L.W. (1998) *Utra-thin whitetopping (UTW): the state-of-the-practice for thin concrete overlays of asphalt*. Paper presented for the TRB meeting of 1998, Washington, D.C.

Pitta, M.R.; Rodolfo, M.P.; Tia, M.; Balbo,J.T.; (2000) *Stresses modelling on UTW based on FEM.* 2nd Inter American Concrete Pavement Congress *(in Portuguese)*. CDrom.

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